The Joy of Matching

Multi-Frequency, Broadband, Reflectionless, and Active Non-Foster Match Network Design

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Abstract

"The Joy of Matching" is a tutorial introduction to the concepts and methods of impedance matching network design. Topics include what impedance is, what impedance functions are, characteristics of passive, active, and antenna impedance functions, the Smith chart, and the advantages and disadvantages of conjugate impedance matching. The author shows how a Smith chart allows one to visualize and predict a network's impedance-transforming behavior without math. We start with simple L-networks made of lumped elements or stubs, and progress through ladder networks to more complicated network topologies. We show solutions to a variety of impedance matching problems: matching at a single frequency, matching at several frequencies simultaneously, and matching a continuous band of frequencies. The Fano limit on the match bandwidth of passive loads like antennas is explained. Two tricks are shown for exceeding the Fano limit: passive reflectionless match networks and active non-Foster match networks. Free or inexpensive software for match network design is recommended.

Speaker's Biography

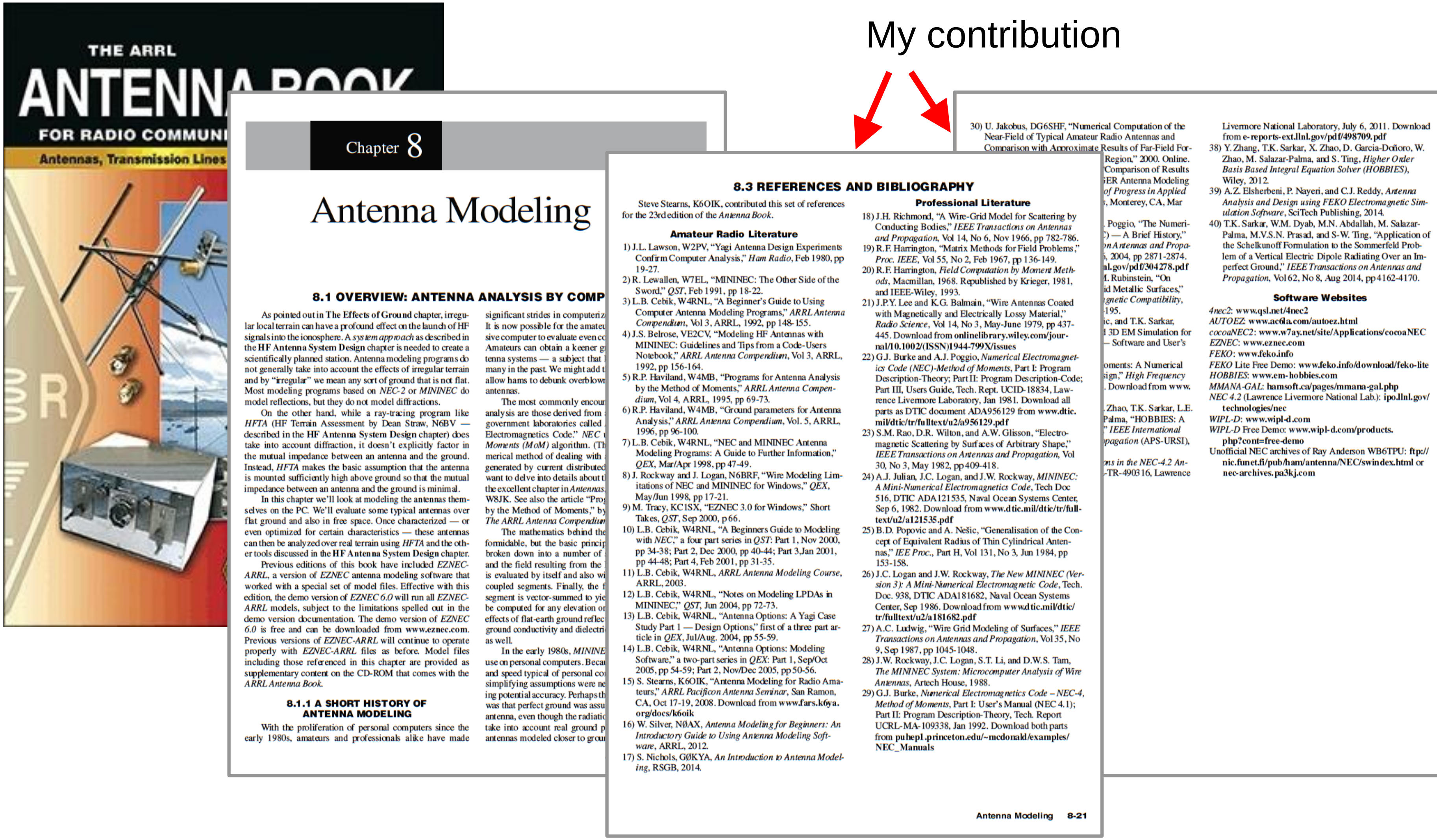
- **Stephen D. Stearns**
- **Technical Fellow, ret., Northrop Grumman Corp.**
- **40 years experience in electronic systems**
  - Northrop Grumman, TRW, GTE Sylvania, Hughes Aircraft
  - Electromagnetic and signal processing systems for communications and radar surveillance, cochannel signal separation, measurement, identification, characterization, polarimetric array signal processing of ionospheric skywave signals for precision geolocating HF emitters
  - Recent work: Vector-sensor antennas; Non-Foster circuits; antennas for radiating localized, non-diffracting, OAM Bessel-Vortex beams
- **FCC licenses**
  - Amateur Radio Extra Class
  - 1st-Class Radiotelephone
  - General Radio Operator License (GROL)
  - Ship Radar Endorsement
- **Education**
  - Stanford – under Prof. T.M. Cover
  - USC – under Profs. H.H. Kuehl and C.L. Weber
  - CSUF – under Profs. J.E. Kemmerly and G.I. Cohn
- **More than 100 publications and presentations, both professional (IEEE) and hobbyist (Amateur Radio)**

ARRL Pacificon Presentations by K6OIK

Year	Title	Archived at http://www.fars.k6ya.org
1999	Mysteries of the Smith Chart	
2000	Jam-Resistant Repeater Technology	
2001	Mysteries of the Smith Chart	✓
2002	How-to-Make Better RFI Filters Using Stubs	
2003	Twin-Lead J-Pole Design	
2004	Antenna Impedance Models – Old and New	✓
2005	Novel and Strange Ideas in Antennas and Impedance Matching	
2006	Novel and Strange Ideas in Antennas and Impedance Matching II	✓
2007	New Results on Antenna Impedance Models and Matching	✓
2008	Antenna Modeling for Radio Amateurs	✓
2010	Facts About SWR, Reflected Power, and Power Transfer on Real Transmission Lines with Loss	✓
2011	Conjugate Match Myths	✓
2012	Transmission Line Filters Beyond Stubs and Traps	✓
2013	Bode, Chu, Fano, Wheeler – Antenna Q and Match Bandwidth	✓
2014	A Transmission Line Power Paradox and Its Resolution	✓
2015	Weird Waves: Exotic Electromagnetic Phenomena	✓
2015	The Joy of Matching: How to Design Multi-Band Match Networks	✓
2016	Antenna Modeling for Radio Amateurs	
2016	The Joy of Matching 2: Multi-Band and Reflectionless Match Networks	

ARRL Antenna Book, 23e, Chapter 8

My contribution



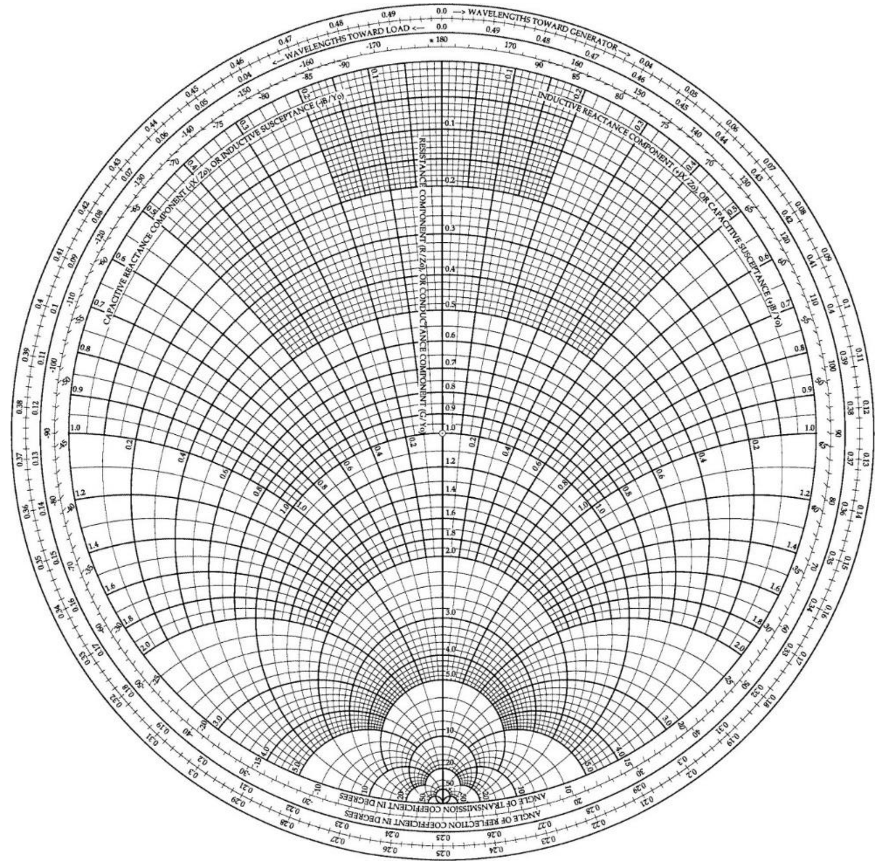
Topics

- **Smith chart**
- **Results from classical network theory 1: passive**
- **Impedance functions and general equivalent circuits**
- **Single-frequency matching**
  - lumped networks
  - TL sections and stubs
- **Multiple-frequency matching**
  - lumped networks
  - TL sections and stubs
- **Fano bound**
- **Broadband matching**
- **Reflectionless matching**
  - Problems with Q formulas
- **Results from classical network theory 2: active**
- **Active non-Foster matching**

The Smith Chart

The Smith Chart

Phillip Hagar Smith, 1905-1987



Developed by Phillip H. Smith at Bell Labs 1936.
Published in *Electronics*, Jan. 1939 and Jan. 1944.

Mrs. Smith sold copyright to IEEE MTT-S in 2015.

Where to Find Smith Charts?

Where to Find Smith Charts?

Drink Coaster

Where to Find Smith Charts?

Campus Sundial

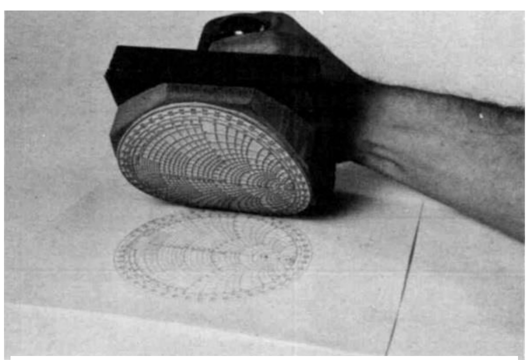
Where to Find Smith Charts?

International Microwave Symposium
17-22 May 2015, Phoenix

IEEE Conference Logo

Where to Find Smith Charts?

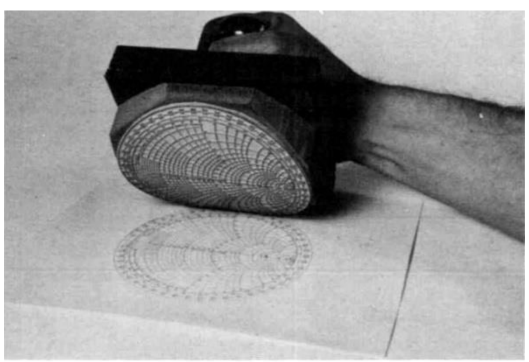
International Microwave Symposium
17-22 May 2015, Phoenix



Giant Rubber Stamp

Where to Find Smith Charts?

International Microwave Symposium
17-22 May 2015, Phoenix

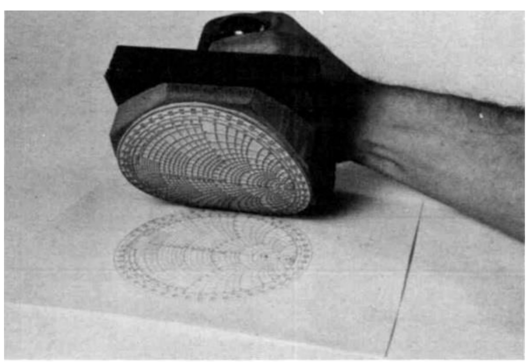


Smith chart rubber stamp is 10 cm in diameter.

Tatoo in Berkeley

Where to Find Smith Charts?

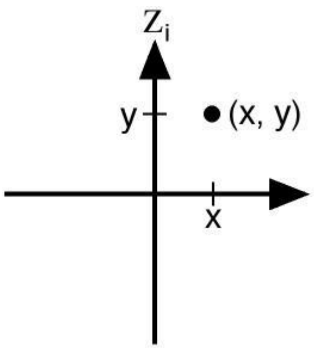
International Microwave Symposium
17-22 May 2015, Phoenix

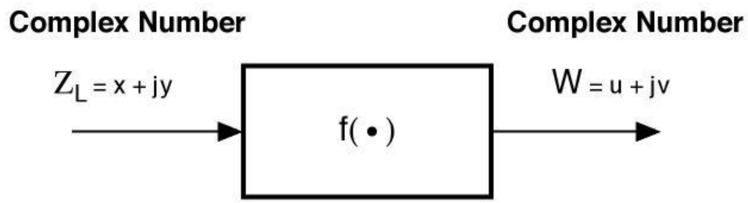


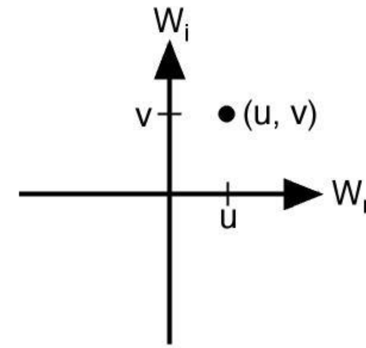
Smith chart rubber stamp is 10 cm in diameter.

Bicycling Attire in Los Angeles

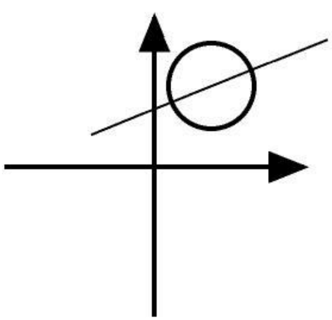
Complex Functions

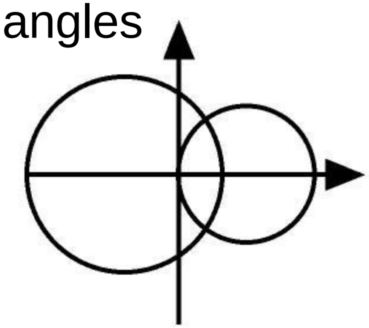






- **Basic types of complex functions**
  - Global Properties
    - Linear – lines map to lines
    - Bilinear – circles map to circles
  - Local Properties
    - Conformal – right angles map to right angles



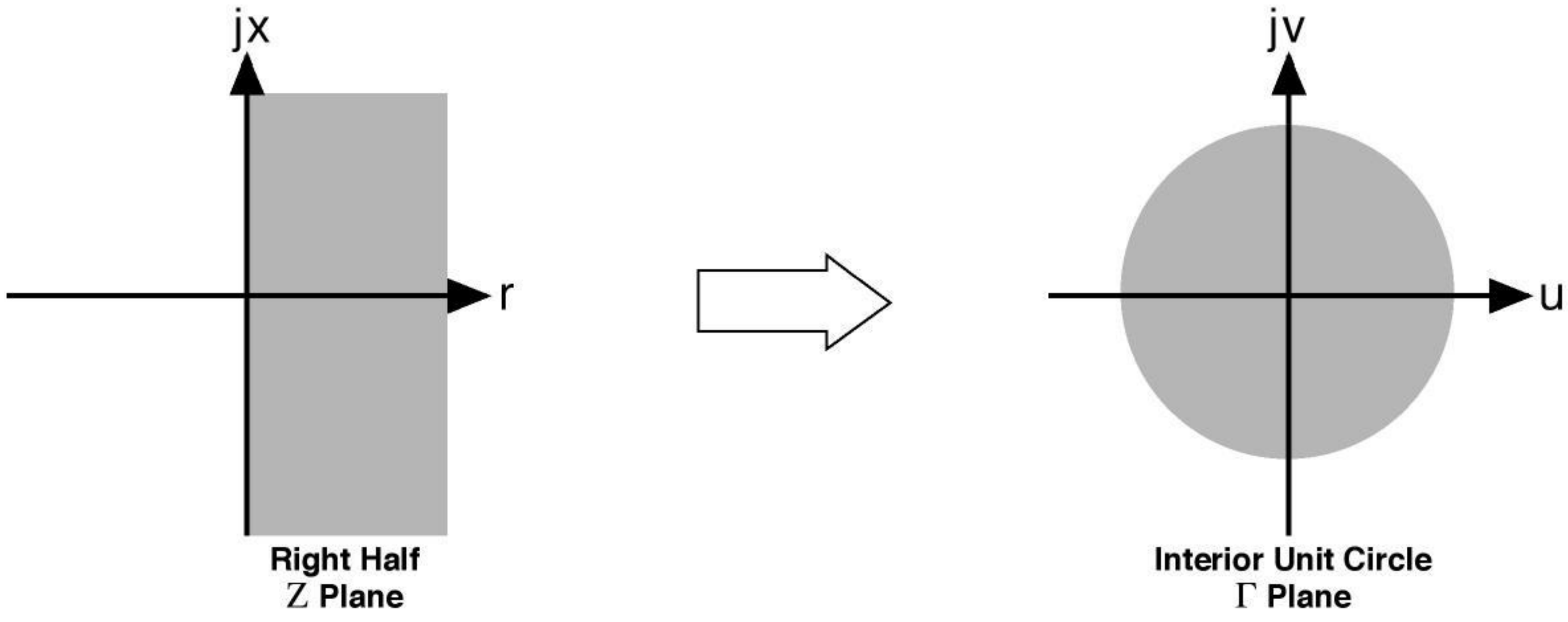


Mathematical Basis of the Smith Chart

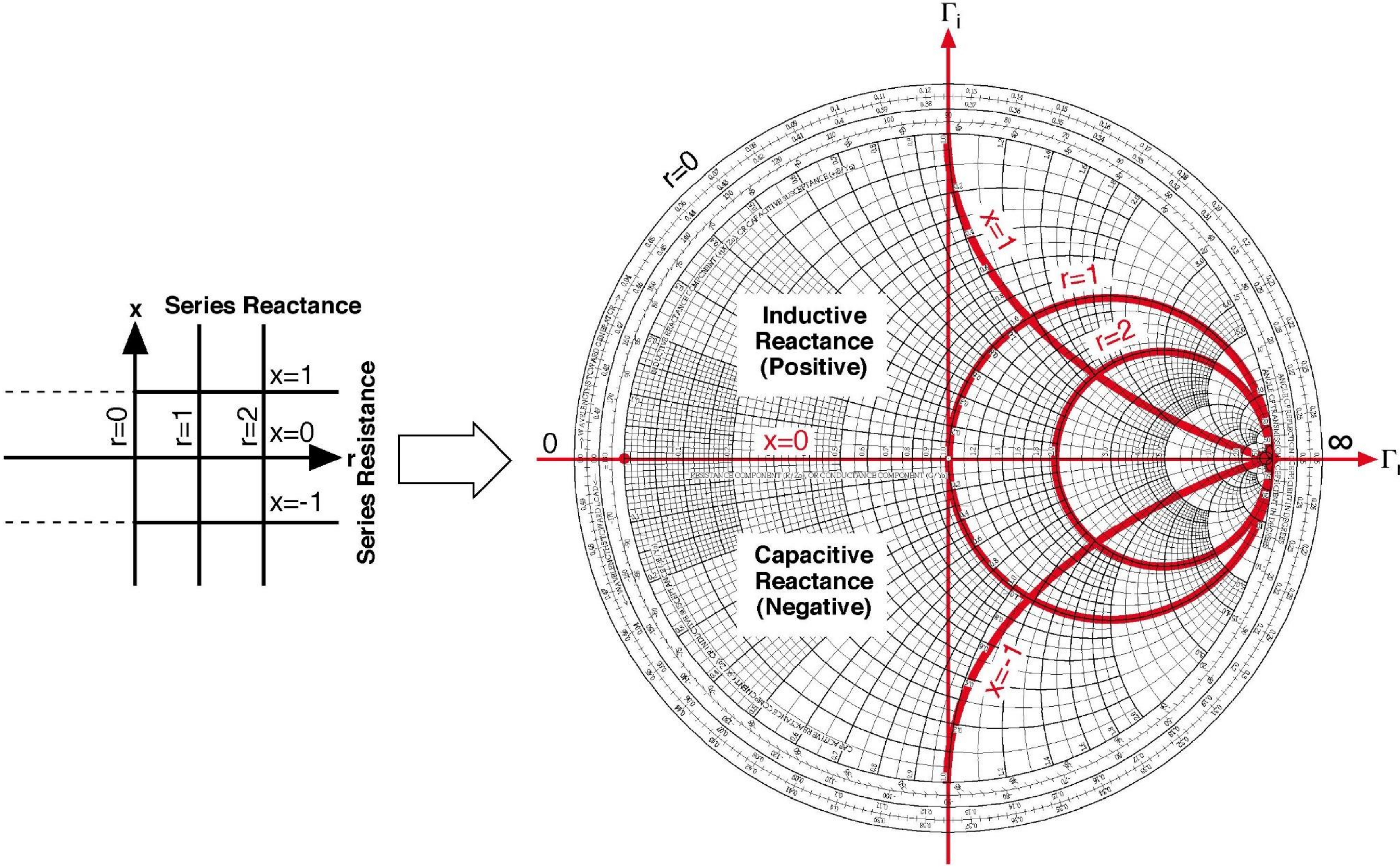
$$\Gamma = \frac{z-1}{z+1}$$

$$u + jv = \frac{(r-1) + jx}{(r+1) + jx}$$

- A bilinear conformal complex function of a complex variable
- Maps normalized impedance z to complex reflection coefficient Γ



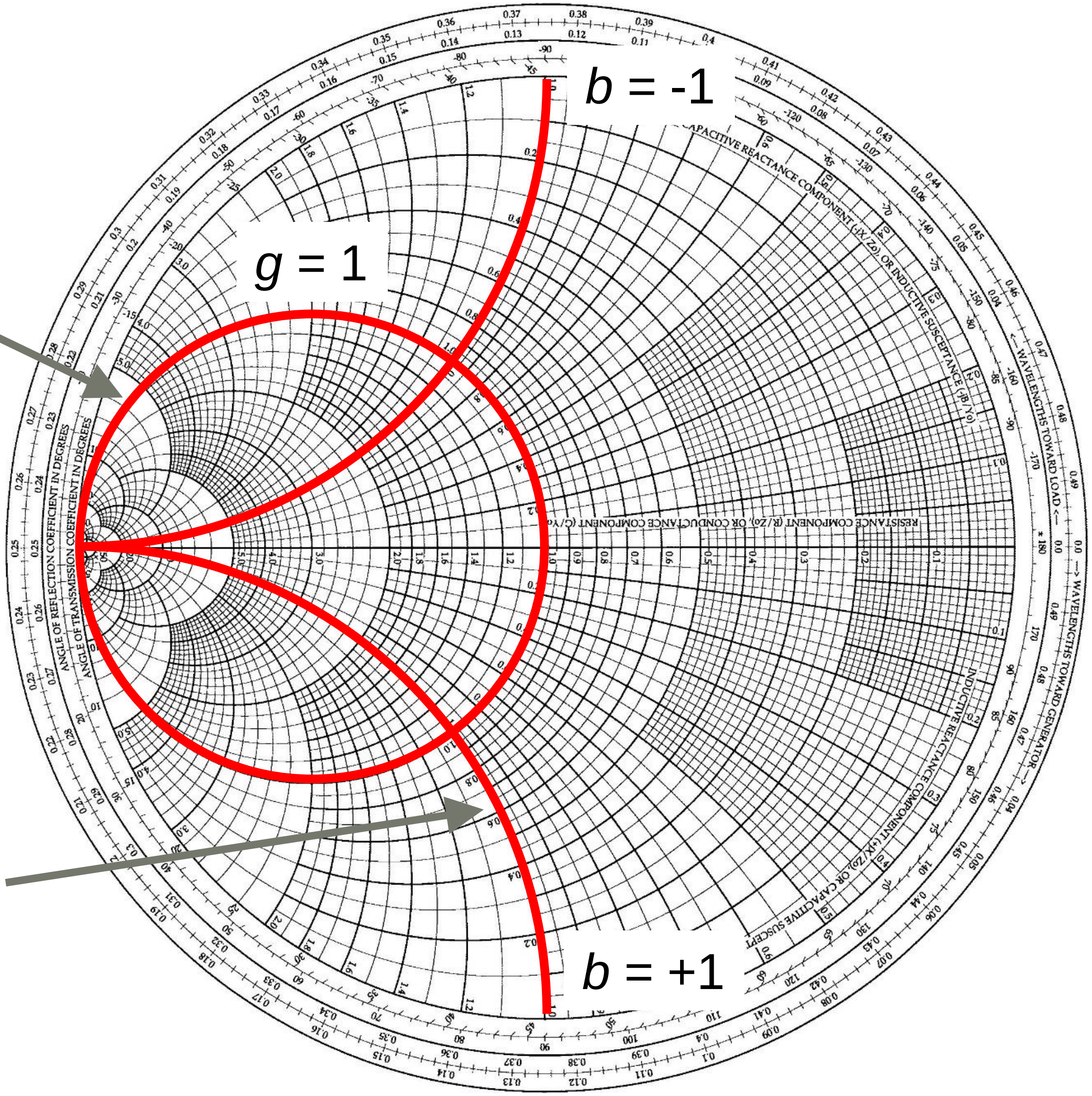
Smith Chart: Impedance Coordinates



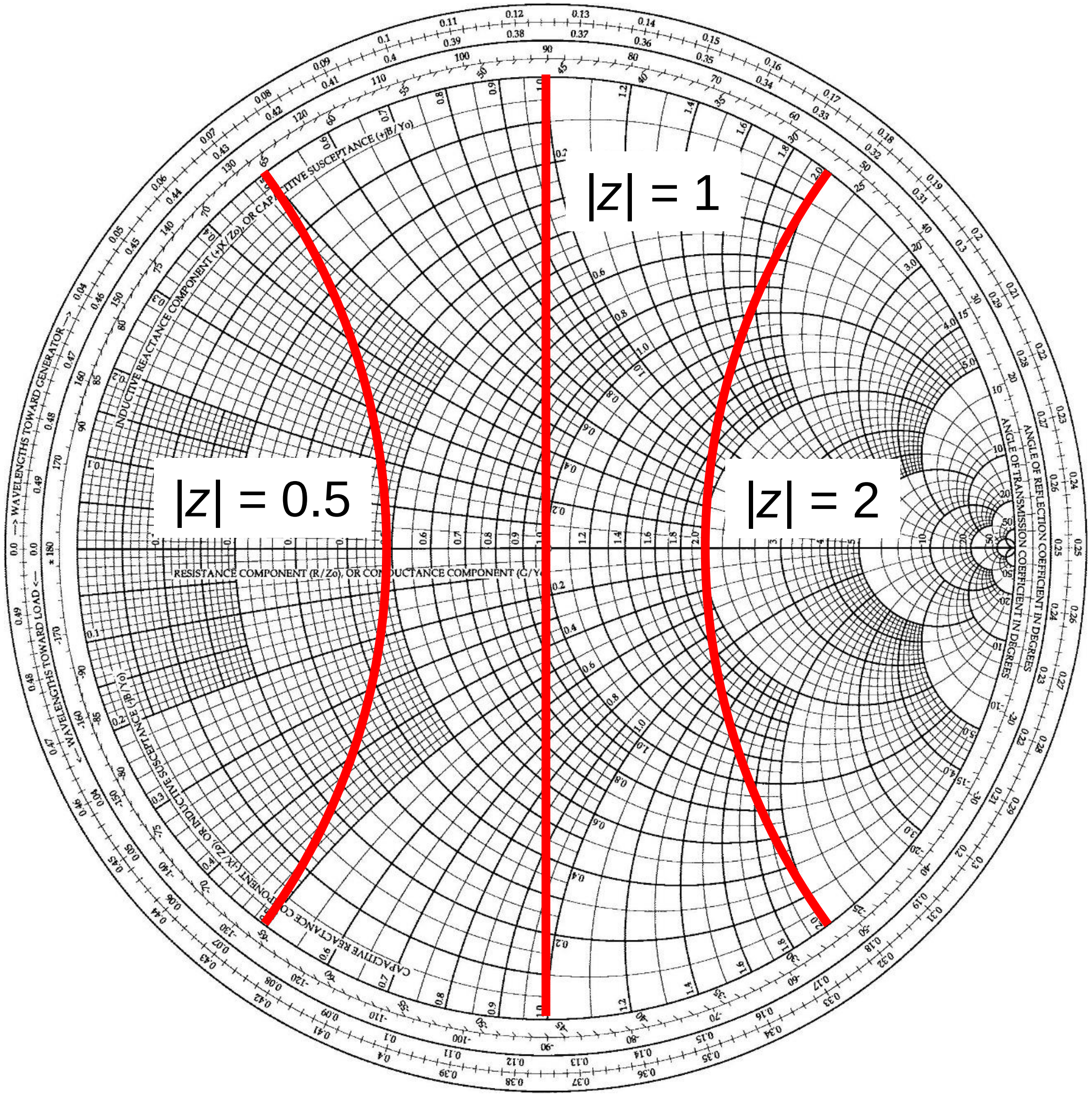
Admittance Coordinates

Constant conductance circles

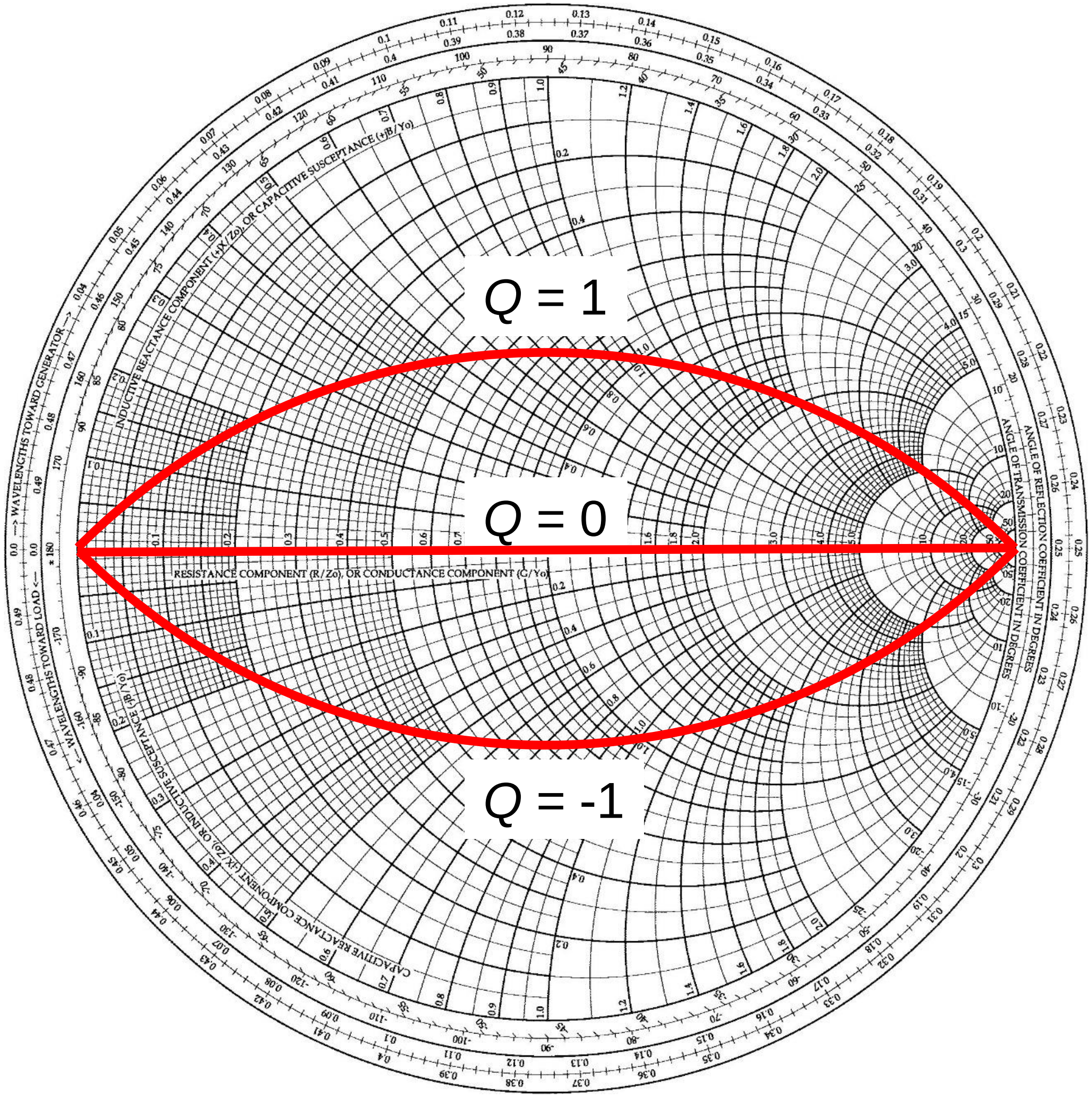
Constant susceptance arcs



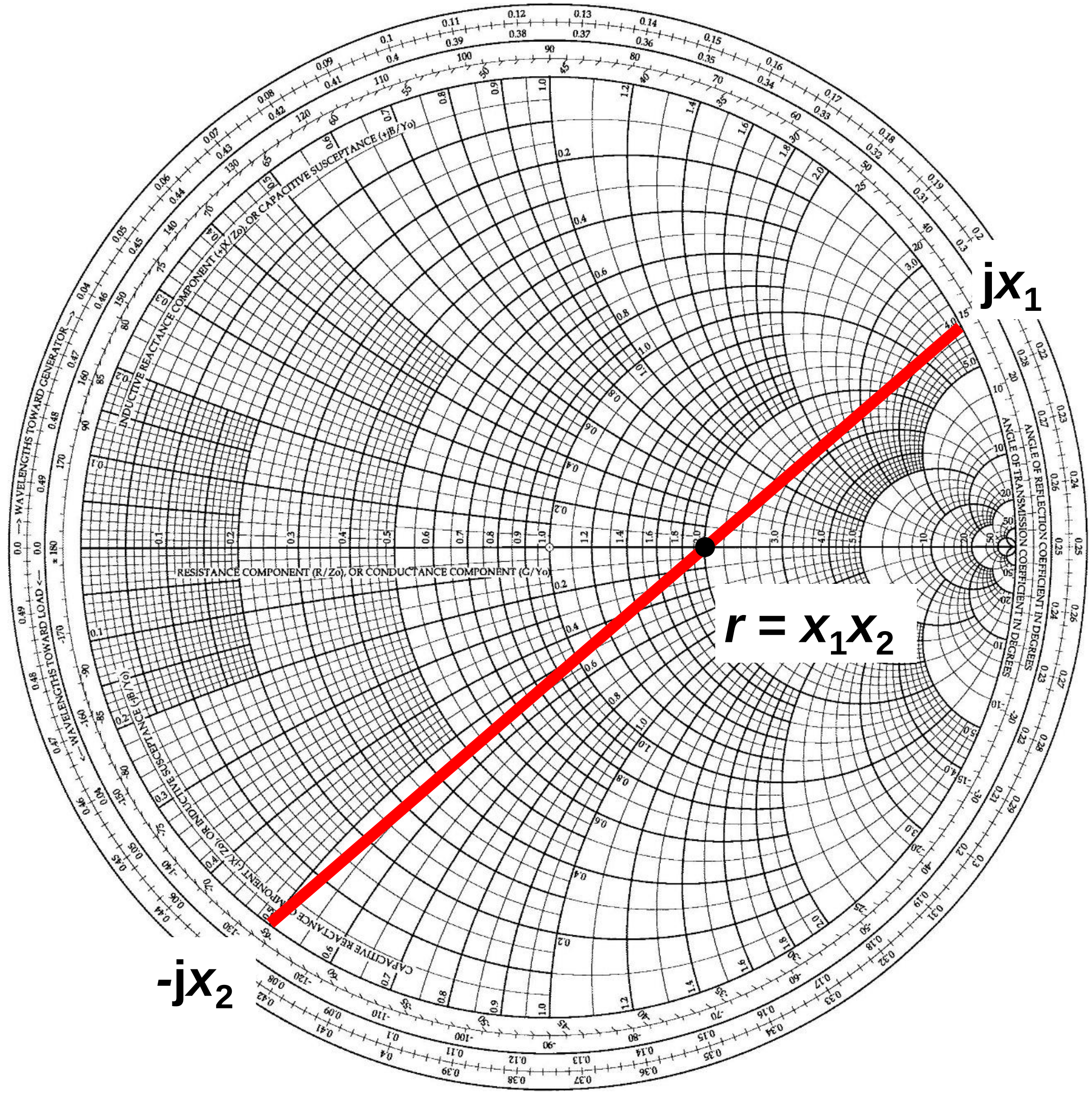
Constant Immittance Magnitude Arcs



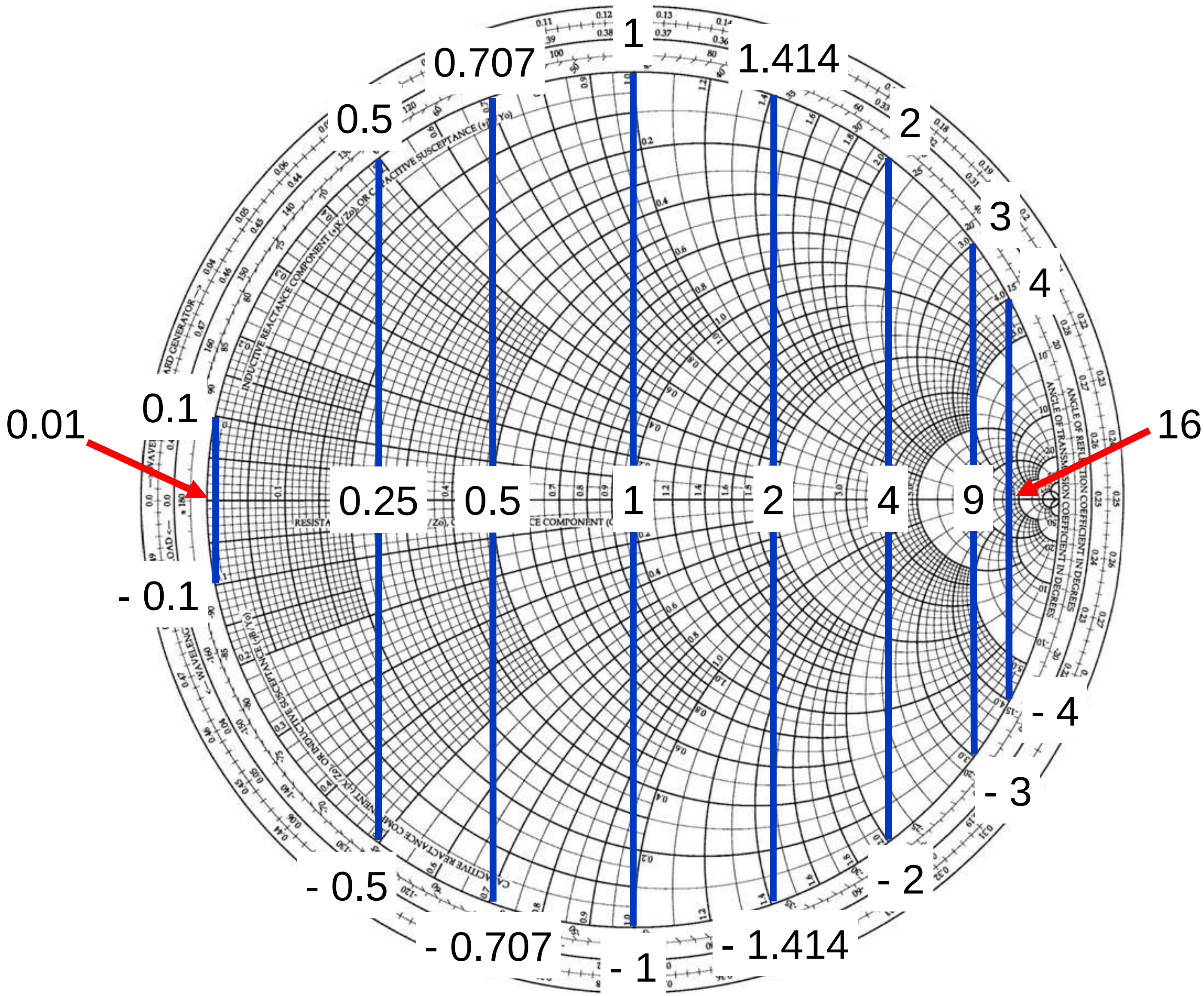
Constant Q Arcs



Multiplication and Division



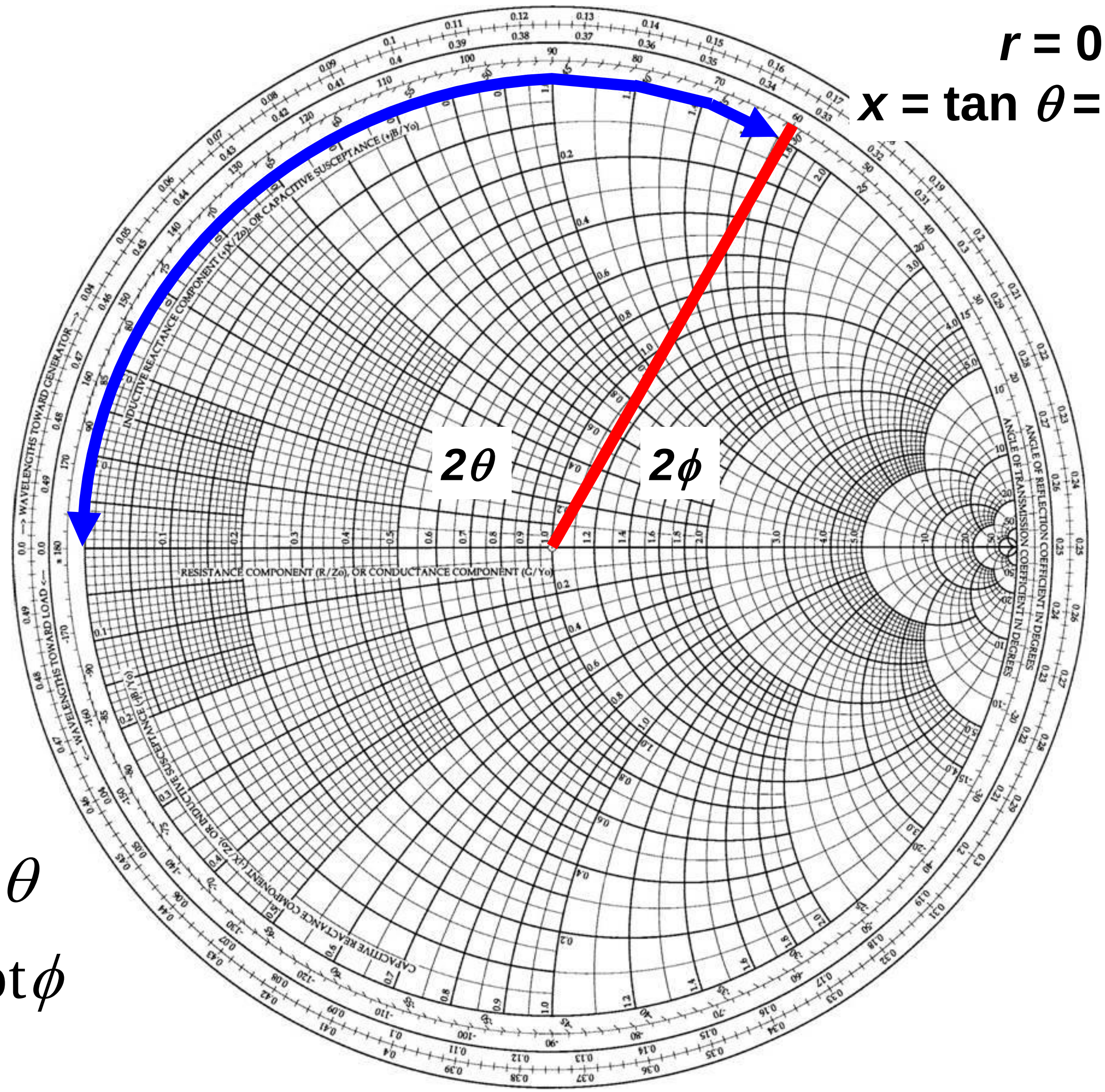
Squares and Square Roots



Computing Tangents and Cotangents

$r = 0$

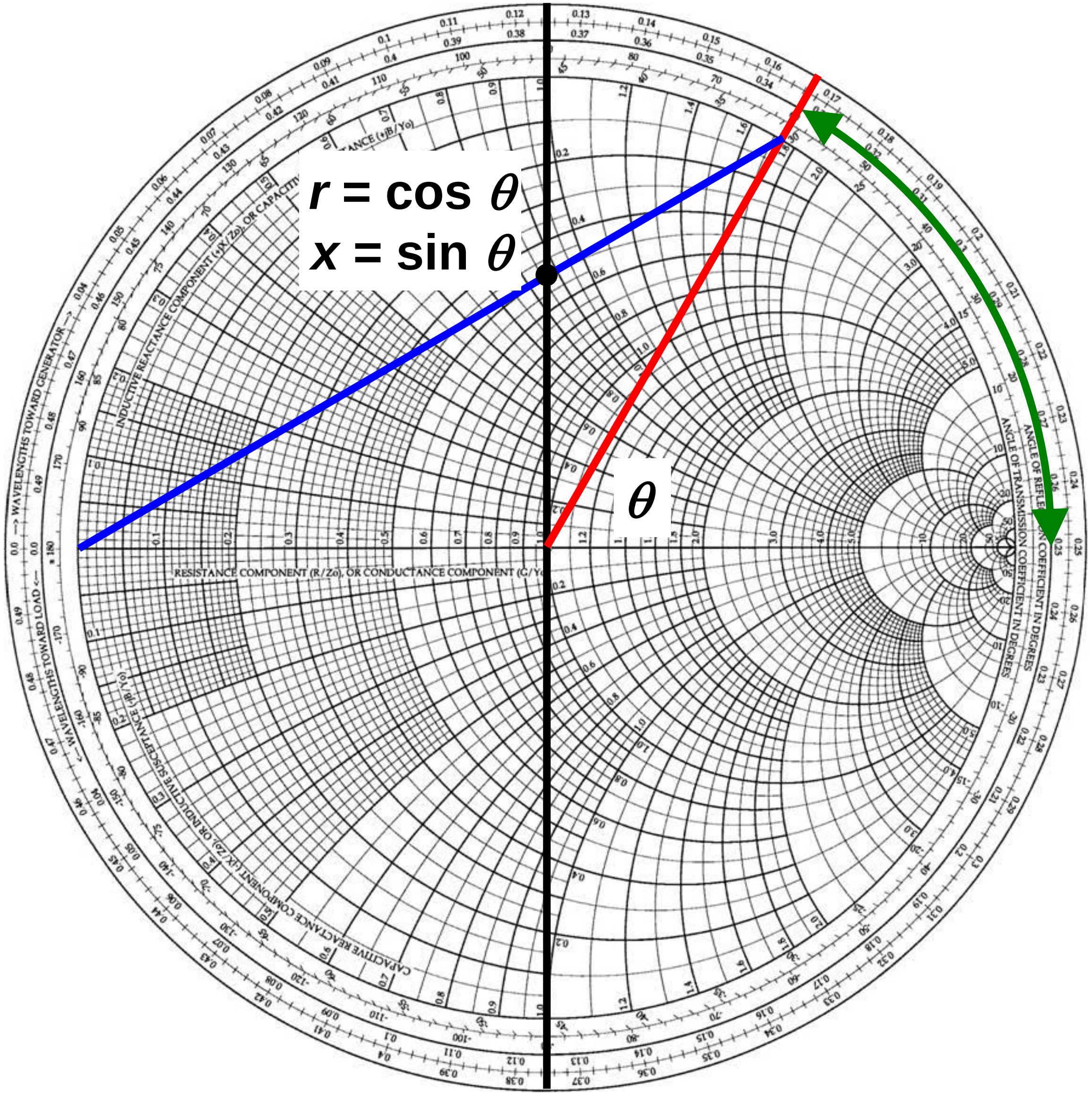
$x = \tan\theta = \cot\phi$



$$z_{short} = j\tan\theta$$

$$z_{open} = -j\cot\phi$$

Computing Sines and Cosines



Network Theory

Key Dates in Passive Network Theory

1893	"Impedance" – A.E. Kennelly
1893	AC circuit theory, complex numbers or phasors, "reactance" – C.P. Steinmetz
1924	Reactance theorem, 1-port synthesis by partial fractions – R.M. Foster
1926	1-port synthesis by continued fractions – W. Cauer
1931	1-port synthesis using RLCM – O. Brune
1939	1-port synthesis using a single R – S. Darlington
1946	n-port synthesis using RLCM – Y. Oono
1949	1-port synthesis using RLC without transformers – R. Bott & R.J. Duffin
1957	*Synthesis of Passive Networks* – E.A. Guillemin
1958	*Network Synthesis* – D.F. Tuttle
1962	*Linear Active Network Theory* – L. de Pian
1973	*Network Analysis and Synthesis* – B.D.O. Anderson and S. Vongpanitlerd
2009	All singular elements found – A.M. Soliman
2015	*Theory and Synthesis of Linear Passive Time-Invariant Networks* – D.C. Youla

Key Dates in Broadband Impedance Matching

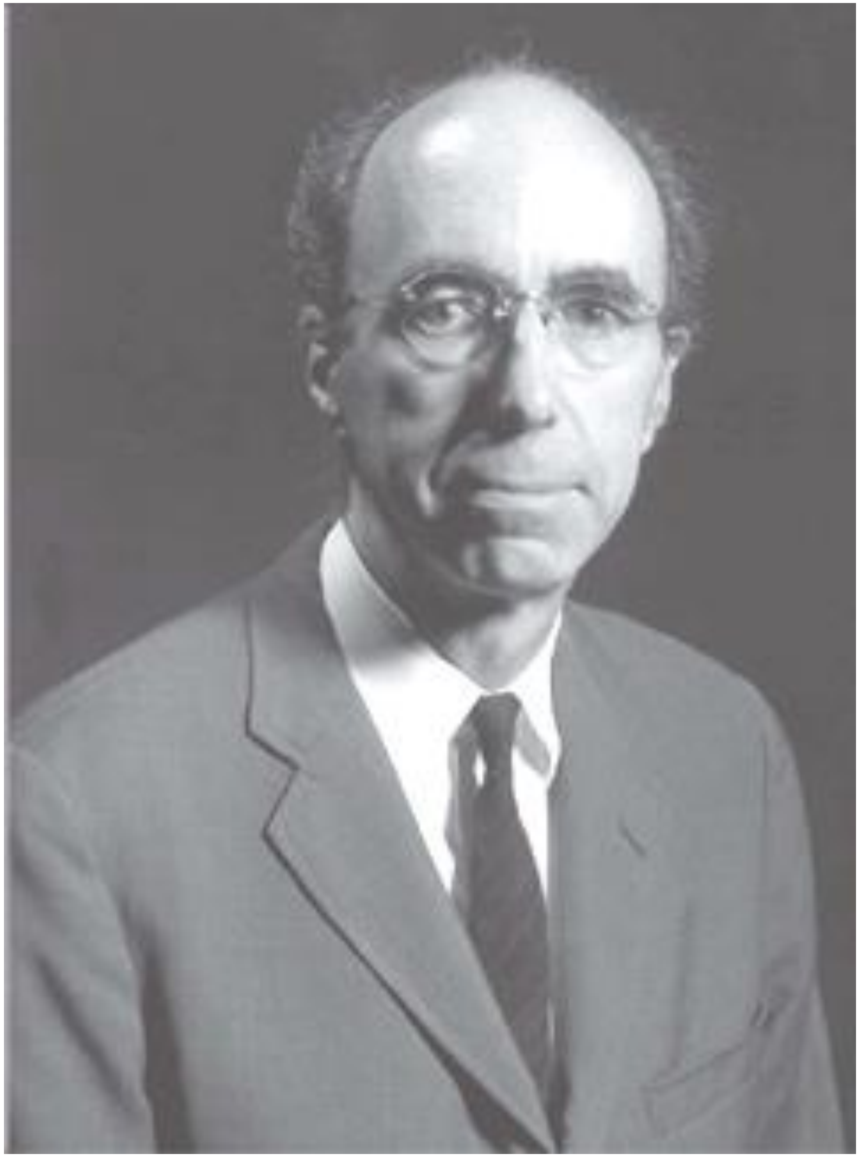
1928	**Constant resistance reflectionless networks – O.J. Zobel**
1937	**Constant resistance multiplexers – E.L. Norton**
1947	**Bound on return loss bandwidth for lossless networks – R.M. Fano**
1952	**Bound on insertion gain bandwidth for reflectionless networks – H.J. Carlin & R. LaRosa**
1964	**New theory of broadband matching – D.C. Youla**
1976	***Theory and Design of Broadband Matching Networks* – W-K. Chen**
1977	**Real frequency method – H.J. Carlin**
1982	**Simplified real frequency technique – B.S. Yarman & H.J. Carlin**
1986	**RFT matching for 2-9 MHz dipole – O.M. Ramahi & R. Mittra**
1994	**SRFT matching for 3 GHz microstrip antenna – H. An, B.K.J.C. Nauwelaers, & A.R. Van de Capelle**
2003	**SRFT matching for loaded wire HF antenna – K. Yegin & A.Q. Martin**
2002-10	**Non-Foster active networks for extreme broadband matching – S.E. Sussman-Fort, S.D. Stearns, & others**

Arthur Edwin Kennelly, 1861-1939

Charles Proteus Steinmetz, 1865-1923

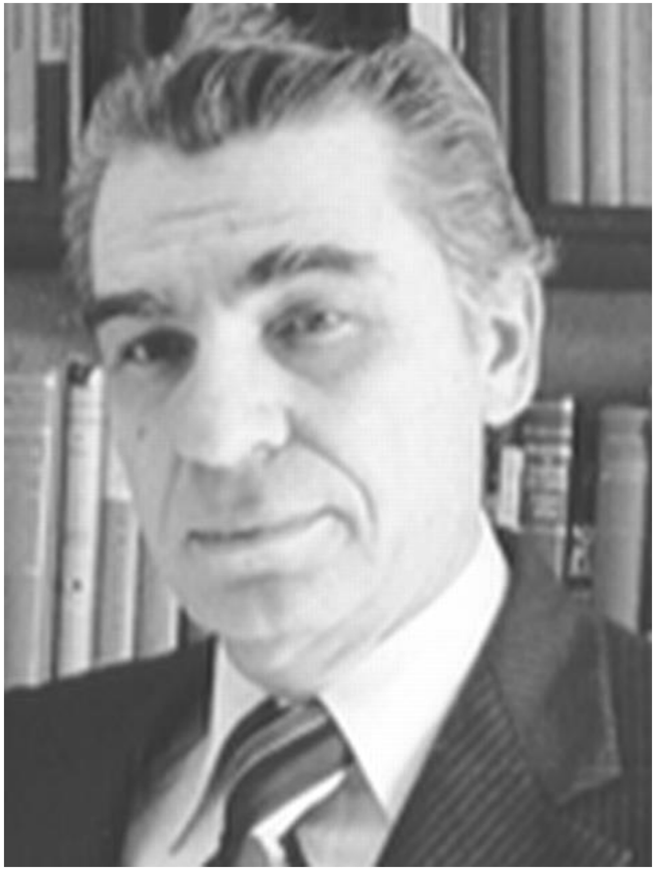
Ronald Martin Foster, 1896-1998

Sidney Darlington, 1906-1997



Earnst Adolph Guillemin, 1908-1970

Dante C. Youla, 1925-



90 Years of Progress in Electrical Network Theory

A Reactance Theorem

By RONALD M. FOSTER

SYNOPSIS: The theorem gives the most general form of the driving-point impedance of any network composed of a finite number of self-inductances, mutual inductances, and capacities. This impedance is a pure reactance with a number of resonant and anti-resonant frequencies which alternate with each other. Any such impedance may be physically realized (provided resistances can be made negligibly small) by a network consisting of a number of simple resonant circuits (inductance and capacity in series) in parallel or a number of simple anti-resonant circuits (inductance and capacity in parallel) in series. Formulas are given for the design of such networks. The variation of the reactance with frequency for several simple circuits is shown by curves. The proof of the theorem is based upon the solution of the analogous dynamical problem of the small oscillations of a system about a position of equilibrium with no frictional forces acting.

AN important theorem[1] gives the driving-point impedance[2] of any network composed of a finite number of self-inductances, mutual inductances, and capacities; showing that it is a pure reactance with a number of resonant and anti-resonant frequencies which alternate with each other; and also showing how any such impedance may be physically realized by either a simple parallel-series or a simple series-parallel network of inductances and capacities, provided resistances can be made negligibly small. The object of this note is to give a full statement of the theorem, a brief discussion of its physical significance and its applications, and a mathematical proof.

THE THEOREM

The most general driving-point impedance S obtainable by means of a finite resistanceless network is a pure reactance which is an odd rational function of the frequency $p/2\pi$ and which is completely determined, except for a constant factor H, by assigning the resonant and anti-resonant frequencies, subject to the condition that they alternate and include both zero and infinity. Any such impedance may be physically

[1] The theorem was first stated, in an equivalent form and without his proof, by George A. Campbell, *Bell System Technical Journal*, November, 1922, pages 23, 26, and 30. By an oversight the theorem on page 26 was made to include unrestricted dissipation. Certain limitations, which are now being investigated, are necessary in the general case of dissipation. The theorem is correct as it stands when there is no dissipation, that is, when all the *R*'s and *G*'s vanish; this is the only case which is considered in the present paper.

A corollary of the theorem is the mutual equivalence of simple resonant components in parallel and simple anti-resonant components in series. This corollary had been previously and independently discovered by Otto J. Zobel as early as 1919, and was subsequently published by him, together with other reactance theorems, *Bell System Technical Journal*, January, 1923, pages 5–9.

[2] The driving-point impedance of a network is the ratio of an impressed electromotive force at a point in a branch of the network to the resulting current at the same point.

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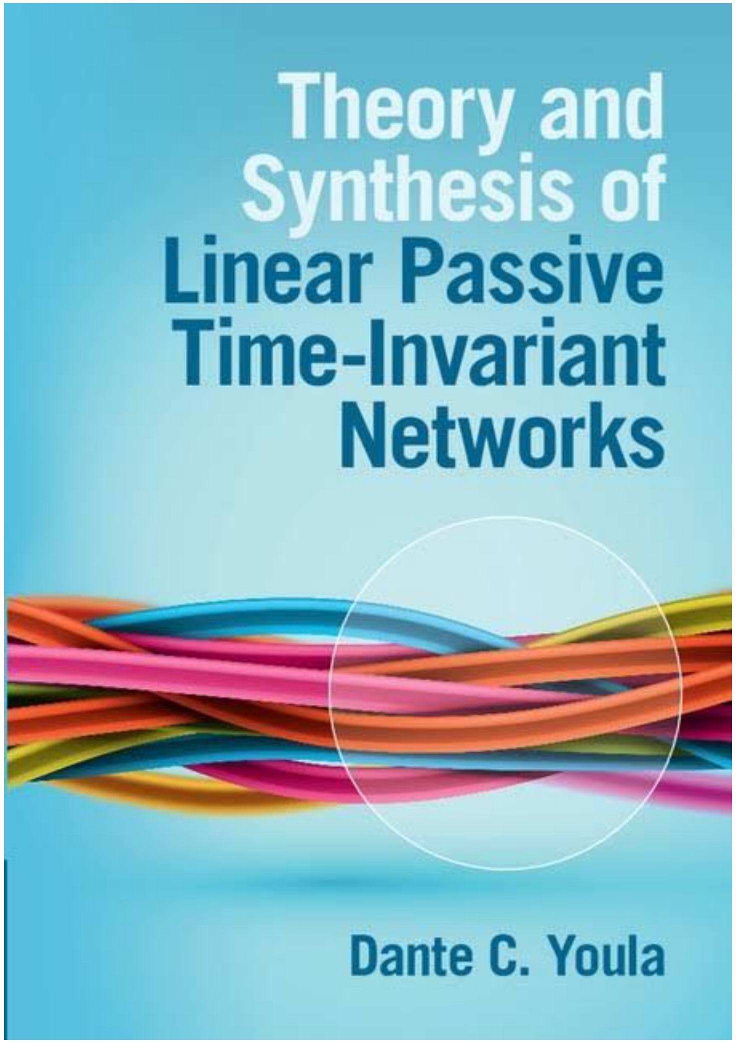


Impedance & Admittance (Immittance) Functions

- **Impedance and admittance functions of passive devices, elements, and networks are positive-real functions of complex frequency $s = \sigma + j\omega$**
- **Analytic in the RHP – no poles or zeros there**
- **Poles and zeros can exist only on $j\omega$ axis and in LHP**
- **Immittance functions of lumped RLC networks**
  - Are rational functions with positive coefficients
  - Numerator and denominator polynomial degrees differ by 0 or 1
  - If the degrees are the same, the network has losses
- **The magnitude of the impedance or admittance function of a passive element or network cannot increase faster than f (1st power of frequency)**
- **The magnitude of the impedance or admittance function of a passive element or network cannot decrease faster than $1/f$ (inverse frequency)**

More Facts from Network Theory

- The imaginary part of the impedance or admittance function of a lossless passive element or network has non-negative derivative with respect to frequency
- The real and imaginary parts of the impedance or admittance function of a passive element or network cannot be specified independently – one determines the other
- Real and imaginary parts are related by Poisson/Schwarz integrals
- An arbitrary resistance function such as $R(f) = R_0 f^{\,2}$ either has reactance or else is not passive
- Antenna models that use non-passive elements are active and therefore problematic with respect to stability
  - Witt (1995)
  - Long, Werner, and Werner (2000)
  - Rudish and Sussman-Fort (2002)
  - Aberle and Romak (2007)
  - Karawas and Collin (2008)
- Similarly, the ideal transformer is passive, yet its π and T equivalent circuits are active non-Foster networks
- Such equivalent circuits are “equivalent” only in certain respects such as impedance on the $j\omega$ axis but may not be equivalent in other respects such as active vs passive or stability behavior

Example of How Real and Imaginary Parts are Related

- **Impedance**

$$Z(s) = \frac{as^3 + bs^2 + cs + d}{es^2 + s}$$

- **Real part**

$$R(j\omega) = \frac{(be - a)\omega^2 + (c - de)}{e^2\omega^2 + 1}$$

- **Imaginary part**

$$X(j\omega) = \frac{ae\omega^4 + (b - ce)\omega^2 - d}{e^2\omega^3 + \omega}$$

General Relation Between Real and Imaginary Parts

- **Poisson/Schwarz integrals**
  - AKA "Hilbert transform" or Kramers-Kronig relations

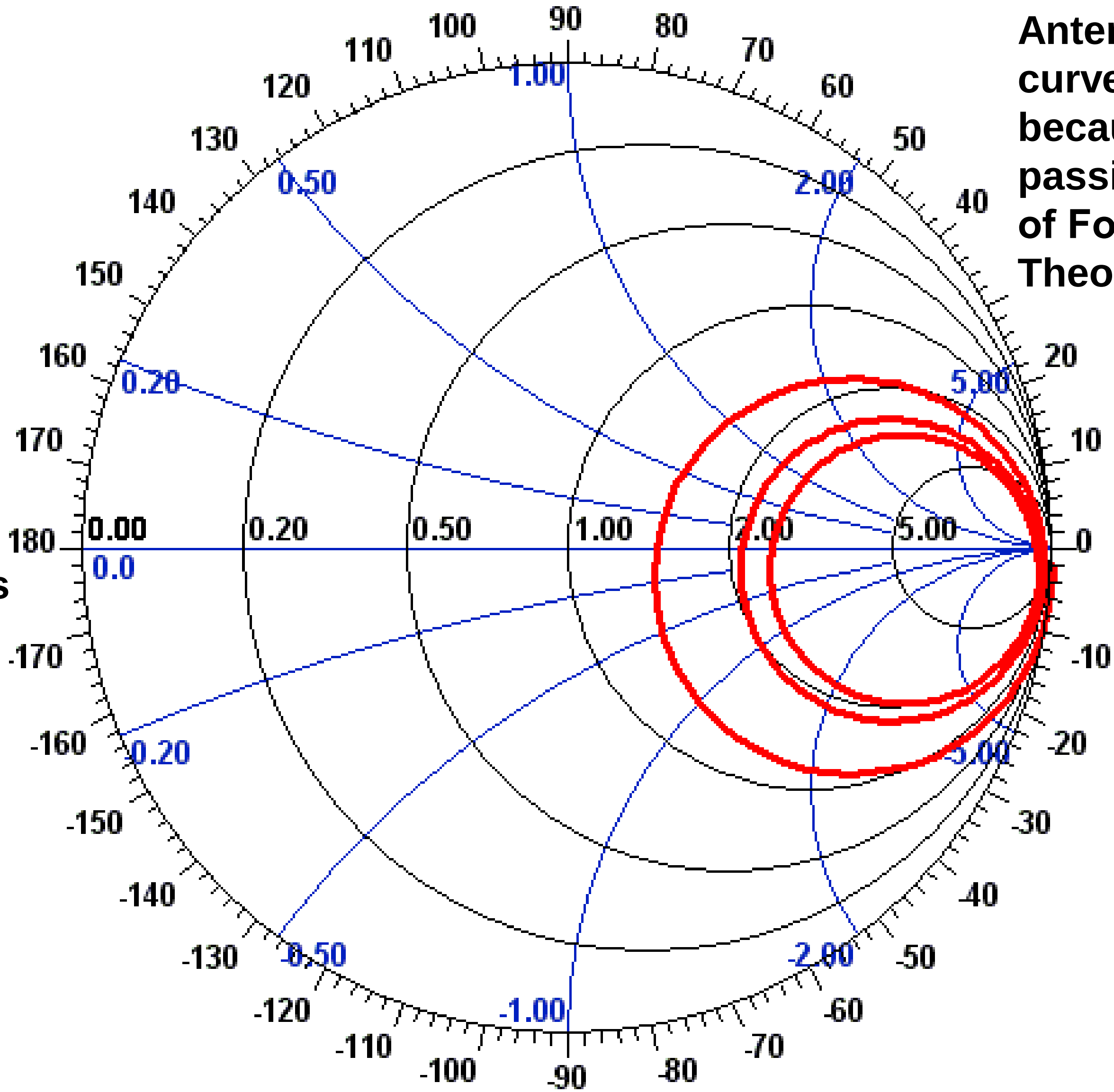
$$R(\omega) = \frac{2}{\pi}\int_0^\infty \frac{u\,X(u)}{\omega^2 - u^2}\,du + R_0$$

$$X(\omega) = \frac{-2\omega}{\pi}\int_0^\infty \frac{R(u)}{\omega^2 - u^2}\,du + X_0(\omega)$$

- **Letting $R(\omega) = R_0\omega^2$, we find that $X(\omega)$ does not exist**
- **Hence $R_0\omega^2$ cannot be the real part of any passive impedance function**
- **No passive device exists that has impedance $R_0\omega^2$**

Positive-Real Impedance Functions Curve Clockwise

At *f* = 0, curve starts on horizontal axis

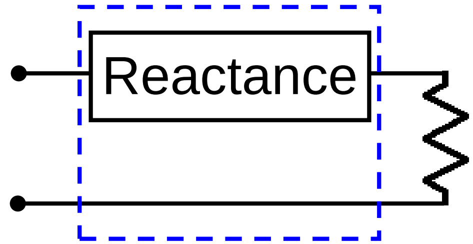


Antenna impedance curves bend clockwise because antennas are passive, not because of Foster's Reactance Theorem.

Darlington Forms (1939)

- **Every immittance function can be realized as a lossless two-port terminated by a resistor**
- **Every antenna impedance function has an equivalent circuit in Darlington form**

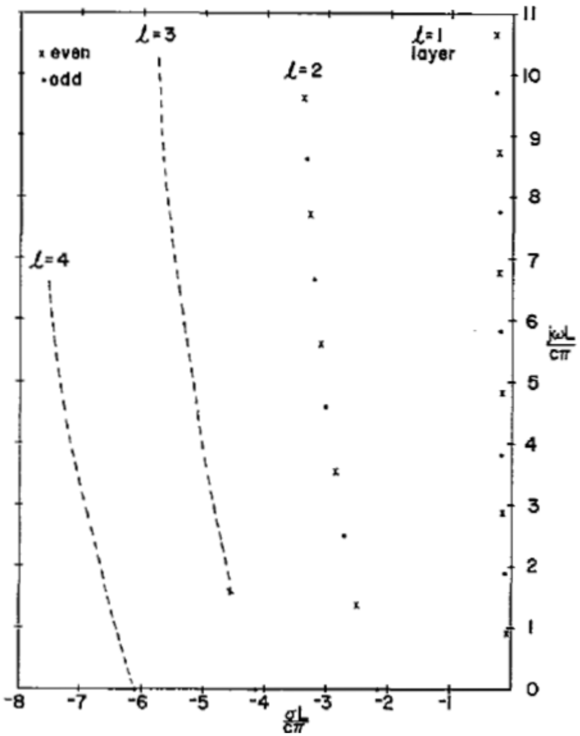
This……

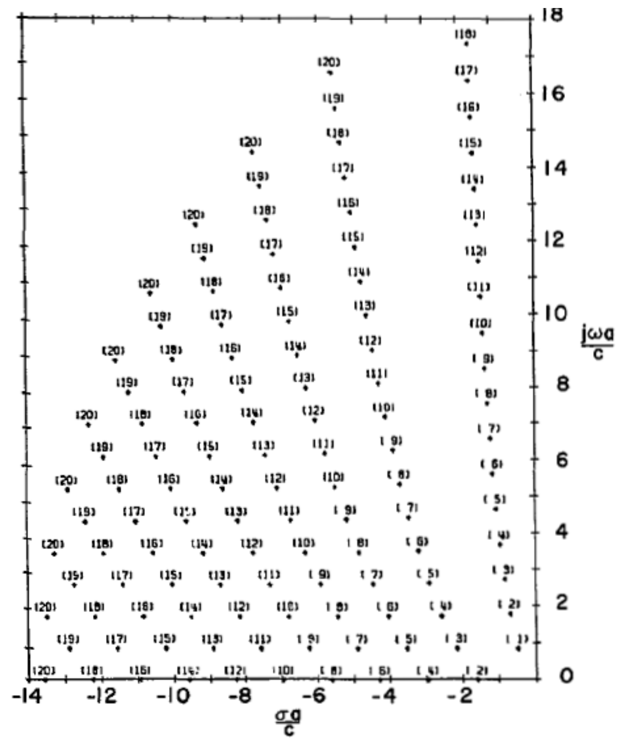


Not This!

- **The Darlington form is the starting point for interesting results in network and matching theory**
  - The Fano (1947) and Carlin-LaRosa (1952) bounds on impedance matching
  - Reflectionless impedance matching networks
  - Non-Foster active impedance matching networks

Antennas Ring at Many Frequencies (Like Bells)





- Distributed and electromagnetic systems are infinite-dimensional linear systems
- Tesche (1973) derived antenna TM modes by SEM
- Immittance functions are transcendental, meromorphic instead of rational because poles and zeros are isolated

Meromorphic Functions

- **A function $f(s)$ is meromorphic if**
  - It can be expressed as a ratio of two analytic functions
  - Singularities are ordinary poles (infinity excepted)
  - Poles are isolated (poles have no accumulation/cluster points)
- **Consider meromorphic immittance functions that satisfy**
  - All poles lie in the left half of the complex s-plane
  - $f^*(s) = f(s^*)$
  - $\mathrm{Re}\{f(j\omega)\}$ is non-negative
  - All poles of $f(s)$ are simple
- **Examples**
  - P.r. rational functions: $P(s)/Q(s)$
  - Complex exponentials: $\exp(s)$, $\exp(-s)$
  - Trigonometric functions: $\sin(s)$, $\cos(s)$, $\tan(s)$
  - Hyperbolic functions: $\sinh(s)$, $\cosh(s)$, $\tanh(s)$
  - Many special functions of mathematical physics

Mittag-Leffler Theorem

- **Mittag-Leffler states a convergent series for *f*(*s*)**

$$f(s) = f(0) + \lim_{N\to\infty} \sum_{n=-N}^{N} \left(\frac{A_n}{s-s_n} - \frac{A_n}{0-s_n} \right)$$
$$= f(0) + \lim_{N\to\infty} \left(P_N(s) - P_N(0) \right)$$

- **where**

$$P_N(s) = \sum_{n=-N}^{N} \frac{A_n}{s-s_n}$$

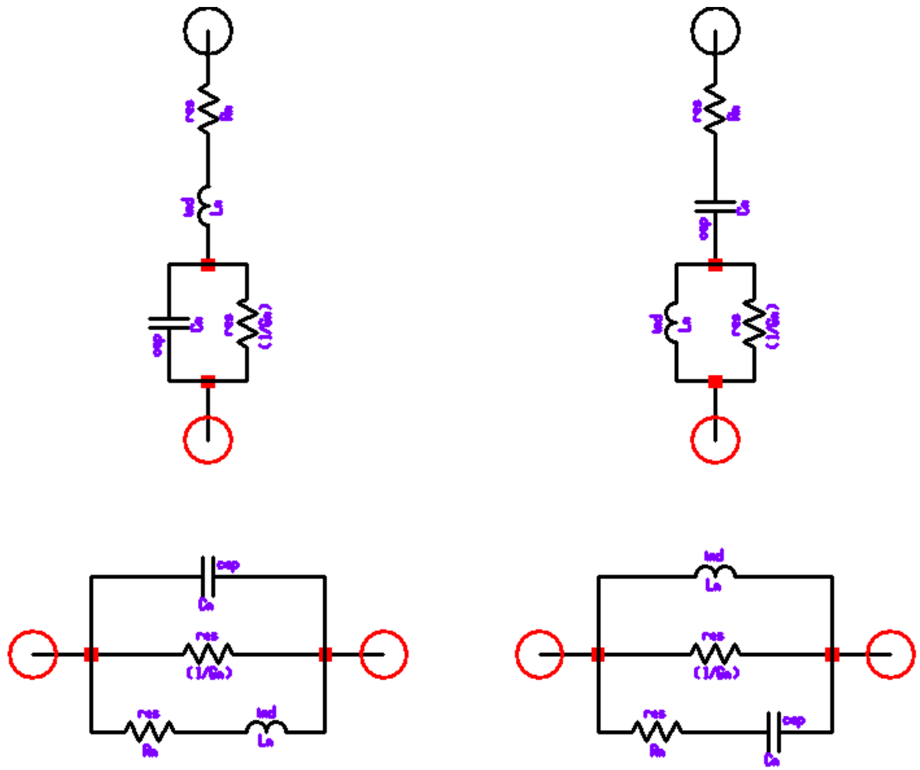
- **Yields a design recipe for a network that realizes *f*(*s*)**
  - Step 1: Determine the poles of s_n of $f(s)$
  - Step 2: Determine the residues A_n

$$A_n = \lim_{s\to s_n} (s-s_n) f(s)$$

Sergei Alexander Schelkunoff, 1897-1992

Equivalent Circuits for Transcendental Immittances

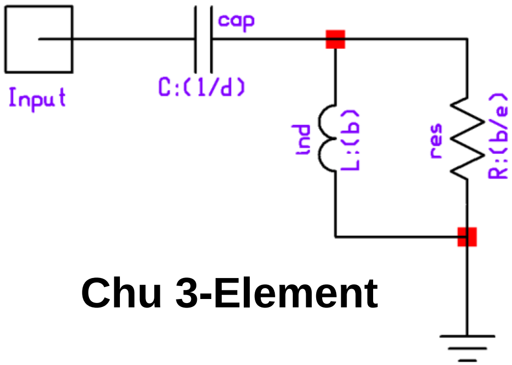
- **Schelkunoff (1944), Zinn (1952): Transcendental immittances of continuous electromagnetic structures can be represented by ladder networks made of one of four subcircuits**
  - Poles and zeros at $f = 0$ and ∞ help determine which one



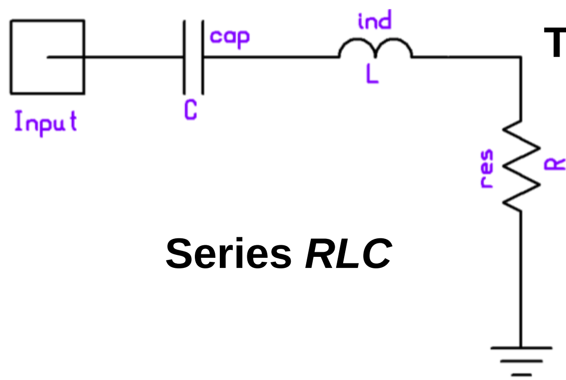
Circuit Models for Antenna Impedance

Broadband Equivalent Circuits

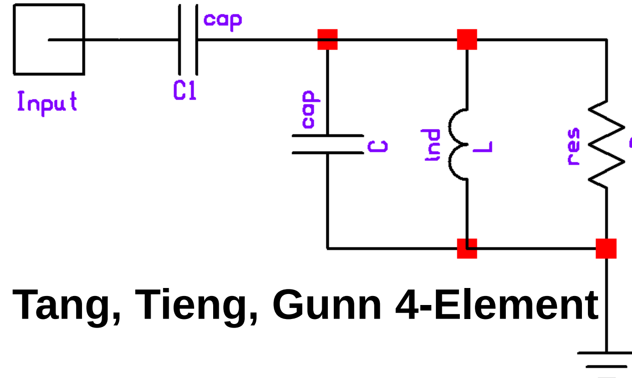
Impedance Models for Electrically-Small Dipoles & Monopoles



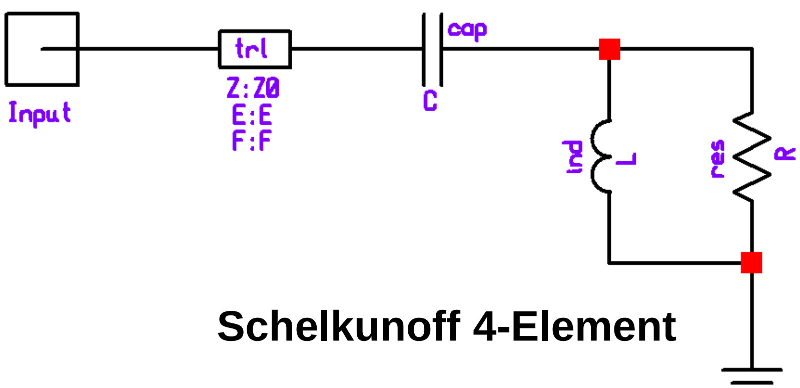
Chu 3-Element



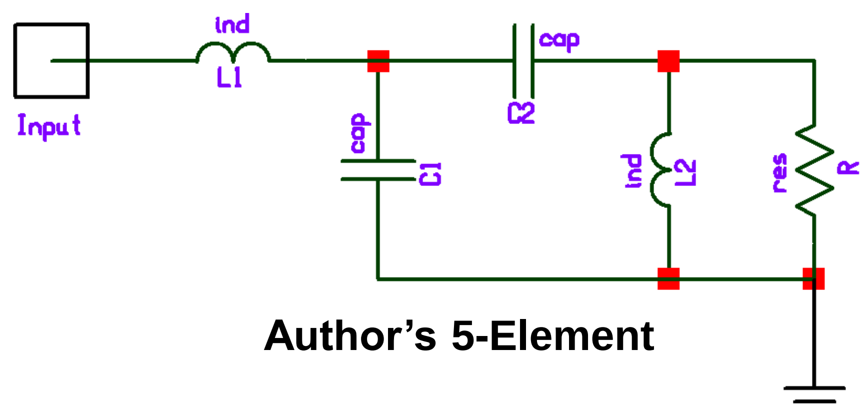
Series *RLC*



Tang, Tieng, Gunn 4-Element

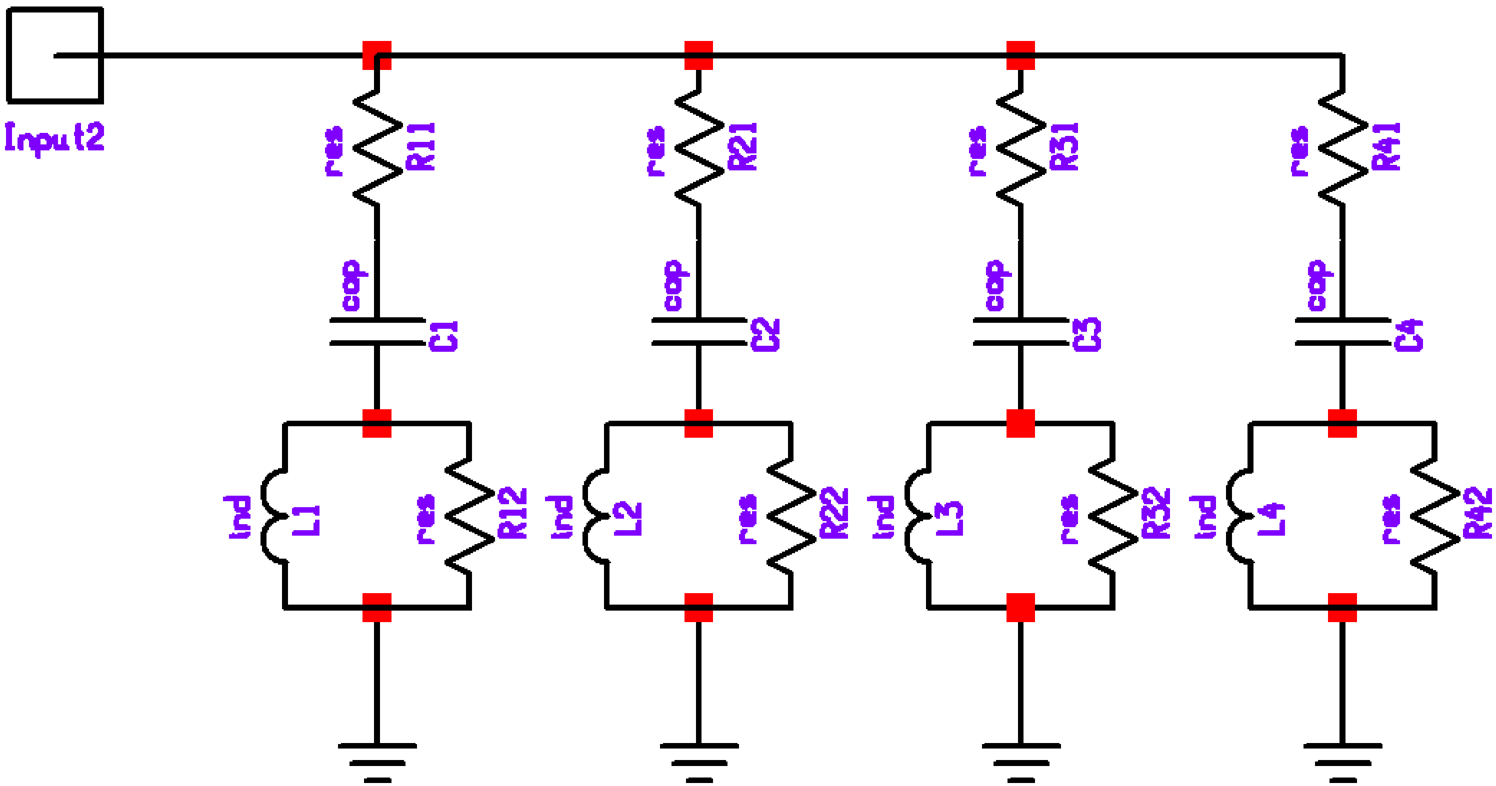


Schelkunoff 4-Element



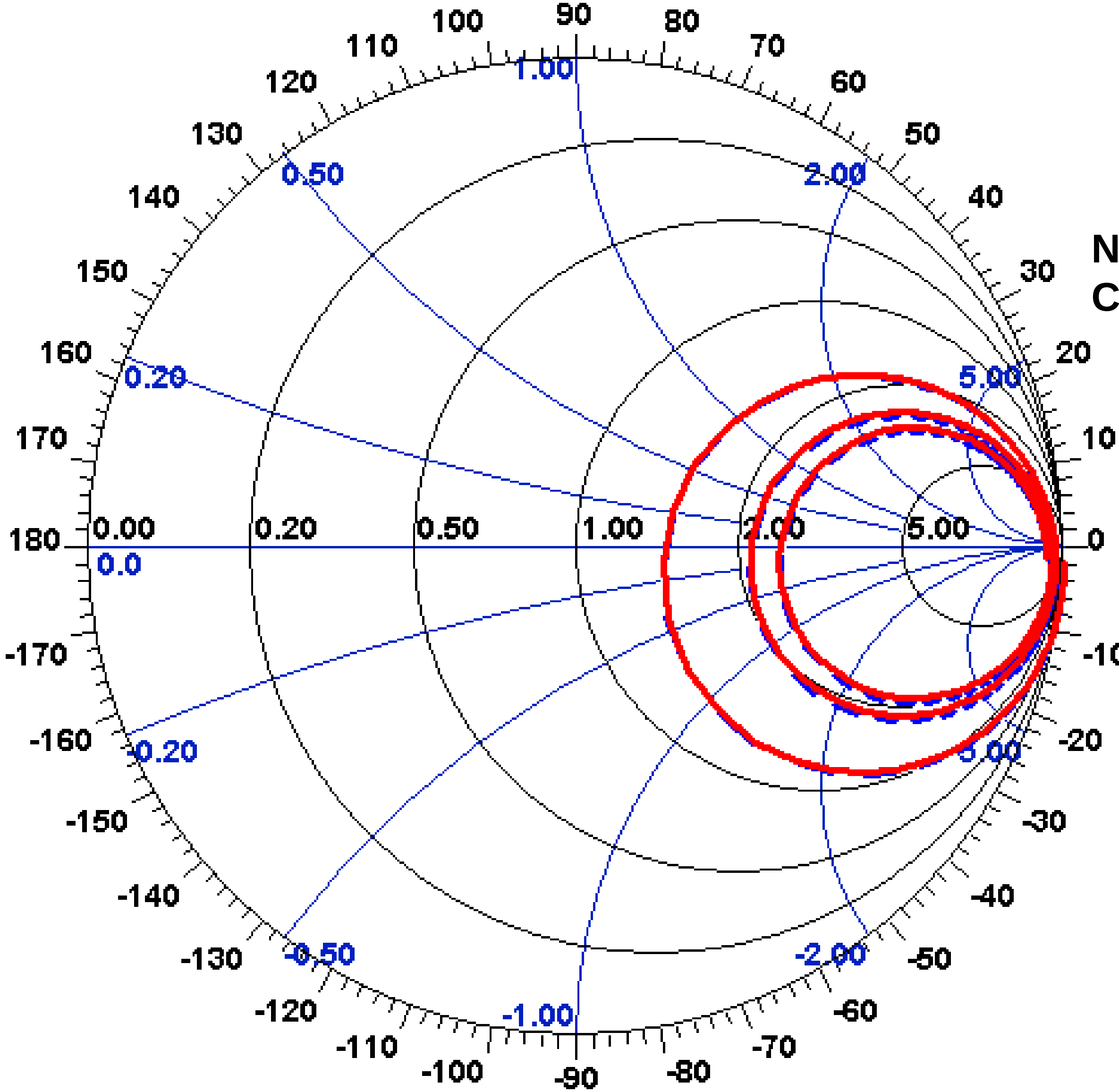
Author's 5-Element

Example 1: K6OIK Broadband Equivalent Circuit for 98.4-ft Dipole (*L* / *d* = 11,013) from DC to 30 MHz



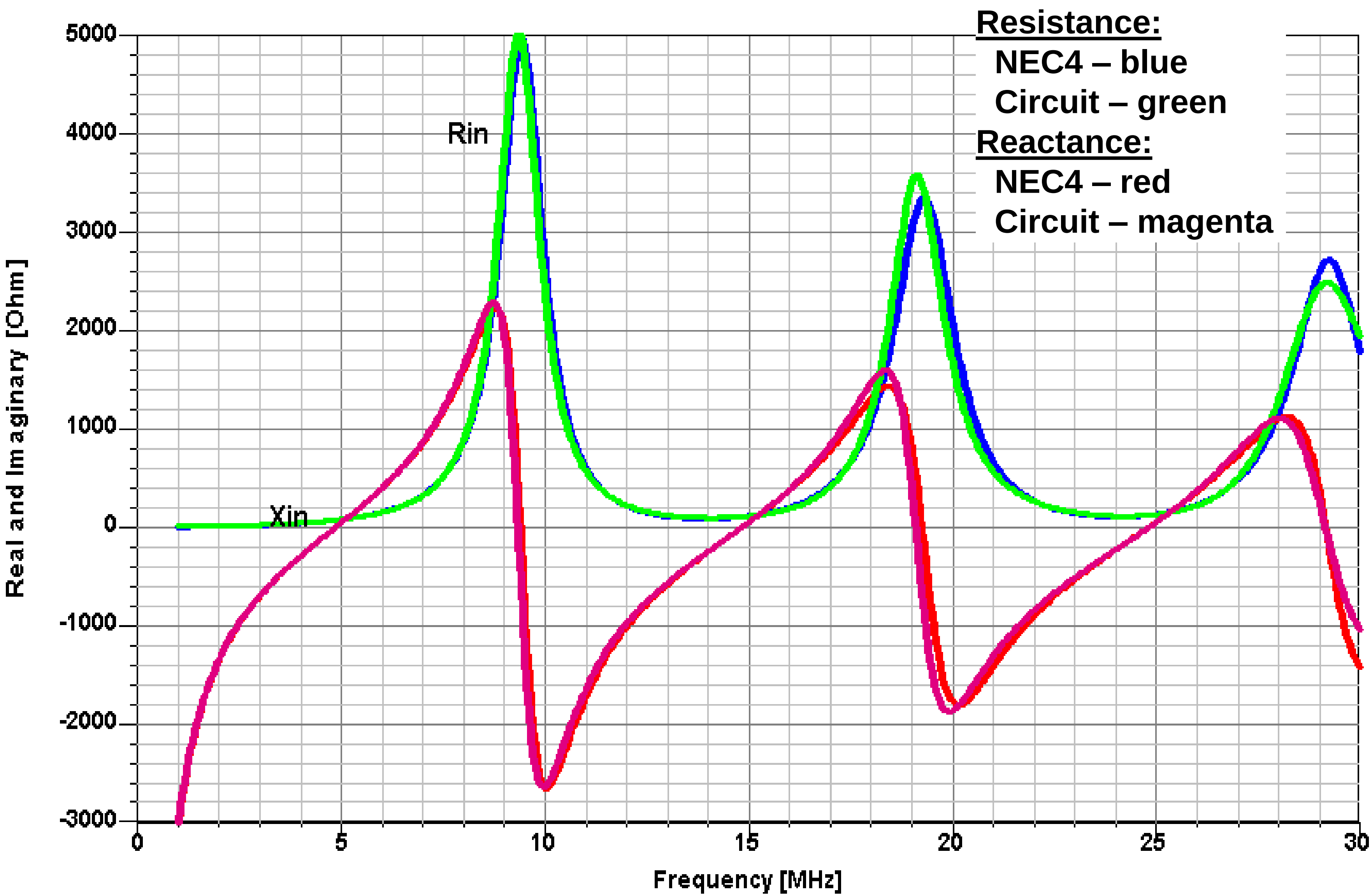
*R*11 = 5.06 Ω	*R*21 = 0 Ω	*R*31 = 25.5 Ω	*R*41 = 0 Ω
*C*1 = 39.9 pF	*C*2 = 4.64 pF	*C*3 = 4.69 pF	*C*4 = 1.68 pF
*L*1 = 27.1 μH	*L*2 = 24.9 μH	*L*3 = 2.26 μH	*L*4 = 24.5 μH
*R*12 = 10.1 kΩ	*R*22 = 50.1 kΩ	*R*32 = 2.68 kΩ	*R*42 = 116 kΩ

Impedance Comparison on Smith Chart

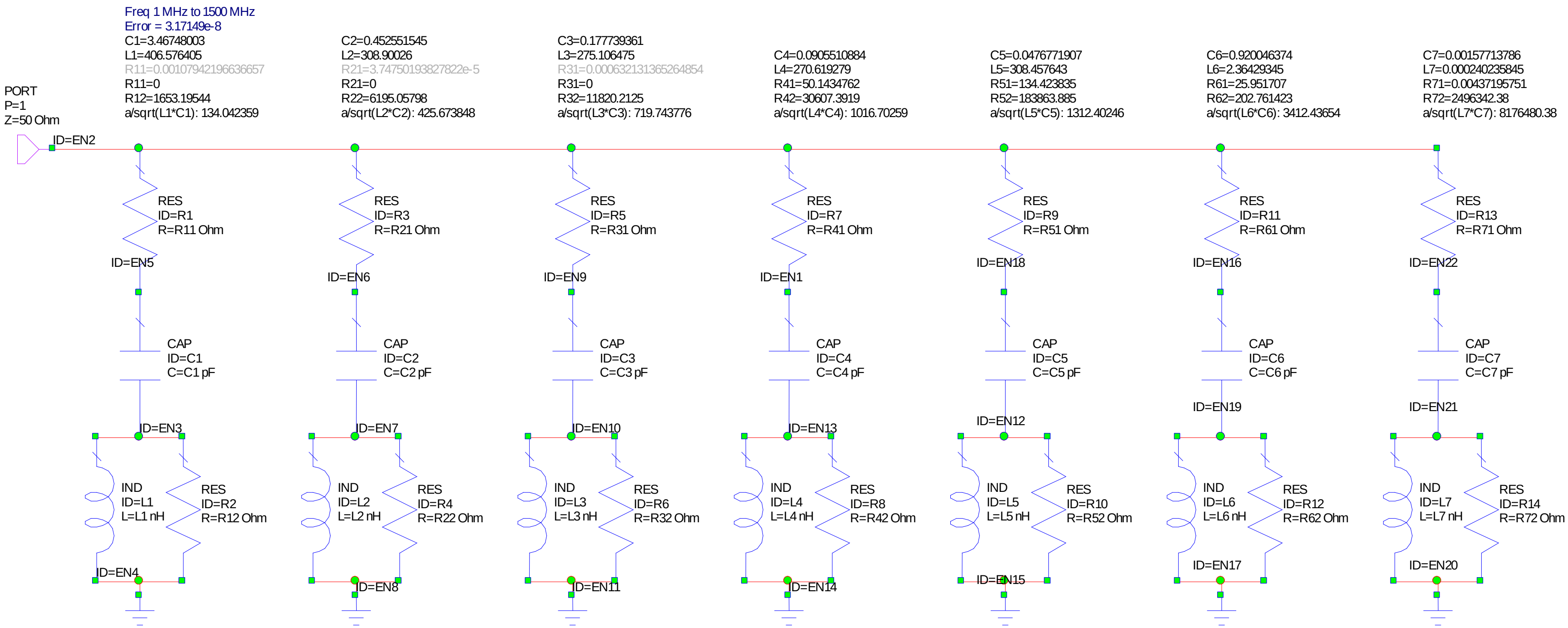


NEC4 – blue dash
Circuit – red

Impedance Comparison: NEC4 vs Equivalent Circuit

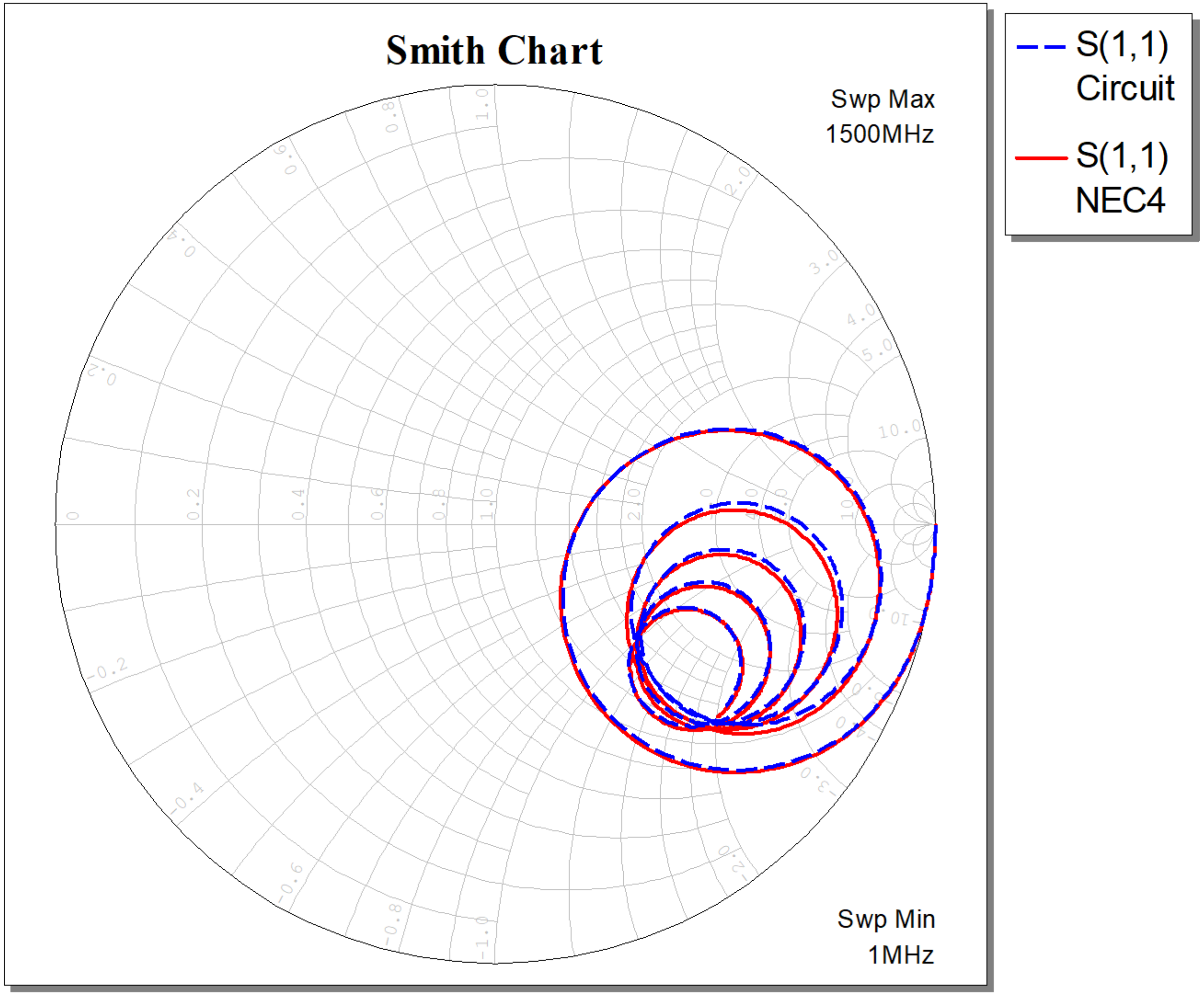


Example 2: K6OIK Broadband Equivalent Circuit for 1-meter Dipole (*L* / *d* = 50) from DC to 1.5 GHz

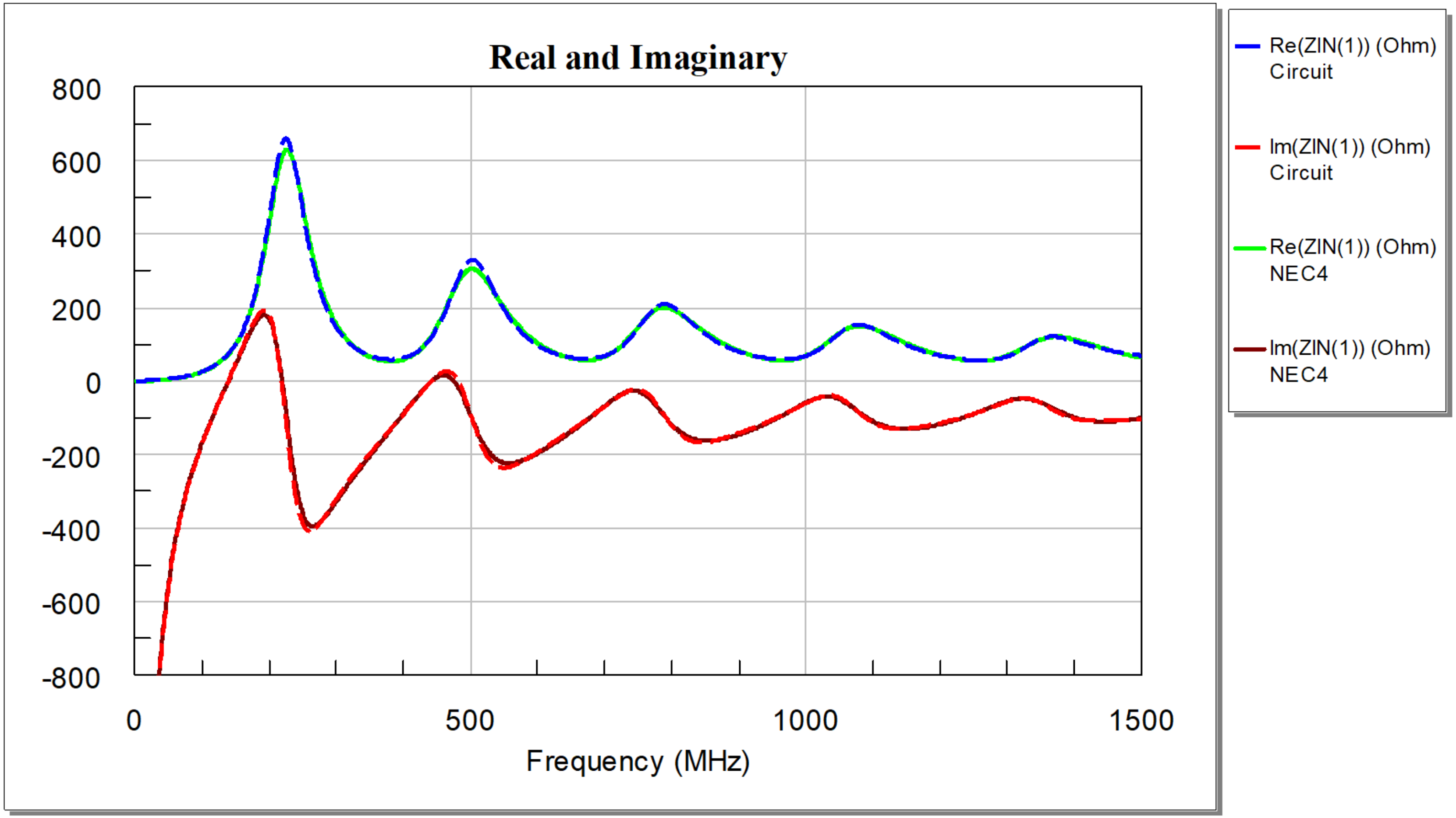


- **Introduced by the author in 2007**
- **Partial fraction expansion of dipole admittance**
- **A modification of Foster's 2nd canonical form**
- **More accurate than other broadband equivalent circuits for dipoles, viz. Hamid-Hamid (1997), Rambabu-Ramesh-Kalghatgi (1999), and Streable-Pearson (1981)**
- **Six stages sufficient to cover *d-c* to 1.5 GHz**

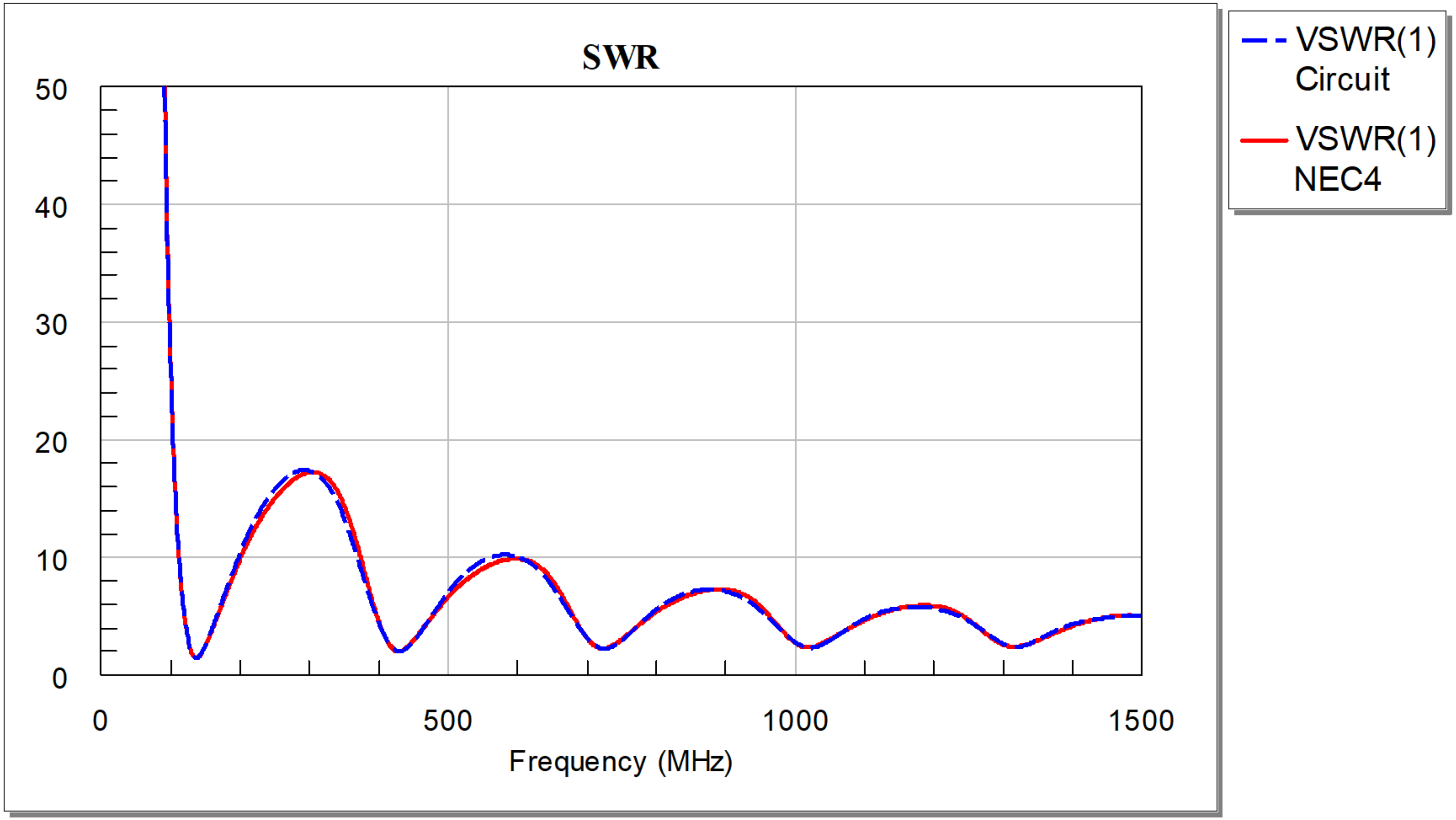
Impedance Comparison on Smith Chart



Impedance Comparison: NEC4 vs Equivalent Circuit



SWR of Equivalent Circuit

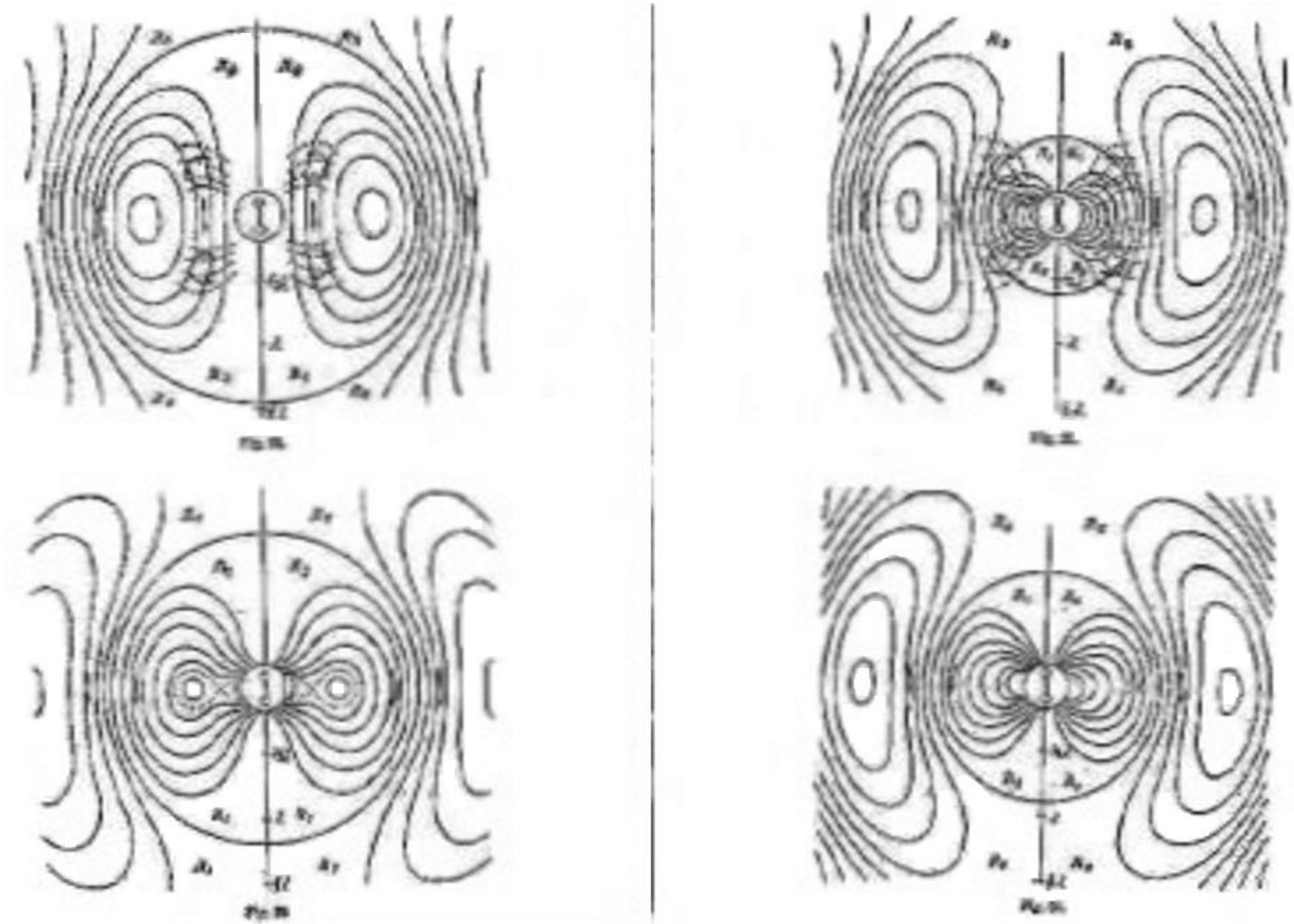


Comparison of Antenna Impedance Models

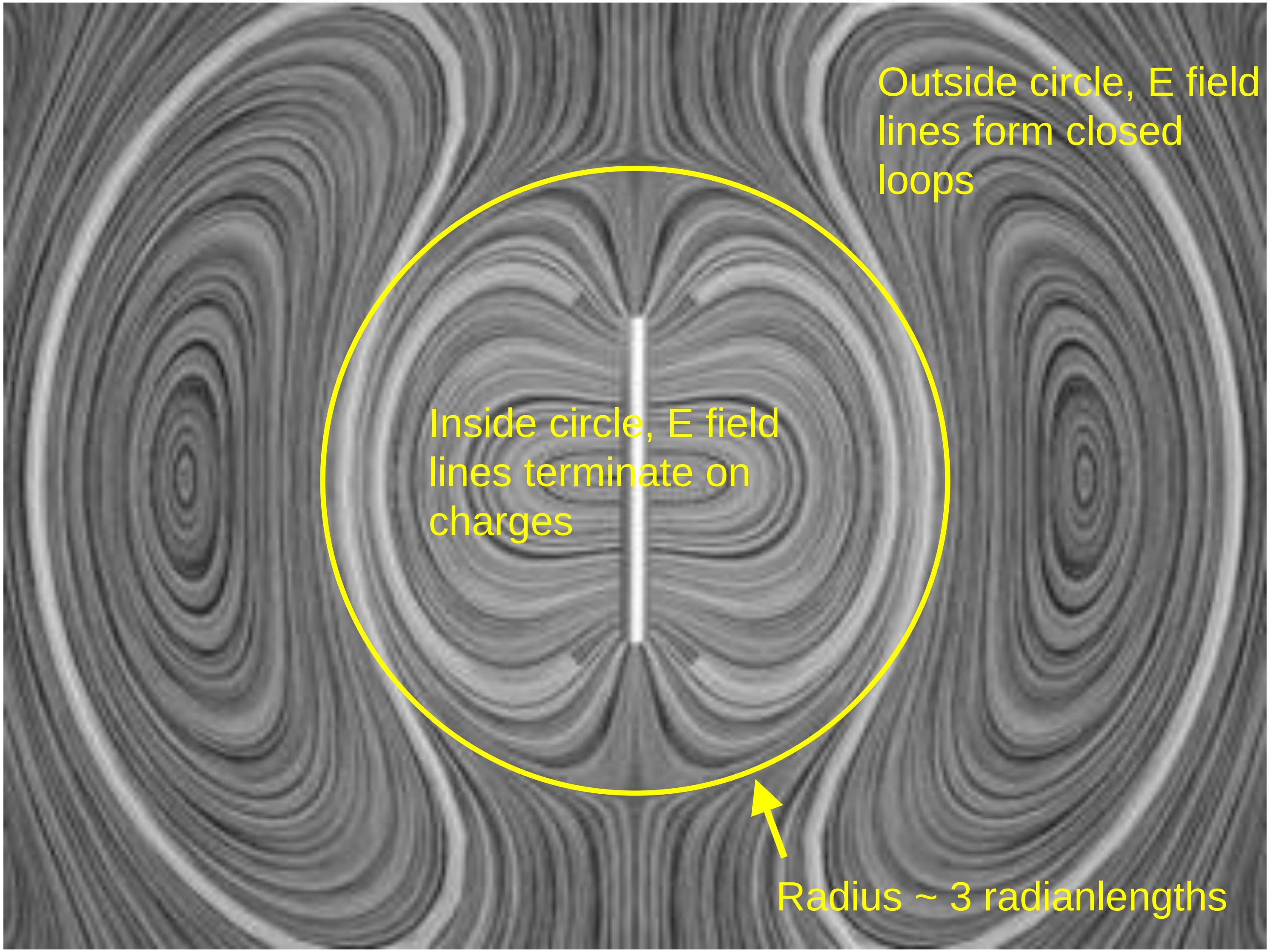
Antenna Impedance Model	Approximation Accuracy	Realizable Equivalent Circuit	Darlington Form	Element Types	Maximum Frequency Range
Series RLC	fair	yes	yes	*R, L, C*	0.94 f_0 to 1.05 f_0
Witt model	fair	no	no	*R*(*f*) and TL stub	0.6 f_0 to 1.2 f_0
Chu 3-Element	good	yes	yes	*R, L, C*	0.90 f_0 to 1.08 f_0
Tang-Tien-Gunn 4-Element	excellent	yes	yes	*R, L, C*	DC to 1.4 f_0
Schelkunoff 4-Element	excellent	yes	yes	*R,L,C*,TL	DC to 1.4 f_0
K6OIK 5-Element	excellent	yes	yes	*R, L, C*	DC to 1.4 f_0
Fosters 1st Form with small losses	poor, best near antiresonances	yes	no	*R, L, C*	no limit
Fosters 2nd Form with small losses	poor, best near resonances	yes	no	*R, L, C*	no limit
Long-Werner-Werner	fair	no	no	*R, C*, TL	5 octaves
Streable-Pearson	good	yes	no	*R, L, C*	no limit
K6OIK Broadband	excellent	yes	no	*R, L, C*	no limit
Schelkunoff TL cascade	fair	yes	yes	*R, L, C, TL*	limited
Spherical TE-TM modes	excellent	yes	no	*R, L, C*	no limit

Antenna Q

Heinrich Hertz's Drawings of Electric Fields of a Dipole circa 1888



Generation of Dipole Fields



Electric and Magnetic Fields of a Dipole

- **Fields of dipole source in free space, or monopole over perfect electrical conductor (PEC) plane**

$$H_r = H_\theta = 0$$

$$H_\phi = j\frac{kI_0 l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$E_r = \eta\frac{kI_0 l\cos\theta}{4\pi r}\left(\frac{2}{kr}\right)\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

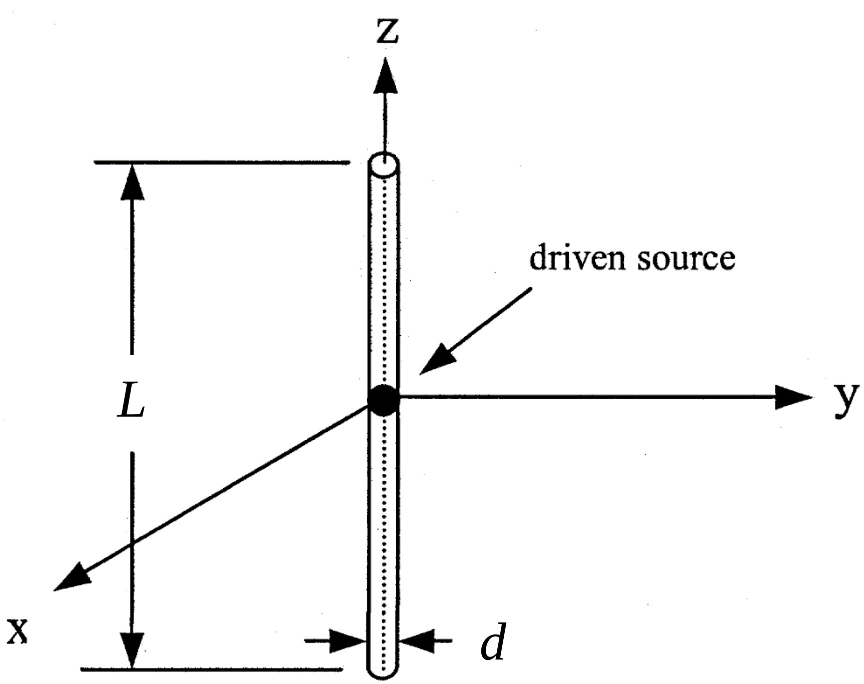
$$E_\theta = j\eta\frac{kI_0 l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr}$$

$$E_\phi = 0$$

Near field terms

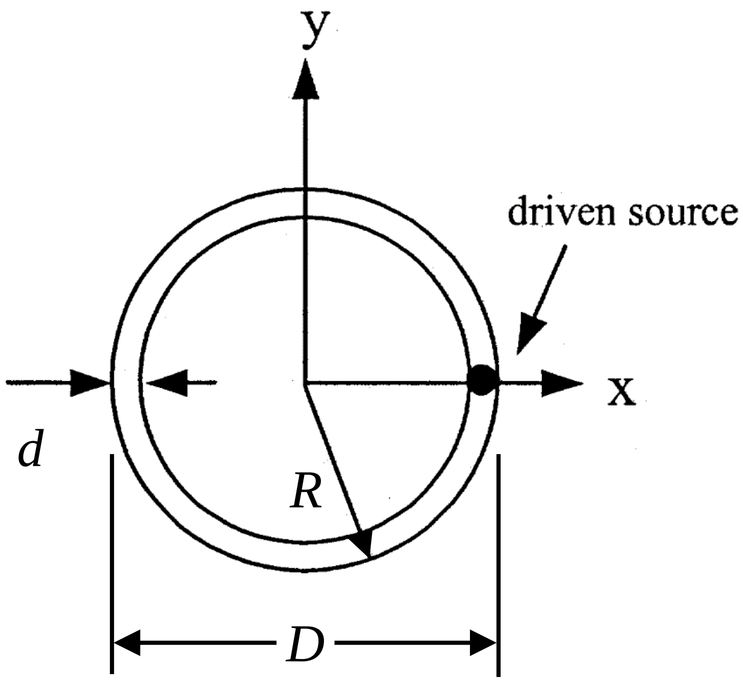
One radianlength defined as $r = 1/k = \lambda/2\pi$ is the distance at which far field and near field terms are equal.

Q of Small Dipole from Electromagnetic Field Analysis



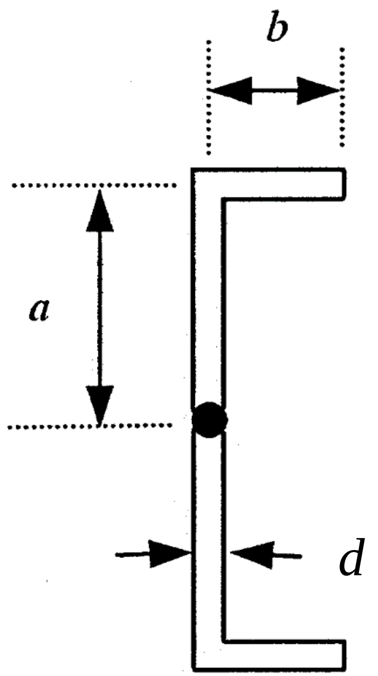
$$Q_{dipole} \approx \frac{6\left[\ln\left(\frac{L}{d}\right)-1\right]}{\pi^3\left(\frac{f\,L}{c}\right)^3}$$

Q of Small Loop from Electromagnetic Field Analysis



$$Q_{loop} \approx \frac{6\ln\left(\frac{D}{d}\right)}{\pi^4\left(\frac{fD}{c}\right)^3}$$

Q of Inverted-L from Electromagnetic Field Analysis



$$Q_{inverted-L} \approx \left(\frac{3}{4}\right) \frac{\left[\ln\left(\frac{2a}{d}\right)-1\right] + \frac{b}{a}\left[\ln\left(\frac{4b}{d}\right)-1\right]}{\pi^3\left(\frac{f}{c}\right)^3 a(a+b)^2}$$

Formulas for Q from Feedpoint Impedance

- **Series RLC equivalent circuit**

$$Q(f) = \frac{|X(f)|}{R(f)}$$

- **Geyi (2000, 2003)**

$$Q(f) = \frac{f}{2R(f)} \left[\frac{dX(f)}{df} \pm \frac{X(f)}{f} \right]$$

- **Yaghjian and Best (2003, 2005)**

$$Q(f) = \frac{f}{2R(f)} \sqrt{\left(\frac{dR(f)}{df} \right)^2 + \left(\frac{dX(f)}{df} + \frac{|X(f)|}{f} \right)^2}$$

- **Hansen (2007)**

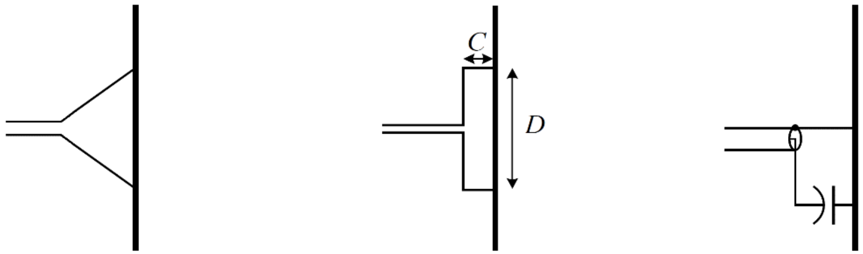
$$Q(f) = \frac{f}{2R(f)} \left| \frac{dX(f)}{df} \right|$$

Counterexamples to these formulas exist. No general formula for computing *Q* from *Z* exists. Antenna *Q* must be computed from field expressions.

Impedance Matching

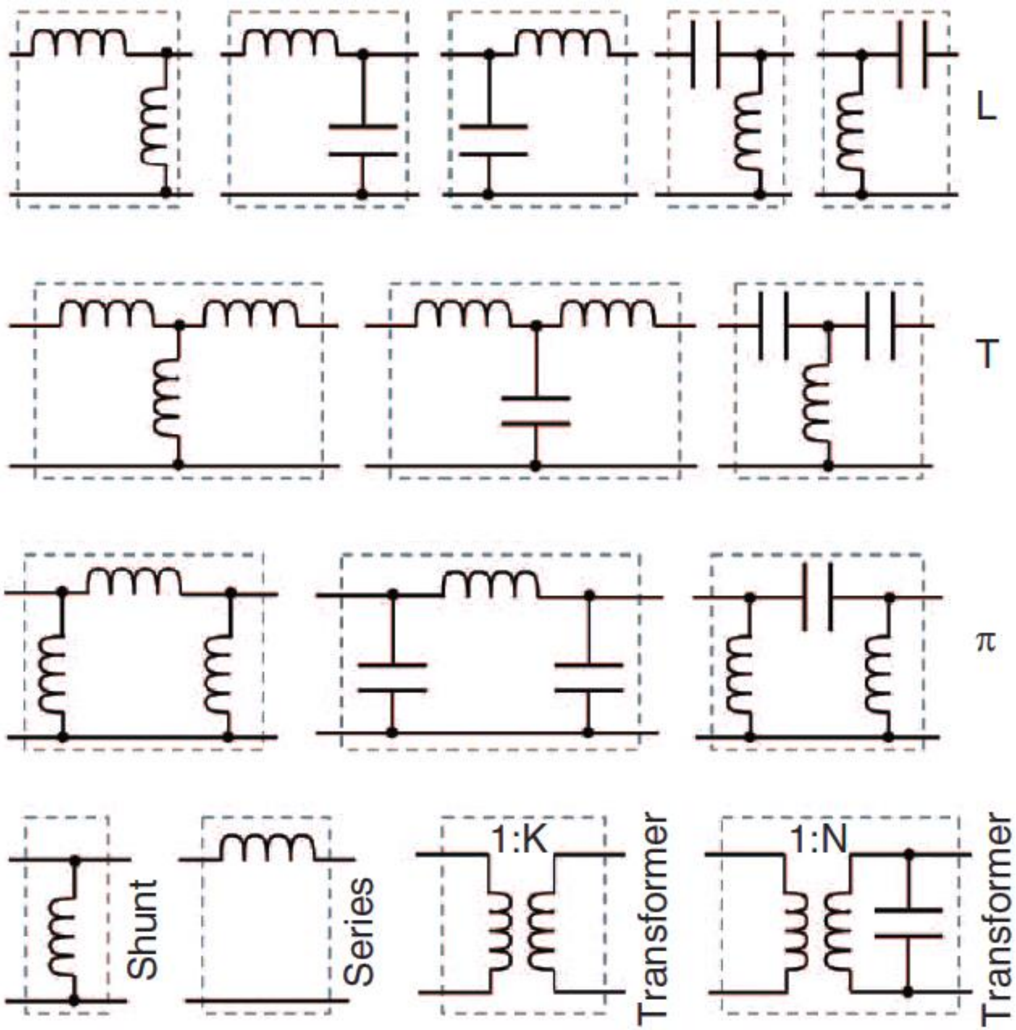
Impedance-Matching Techniques

- **Transmitter ATUs**
- **Feedpoint networks (today's subject)**
  - Lumped-element networks: L, Pi, T, ladder, twin-T, bridged-T, lattice
  - Transmission line networks: cascaded sections and stubs
  - Active non-Foster circuits
- **Structures built into antenna or at feed point**
  - Delta match, Gamma match, T match



- **External structures**
  - Caps and radomes of special materials

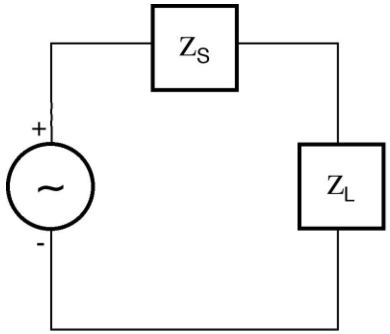
Common Lumped Match Network Topologies



- **Common networks**
  - L networks
    - 4 shown of 8 possible
  - T networks
    - 3 shown of 8 possible
  - Pi networks
    - 3 shown of 8 possible
- **Other lumped networks**
  - General ladder networks
  - Twin-T networks
  - Bridged-T networks
  - Lattice networks

Conjugate Matching versus Z_0 Matching

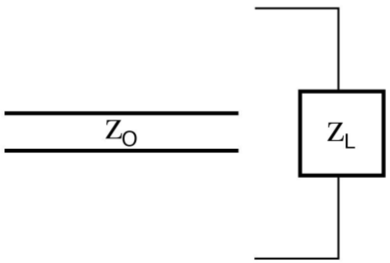
- **Conjugate Impedance Matching**



$$Z_s = Z_L^*$$

- **Maximizes power delivery at a junction – transmitter-to-line or line-to-antenna, but**
- **Does NOT necessarily maximize power delivery at other junctions**
- **Does NOT necessarily prevent reflections at the junction**
- **Inherently single or discrete frequency; broadband not possible unless non-Foster**

- **Z_0 Impedance Matching**

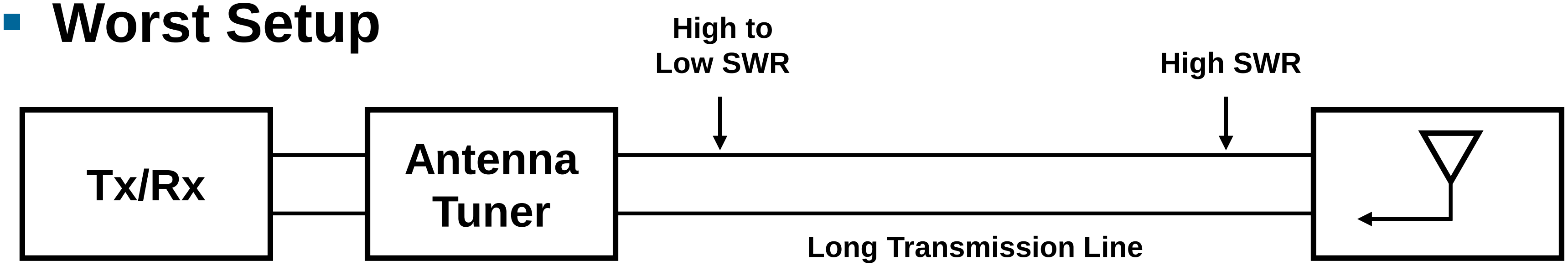


$$Z_0 = Z_L$$

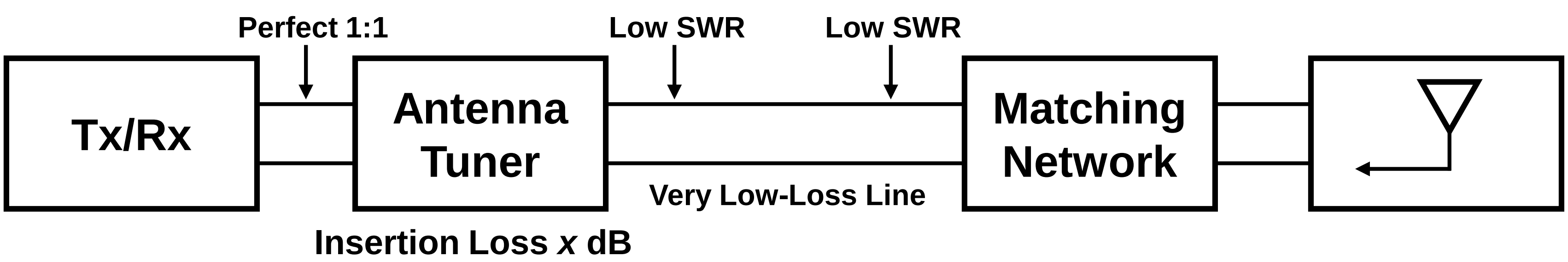
- **Best criterion to use at the load end of a transmission line**
- **Minimizes reflections on the line, thereby avoids additional loss due to SWR**
- **Maximizes system bandwidth**

Where Should a Matching Network Go?

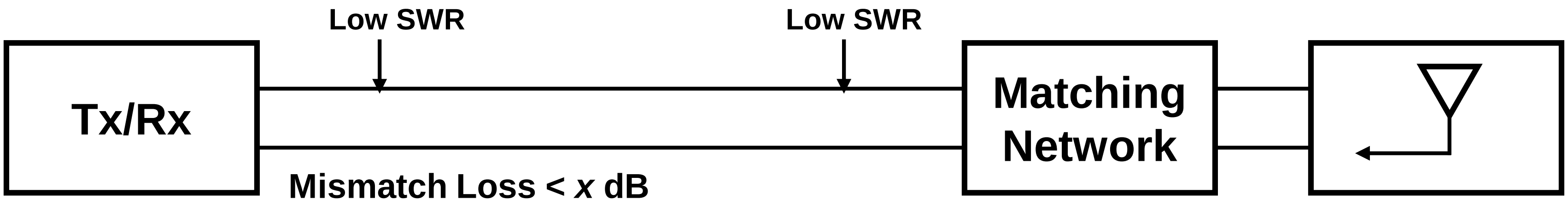
- Worst Setup



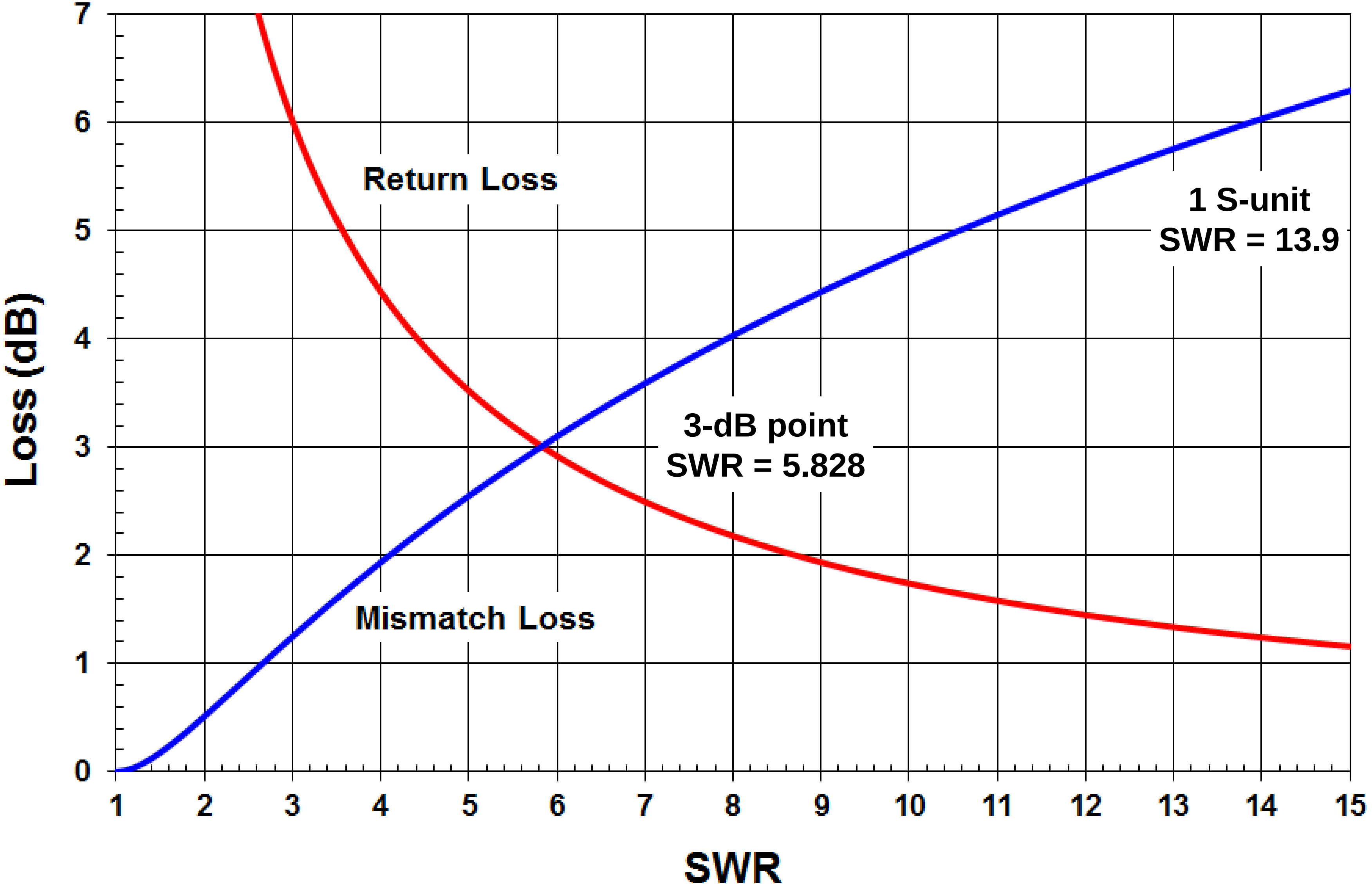
- Good Setup



- Best Setup

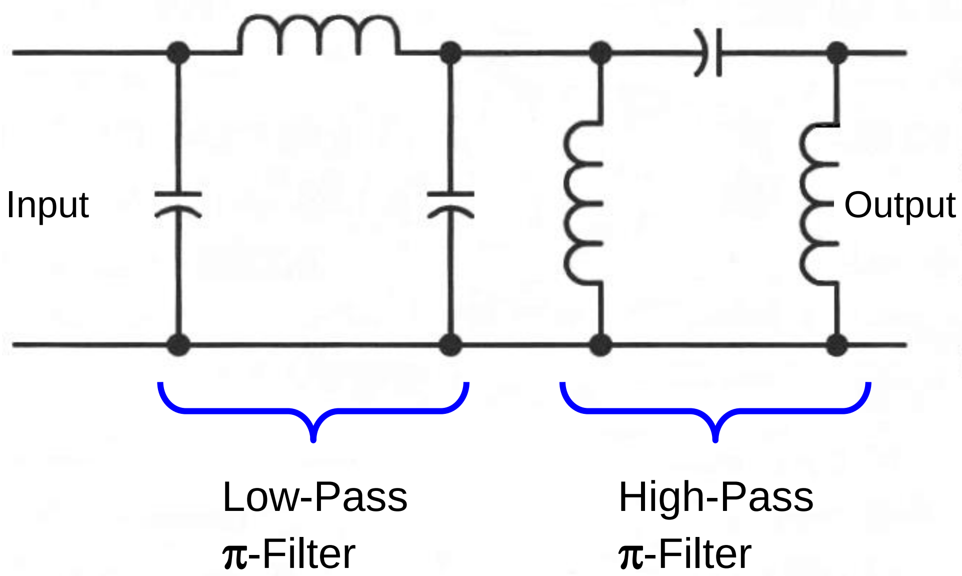


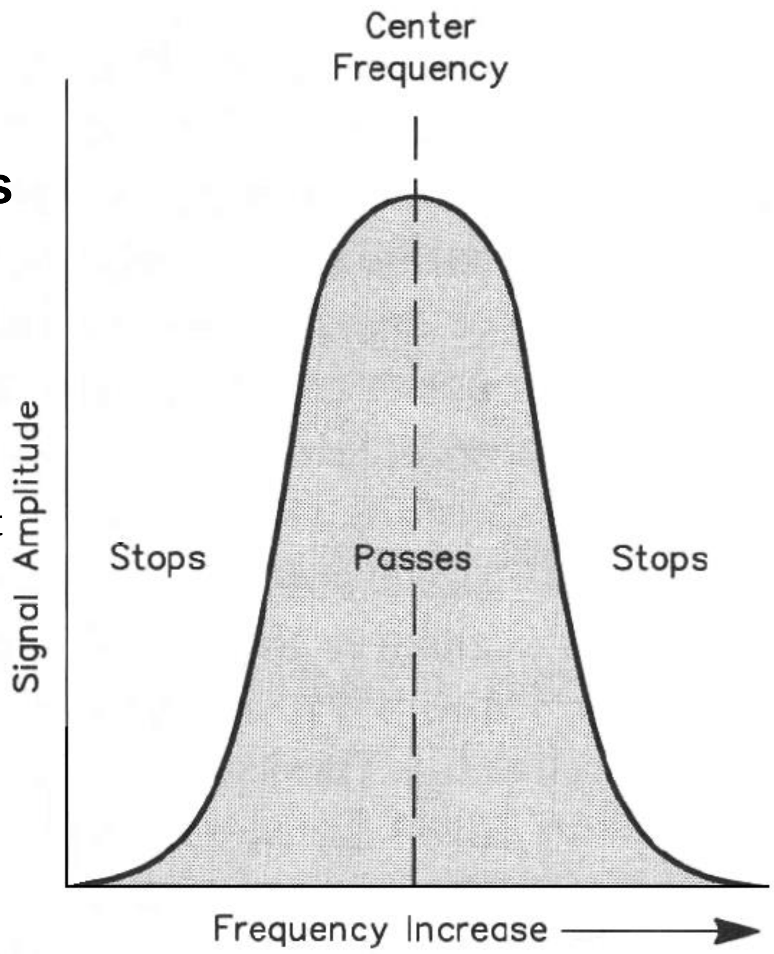
Reflection Losses versus SWR



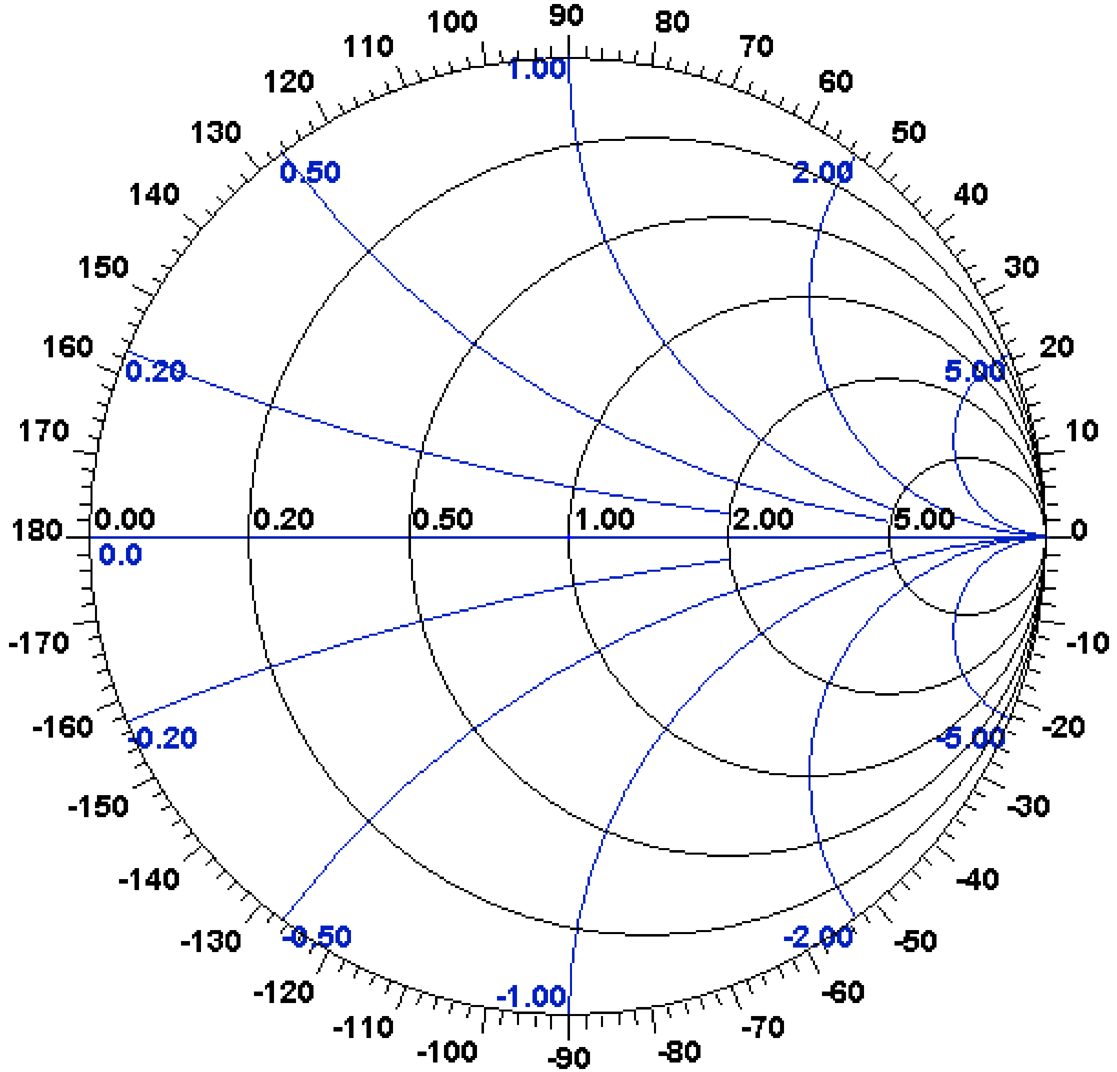
A Band-Pass Reflection Filter

- **Most filters are reflection filters**
- **Filter elements are pure reactances**



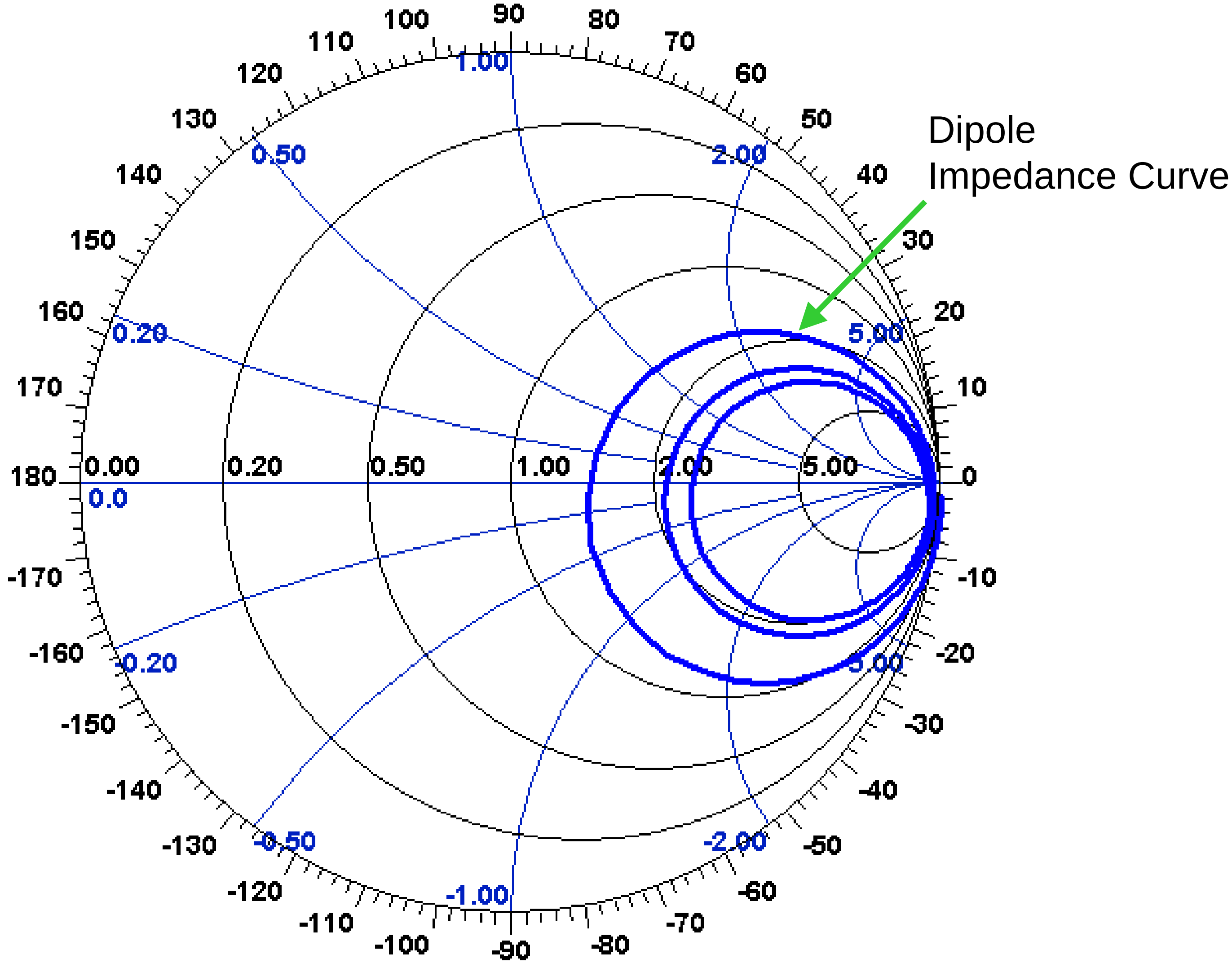


Smith Chart

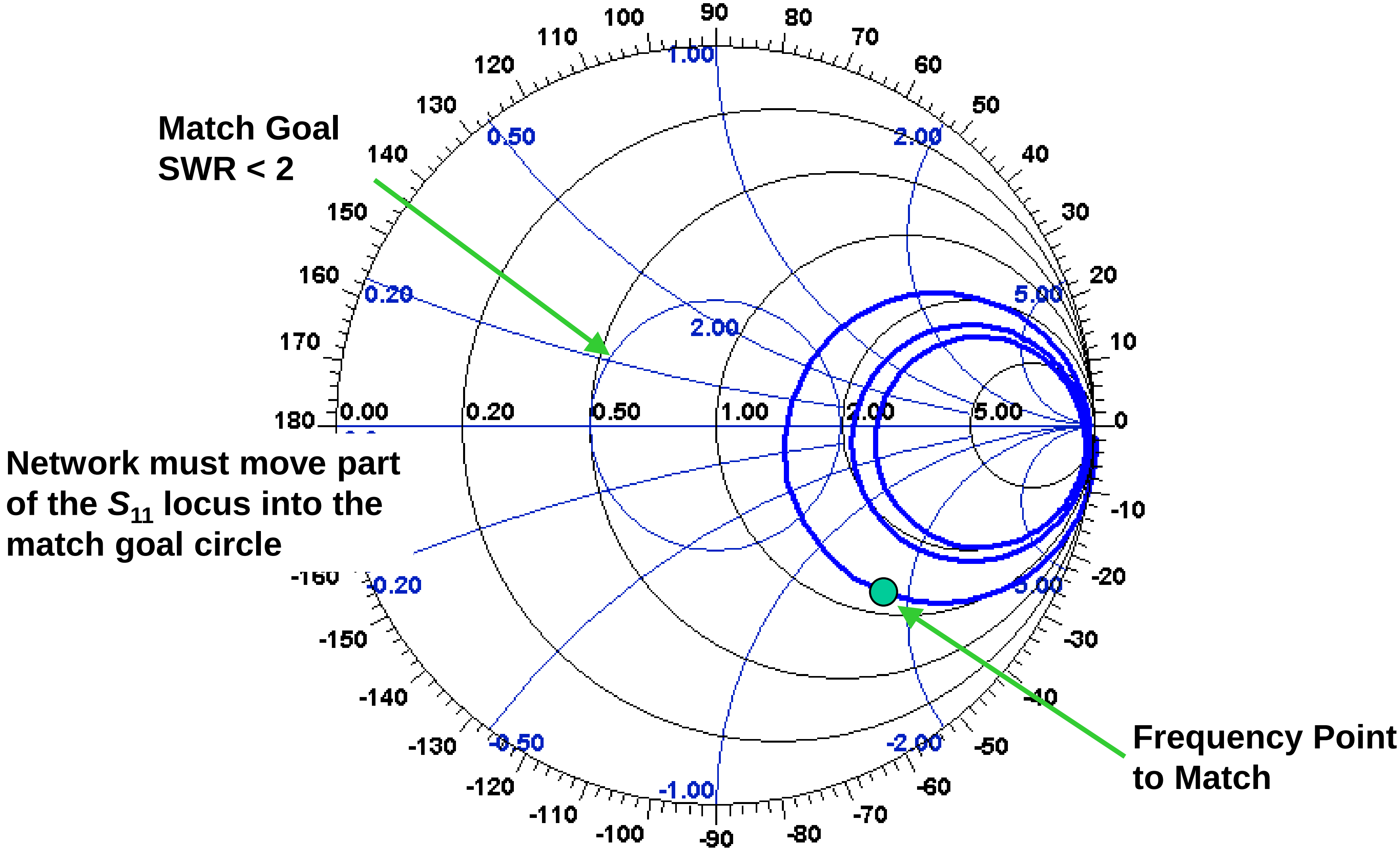


90
100
80
110
70
120
60
130
50
140
40
150
30
160
20
170
10
180
0
-170
-10
-160
-20
-150
-30
-140
-40
-130
-50
-120
-60
-110
-70
-100
-80
-90
1.00
0.50
2.00
0.20
5.00
0.00
0.20
0.50
1.00
2.00
5.00
0.0
-0.20
-5.00
-0.50
-2.00
-1.00

Smith Chart + Data



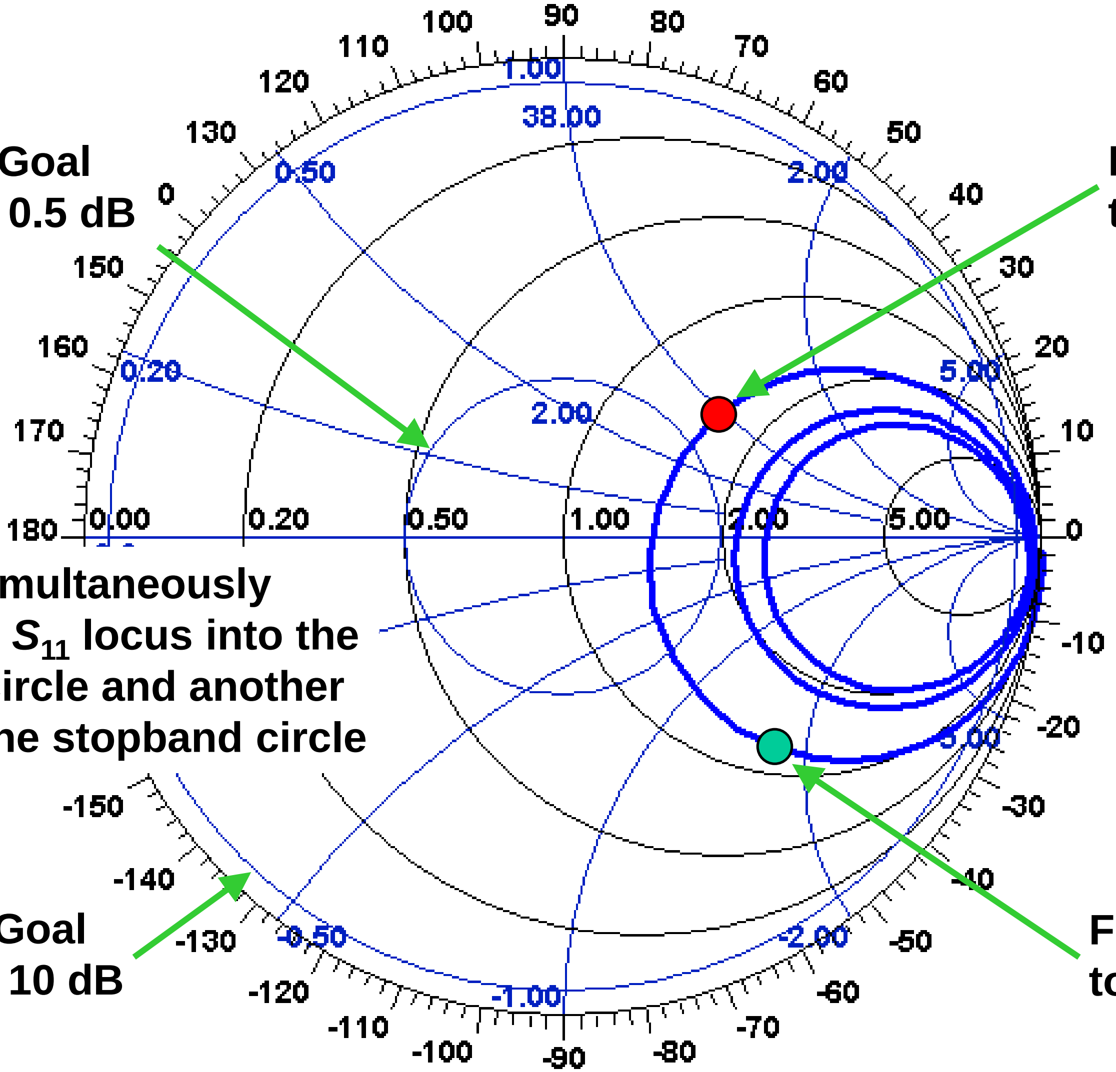
Smith Chart + Data + Match Goal



Smith Chart + Data + Filter Goals

Filter Passband Goal Insertion Loss < 0.5 dB

Frequency Point to Stop



Network must simultaneously move part of the S_{11} locus into the passband goal circle and another part to outside the stopband circle

Filter Stopband Goal Insertion Loss > 10 dB

Frequency Point to Pass

Single-Frequency Matching

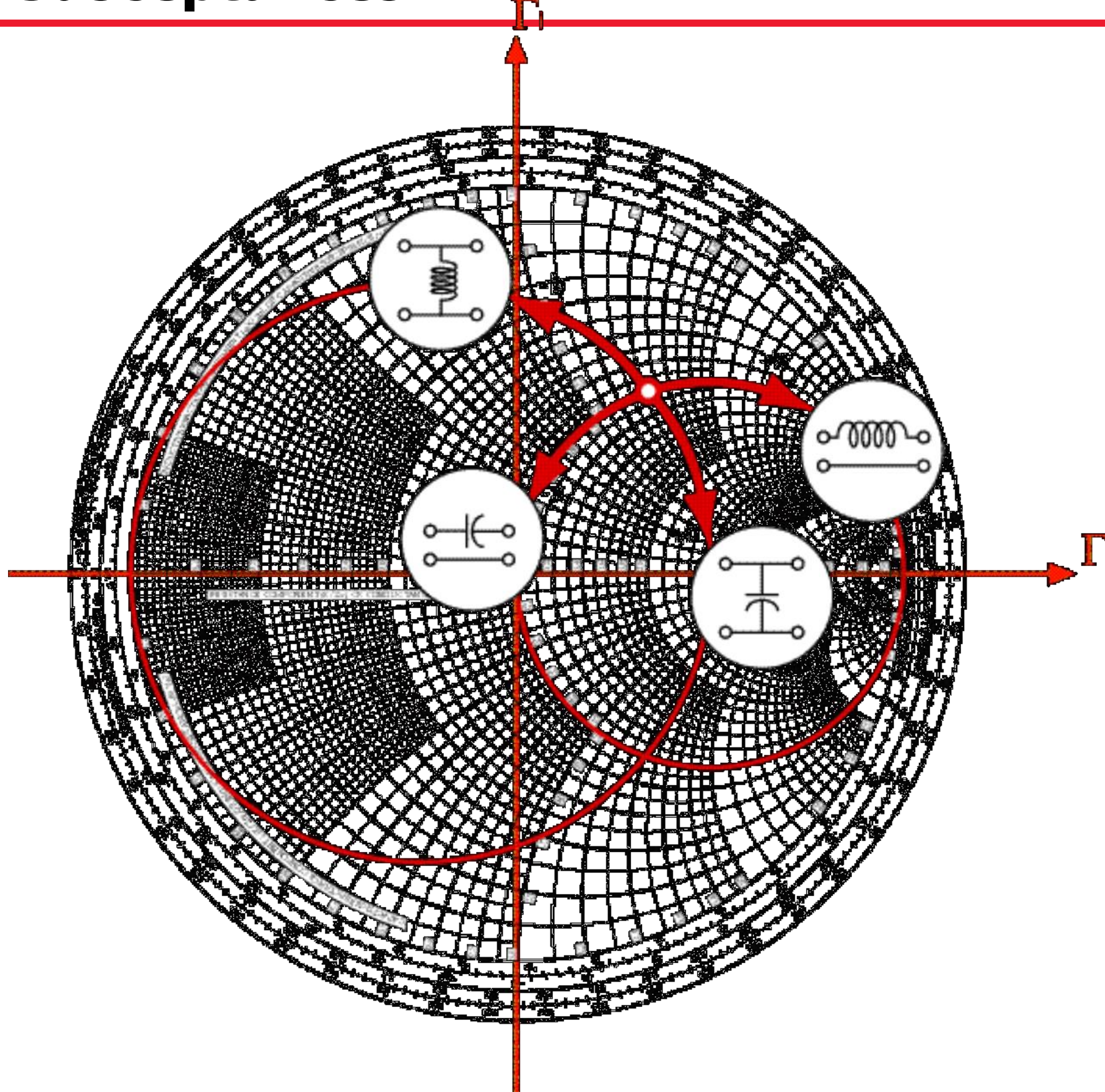
Single-Frequency Matching

- Eight canonical L networks
- Transmission line stubs
- Transmission line sections
- Quarter-wave sections
- Alternated-line, nonsynchronous match

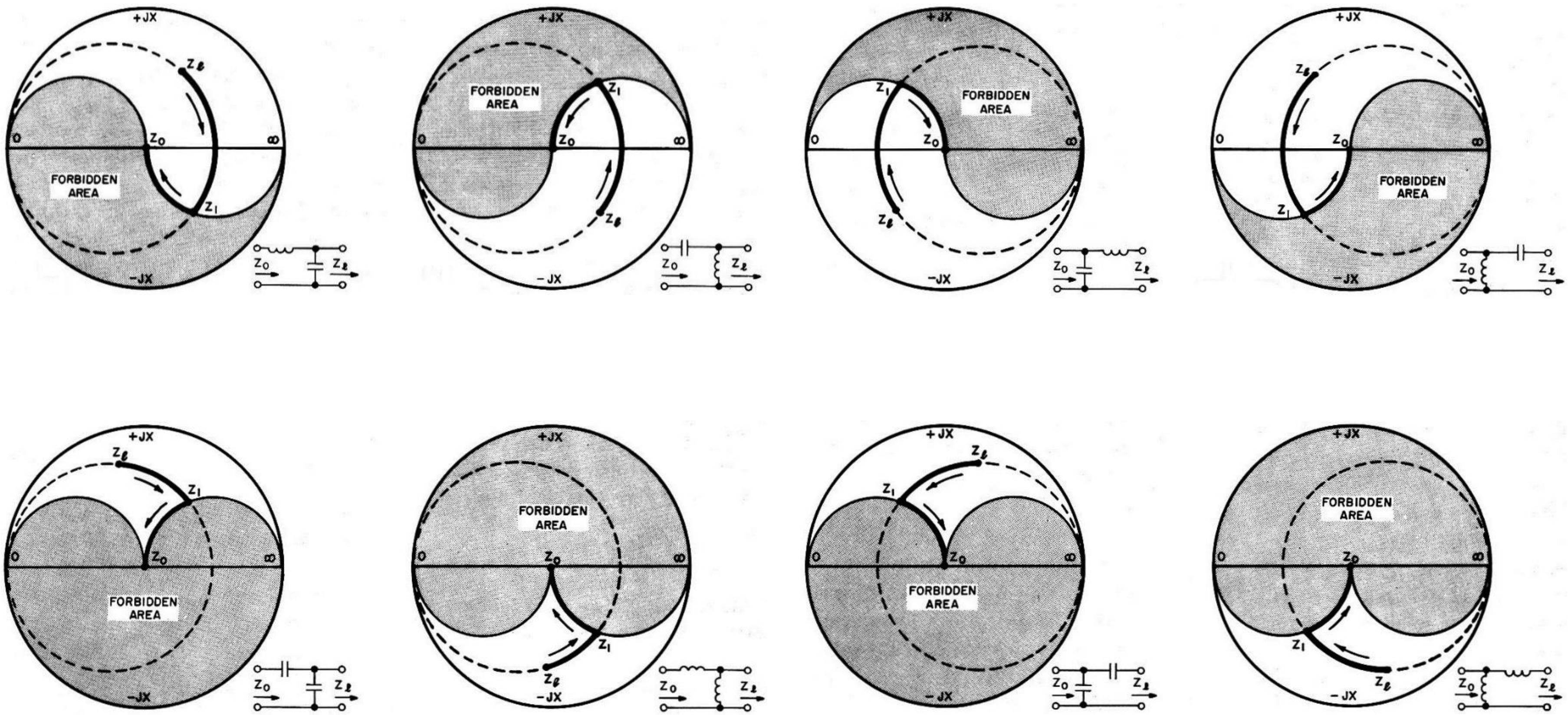
2-Element Ladder Networks

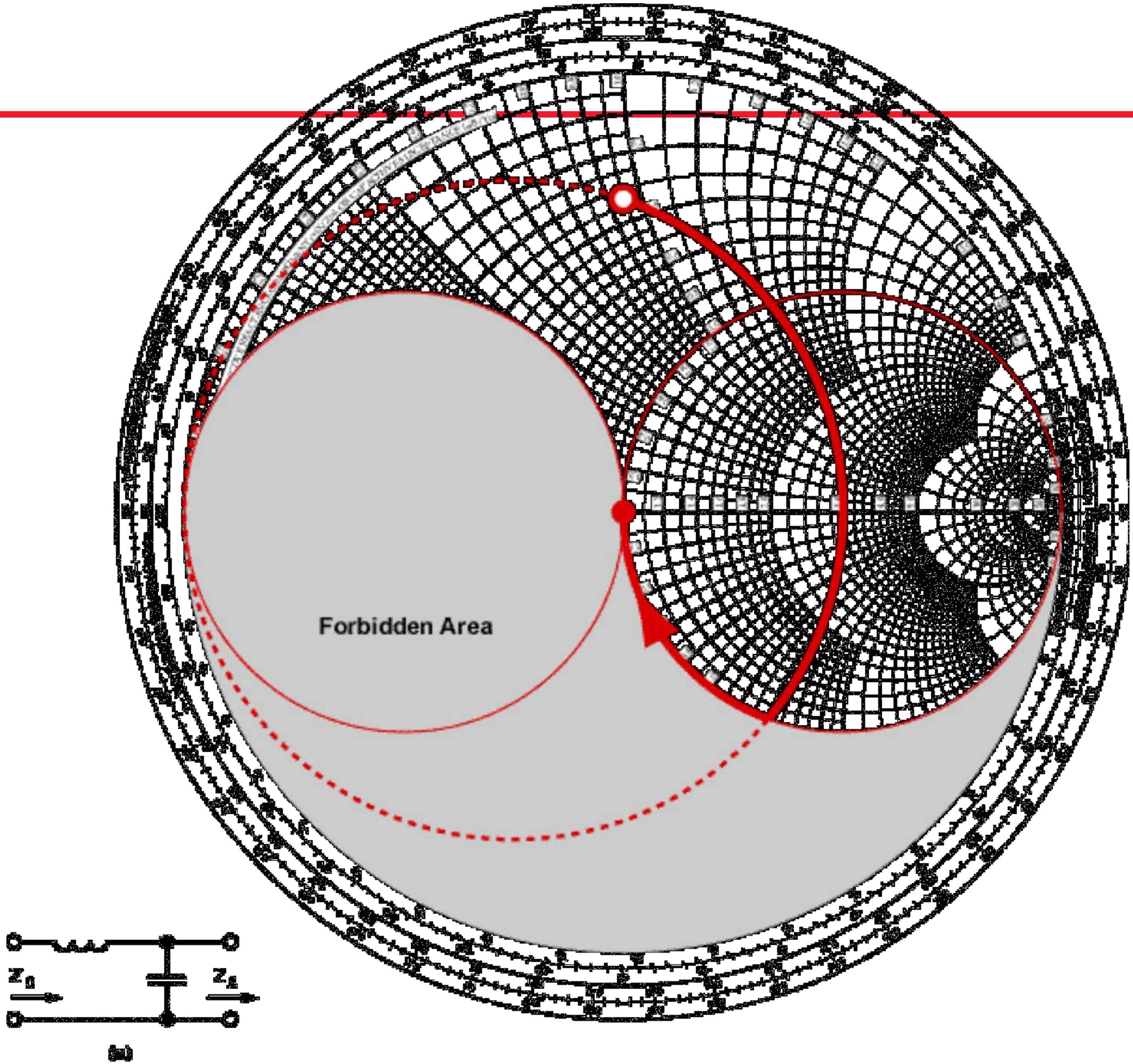
L networks

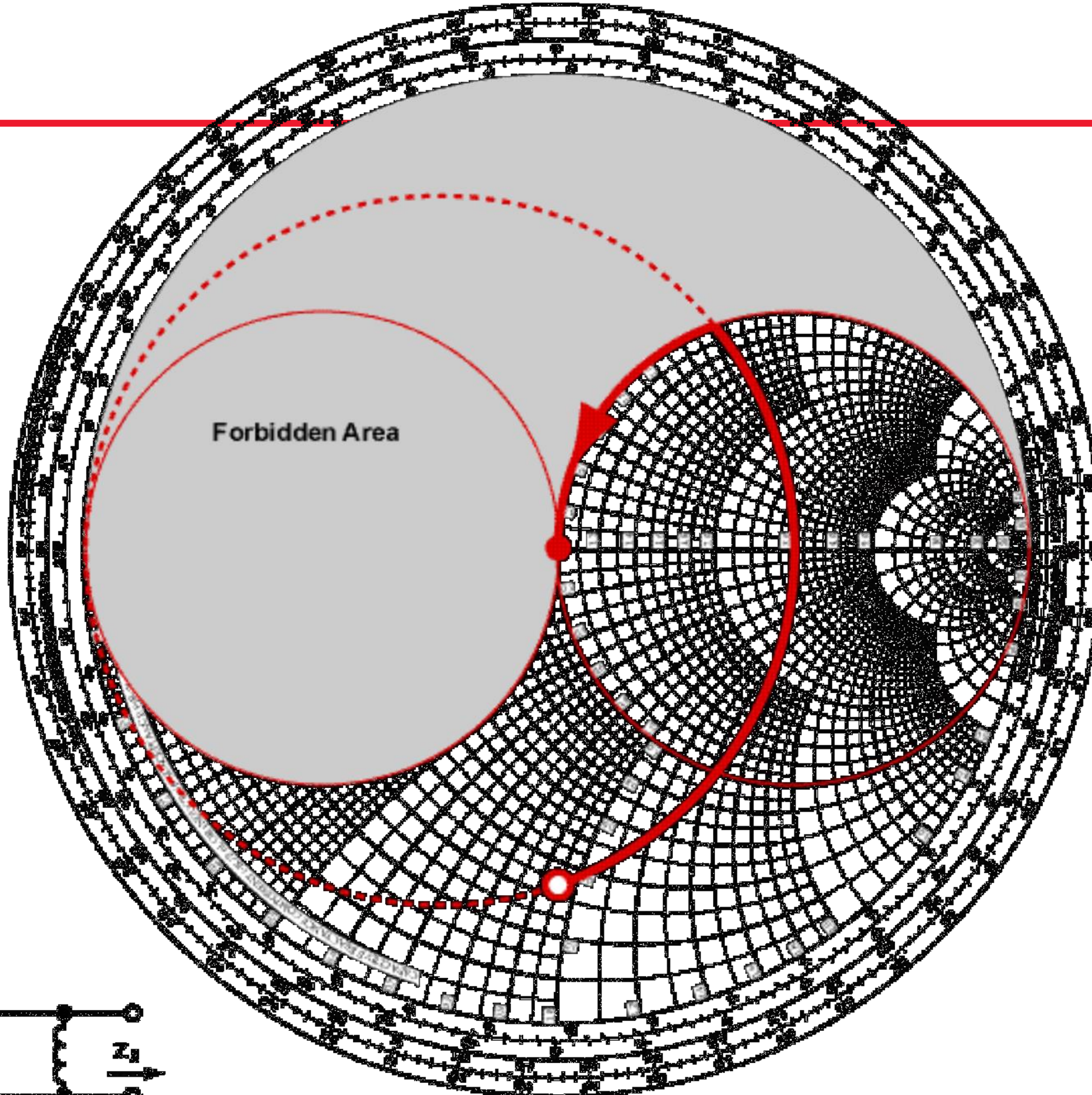
Smith Chart: Effect of Adding Series Reactances or Shunt Susceptances



Eight L Networks and Their Match Regions




Forbidden Area
(a)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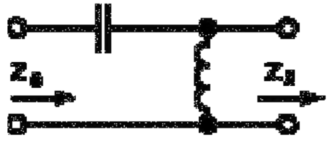

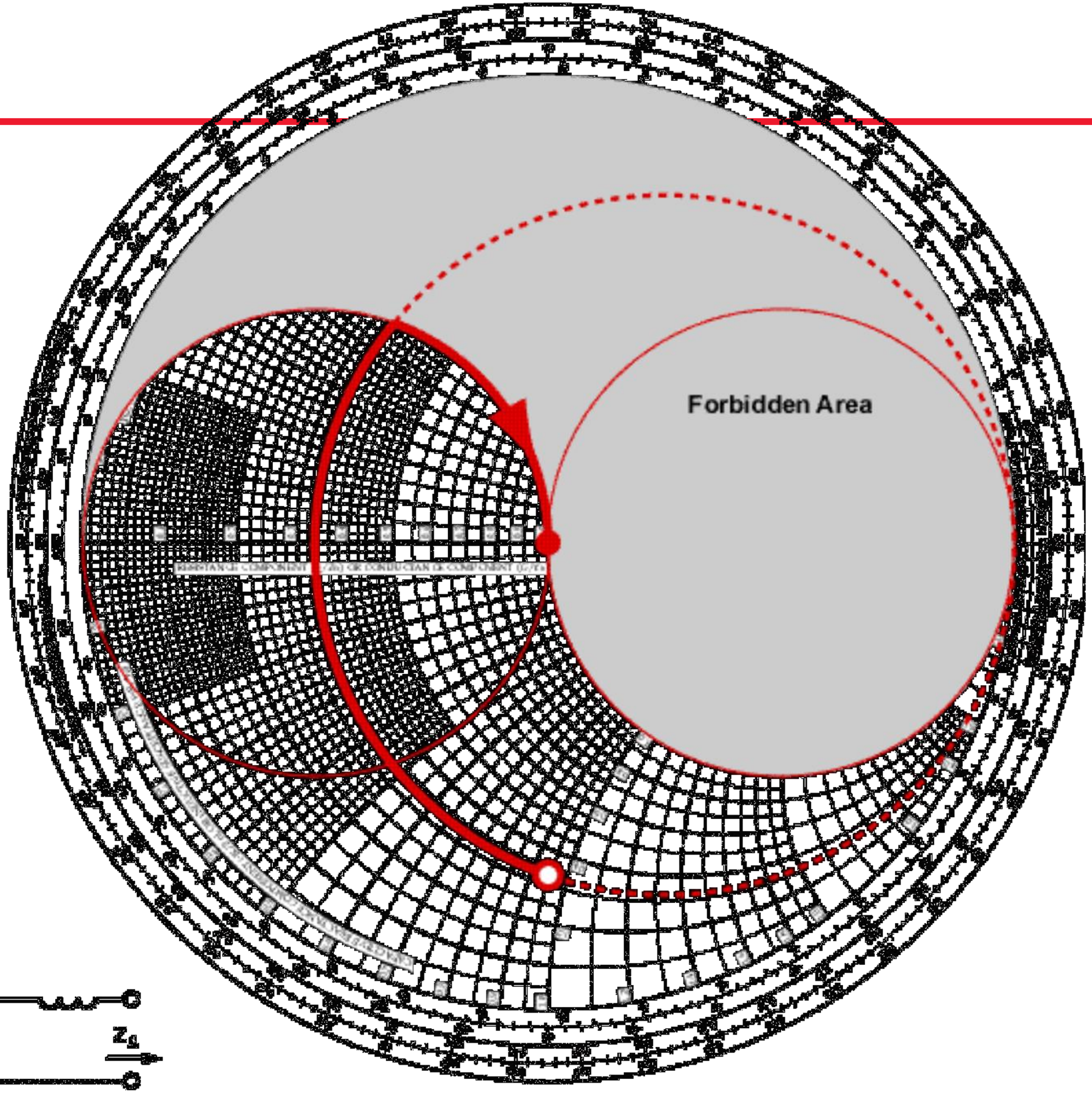
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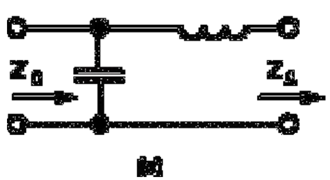

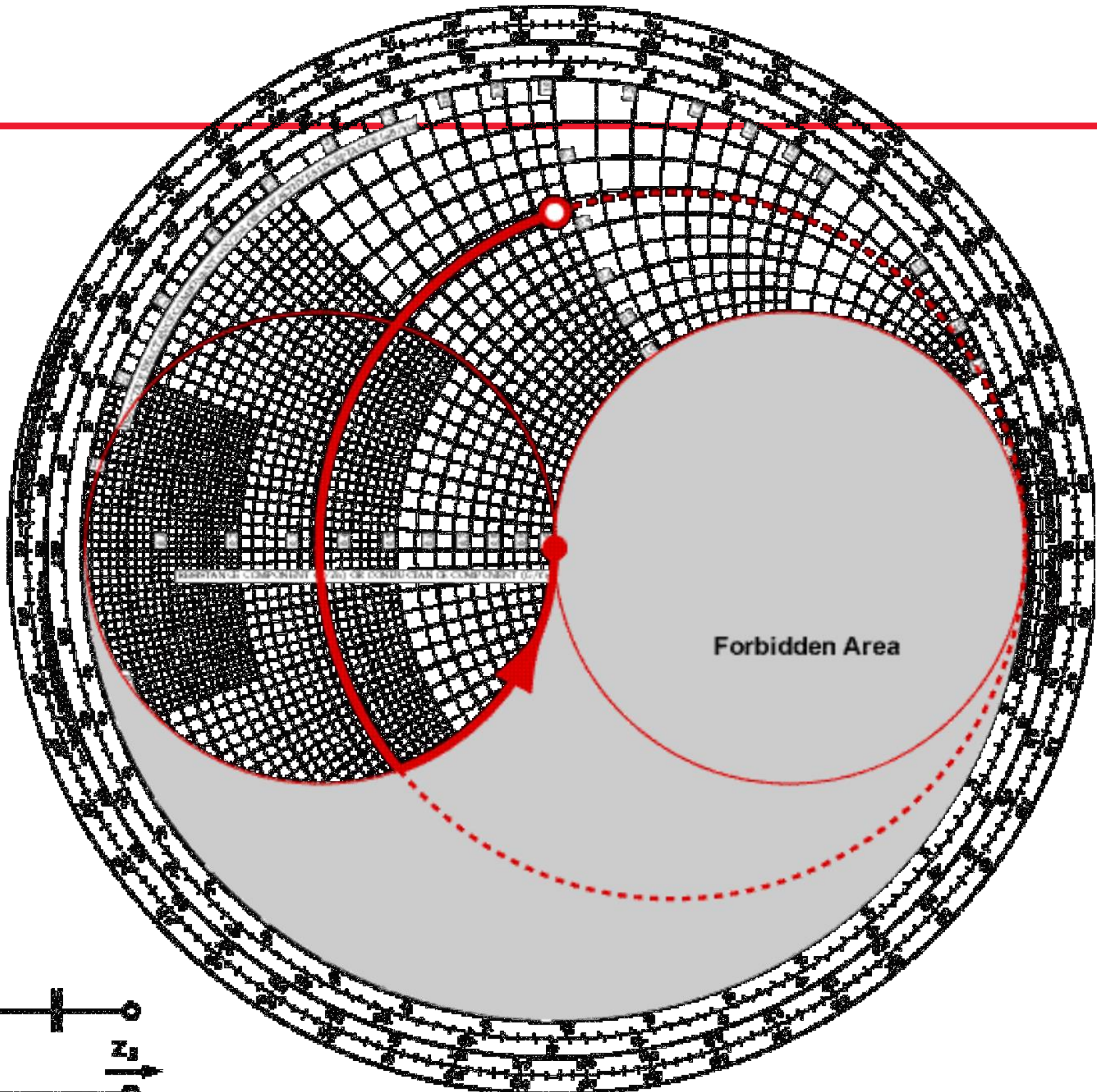
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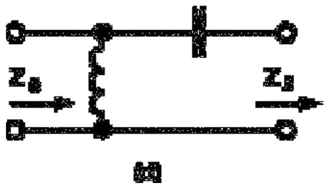
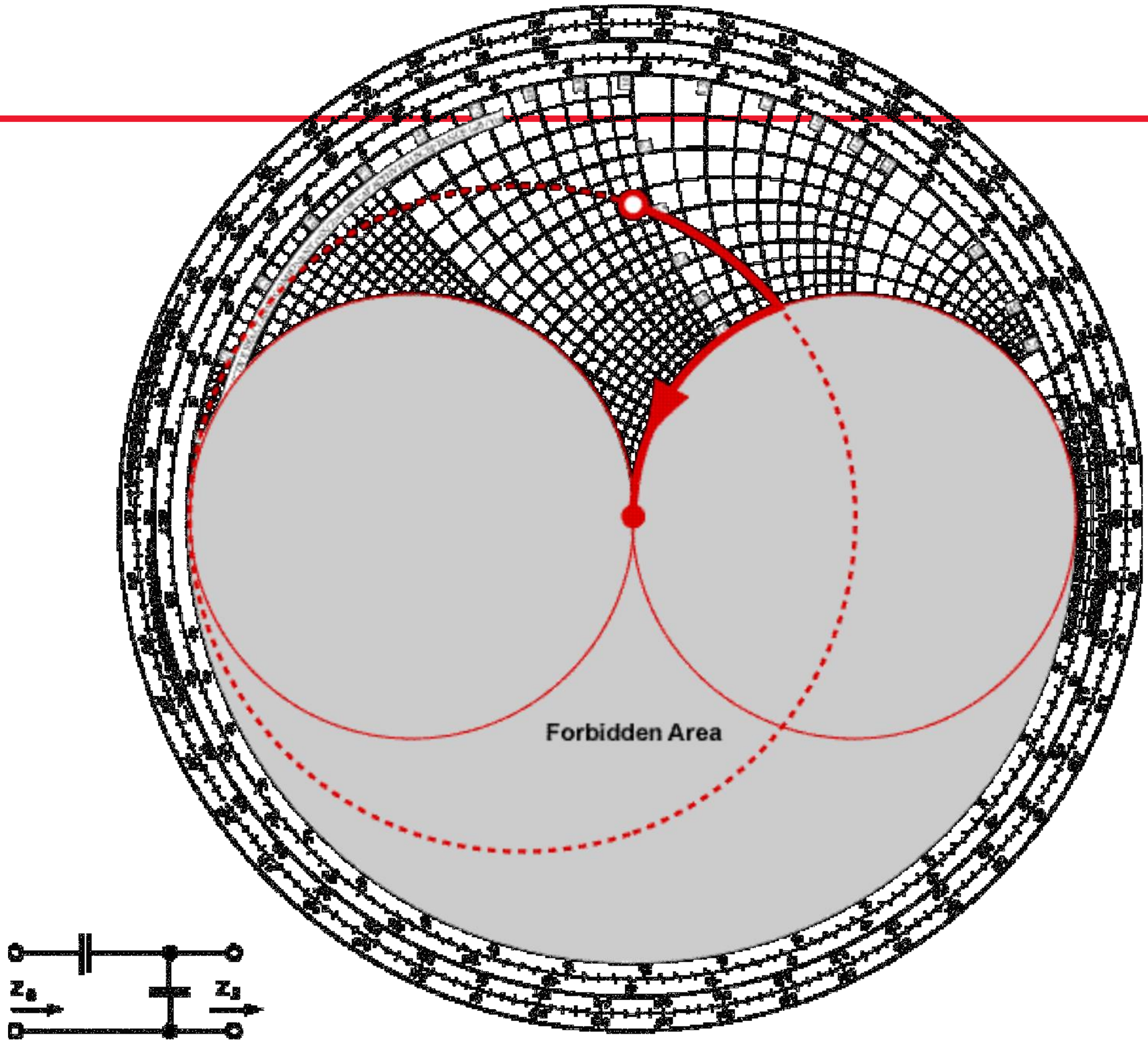
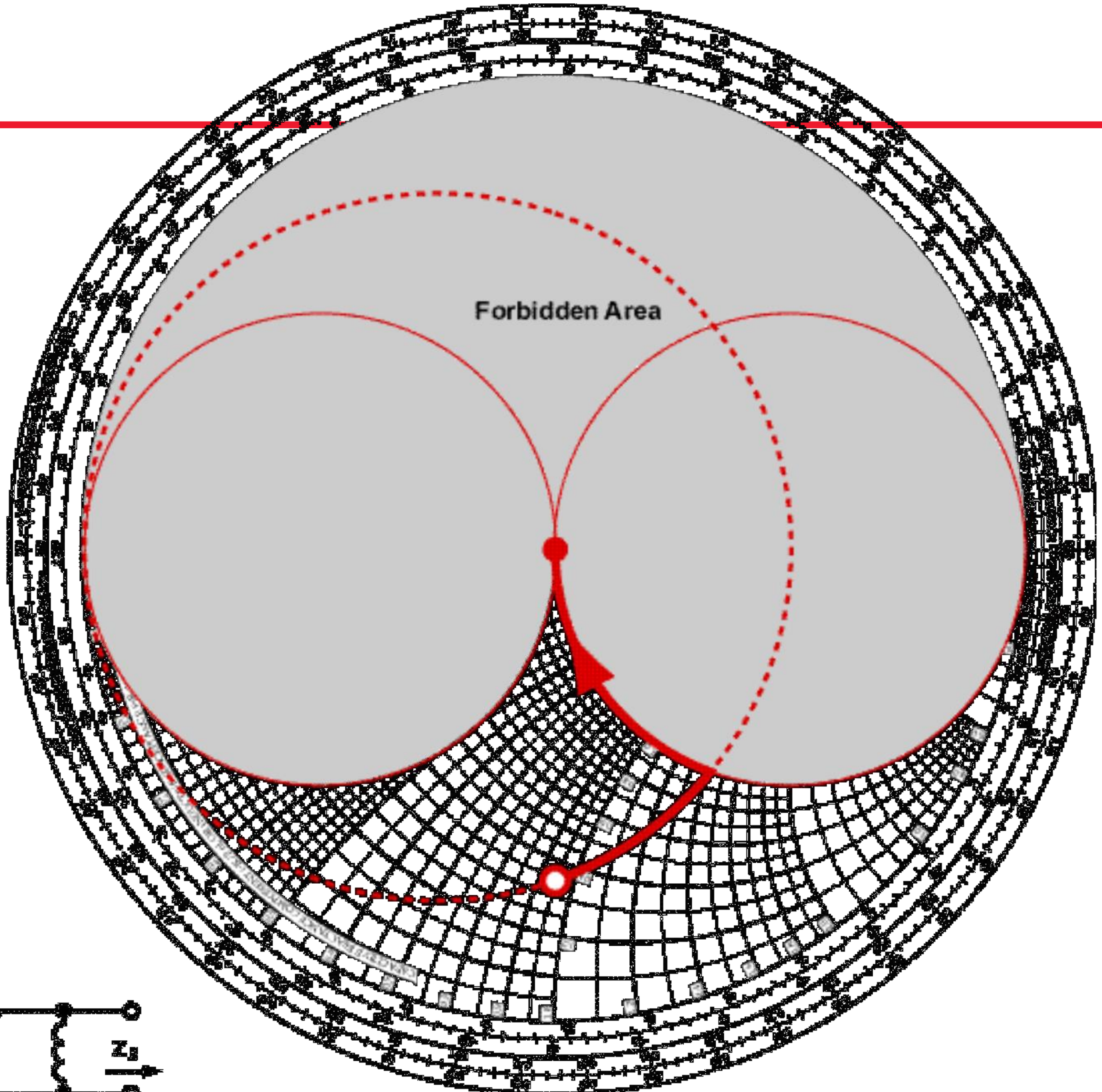




(a)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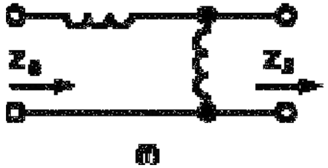

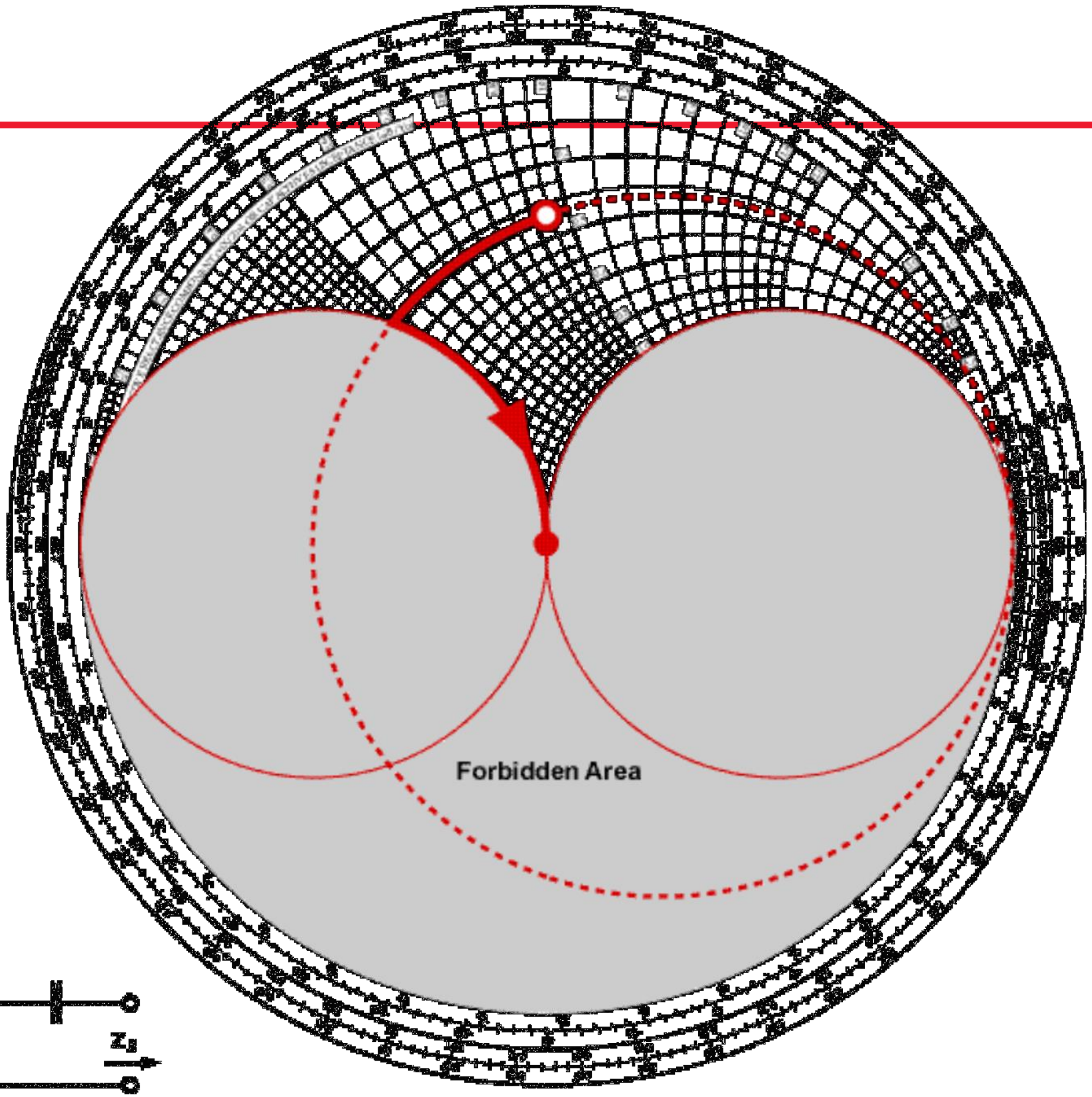
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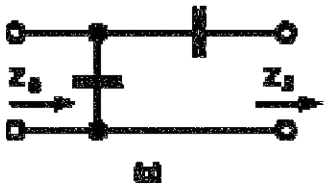

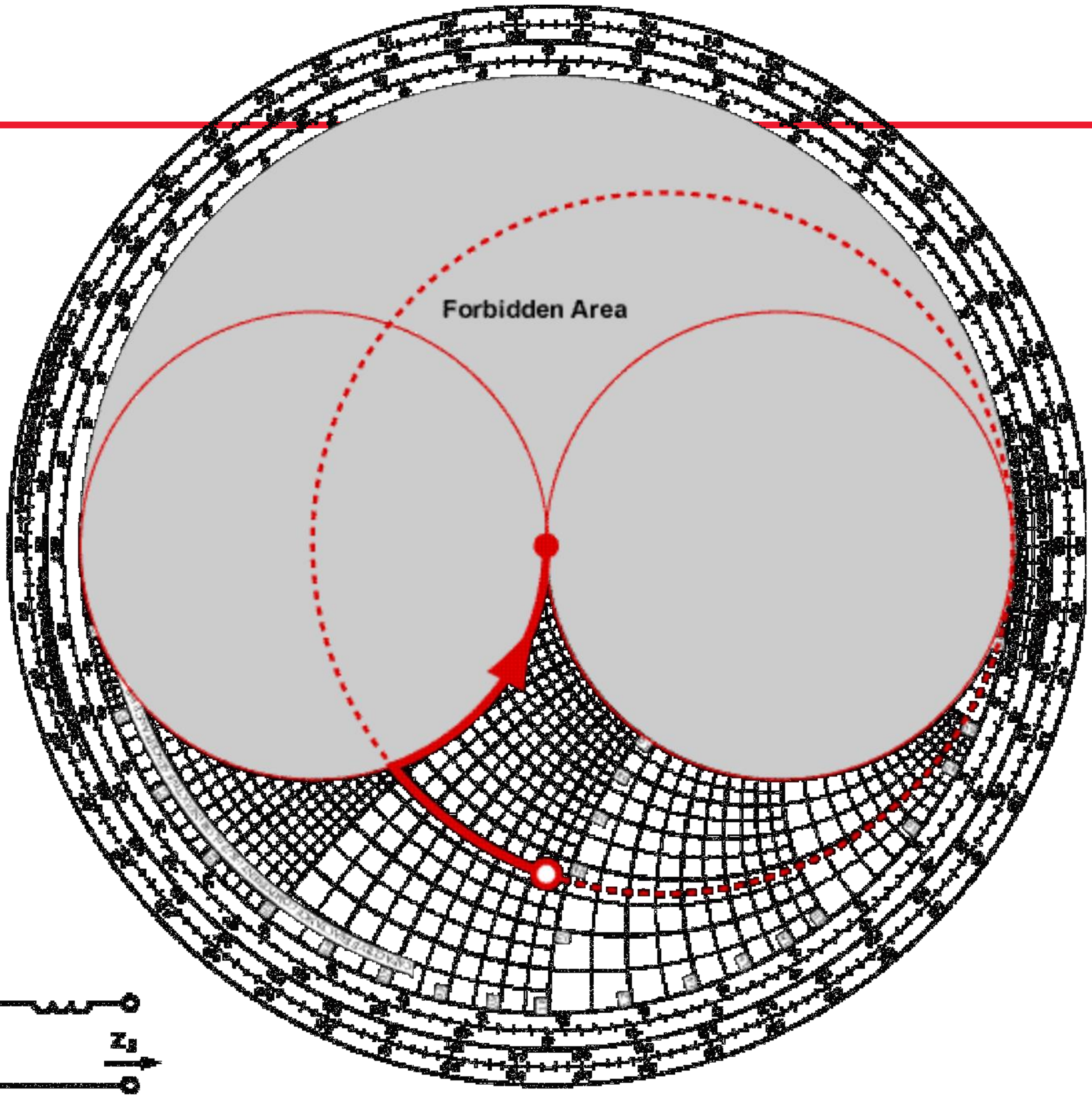
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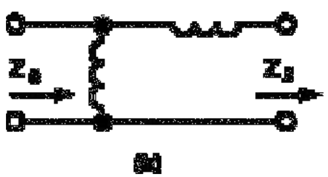
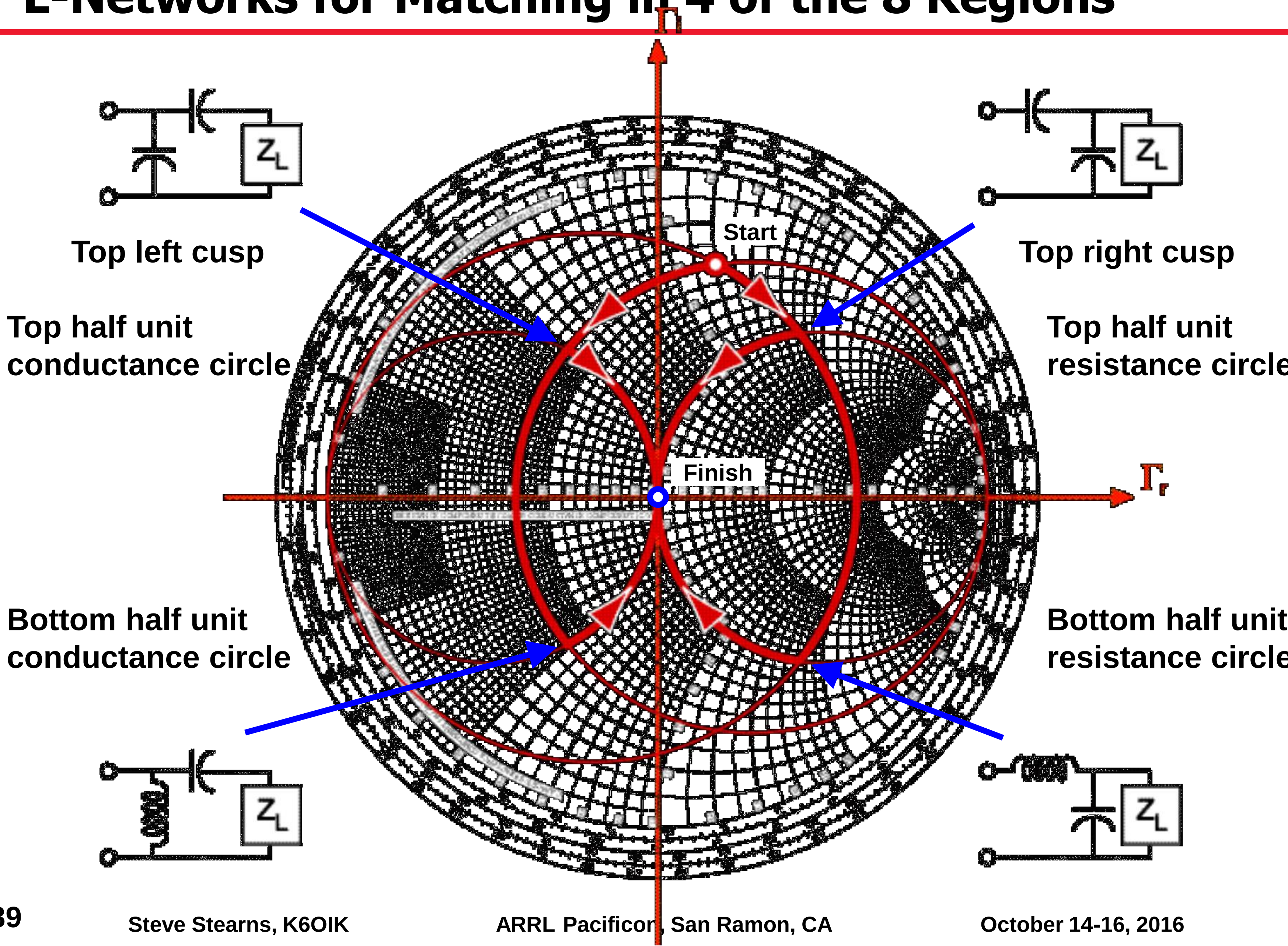
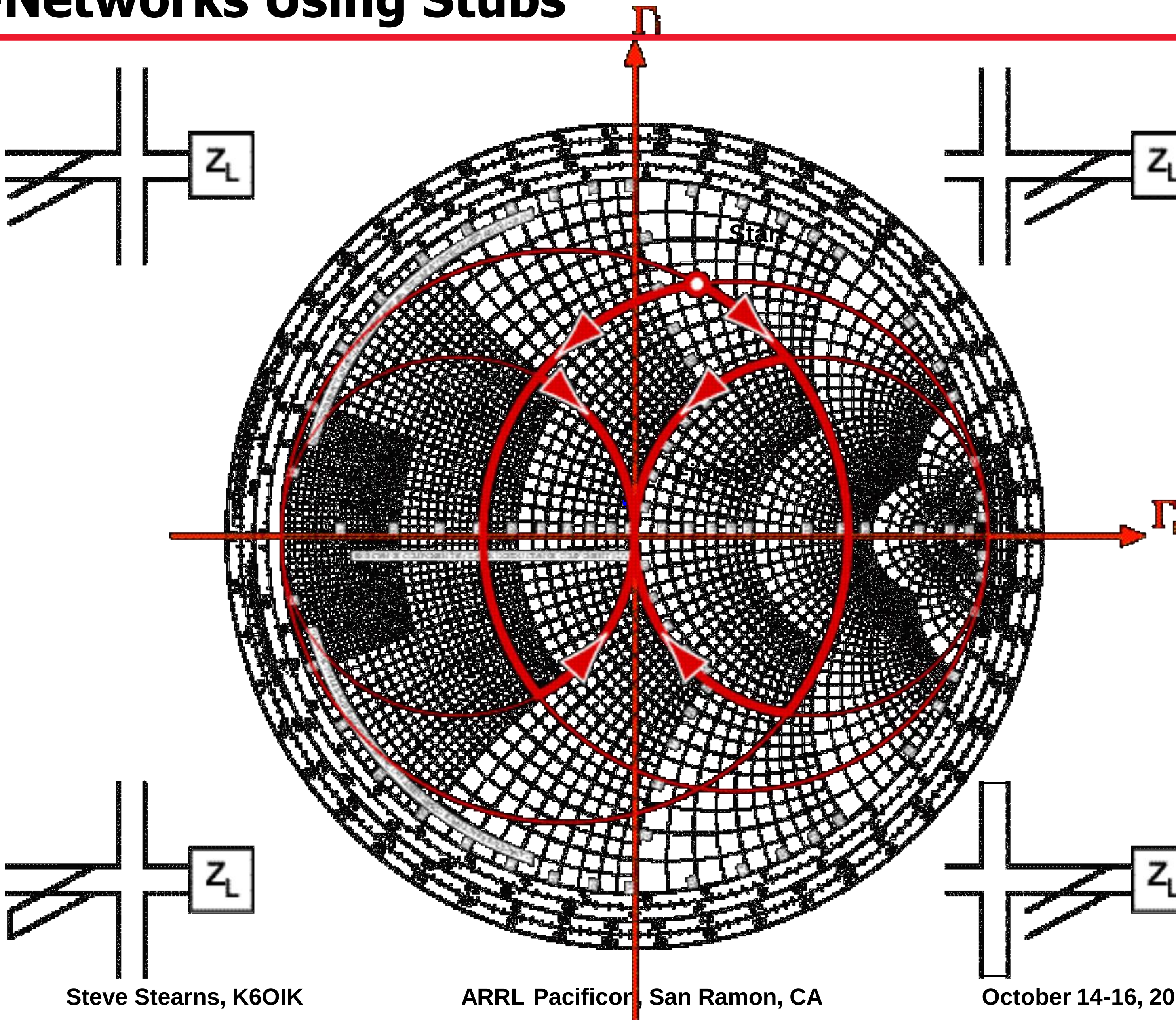


L-Networks for Matching in 4 of the 8 Regions



L-Networks Using Stubs



Electrically-Short Stubs As Capacitors and Inductors

- Open and shorted stubs that are electrically short act like lumped element capacitors and inductors
- Electrically-short short-circuited stubs act as inductors

$$Z_{in} = jZ_0 \tan\frac{2\pi f l}{c} \approx j\,2\pi f\,\frac{Z_0\,l}{c} \qquad \Rightarrow \qquad \boxed{L = \frac{Z_0\,l}{c} = Z_0\,\tau}$$

- Electrically-short open-circuited stubs act as capacitors

$$Z_{in} = -jZ_0 \cot\frac{2\pi f l}{c} \approx \frac{1}{j\,2\pi f\,\dfrac{l}{Z_0\,c}} \qquad \Rightarrow \qquad \boxed{C = \frac{l}{Z_0\,c} = \frac{\tau}{Z_0}}$$

- Stubs may be used in place of capacitors and inductors to make filters

Three Ways to Set Capacitance and Inductance

- Adjust both Z_0 and l
- Adjust Z_0 with l fixed
- Adjust l with Z_0 fixed

Z_0	Short-Circuited Stubs nH per ft	Open-Circuited Stubs pF per ft
12.5	12.7	81.3
25	25.4	40.7
37.5	38.1	27.1
50	50.8	20.3
75	76.2	13.6
100	102	10.2
150	152	6.78
200	203	5.08
300	305	3.38
450	457	2.26

Quarter-Wave Stubs As Resonant Circuits

- Shorted quarter-wave stubs act like parallel resonant circuits

$$Z_{in} = jZ_0 \tan \frac{\pi}{2} \frac{f}{f_0} \quad \Rightarrow \quad \boxed{L = \frac{2Z_0}{\pi^2 f_0} \qquad C = \frac{1}{8Z_0 f_0}}$$

- Open quarter-wave stubs act like series resonant circuits

$$Z_{in} = -jZ_0 \cot \frac{\pi}{2} \frac{f}{f_0} \quad \Rightarrow \quad \boxed{L = \frac{Z_0}{8 f_0} \qquad C = \frac{2}{\pi^2 Z_0 f_0}}$$

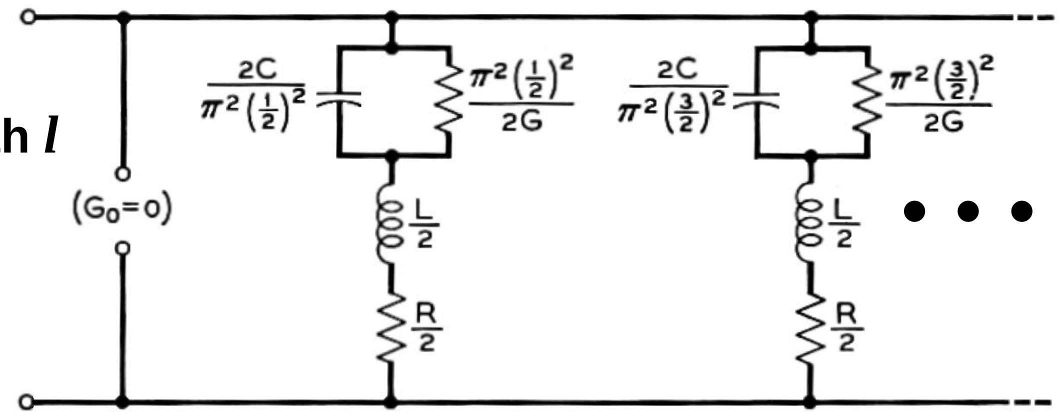
Exact Equivalent Circuits of an Open Stub

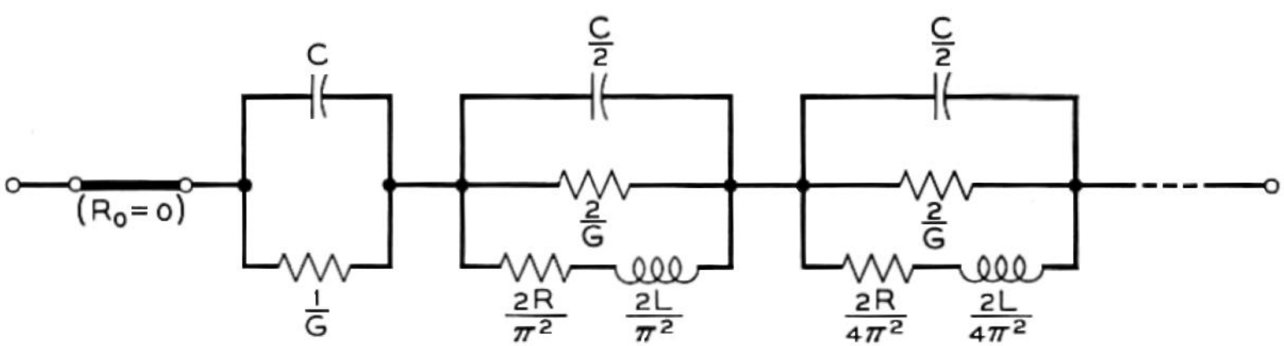
$$Z_{in} = Z_0 \coth \gamma l = Z_0 \coth(\alpha l + j\beta l)$$

- **Frequency-independent loss**
- ***R*, *L*, *G*, *C* are for total line length *l***

$$L = Z_0 \frac{l}{c} = Z_0 \tau$$

$$C = \frac{l}{Z_0 c} = \frac{\tau}{Z_0}$$





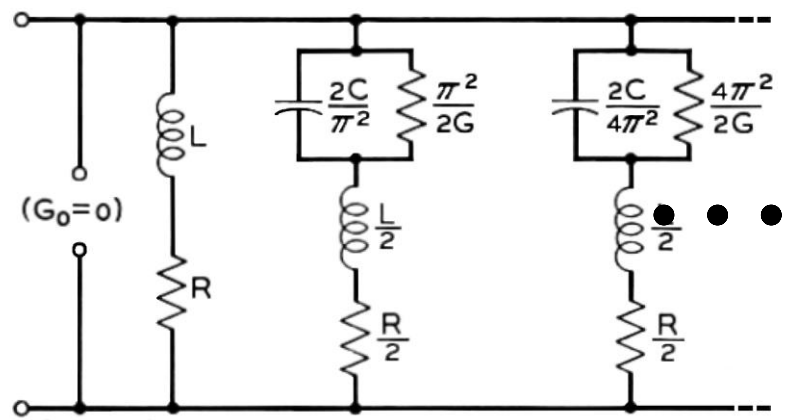
Exact Equivalent Circuits of a Shorted Stub

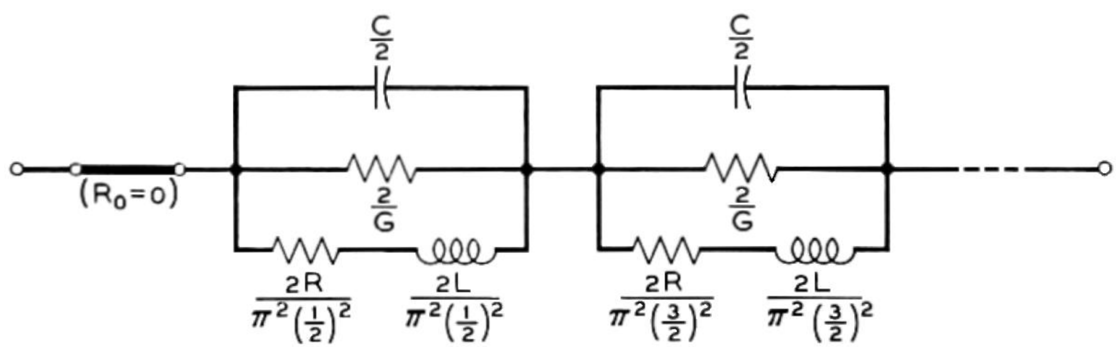
$$Z_{in} = Z_0 \tanh \gamma l = Z_0 \tanh(\alpha l + j\beta l)$$

- **Frequency-independent loss**
- ***R*, *L*, *G*, *C* are for total line length *l***

$$L = Z_0 \frac{l}{c} = Z_0 \tau$$

$$C = \frac{l}{Z_0 c} = \frac{\tau}{Z_0}$$





Comments on Stubs

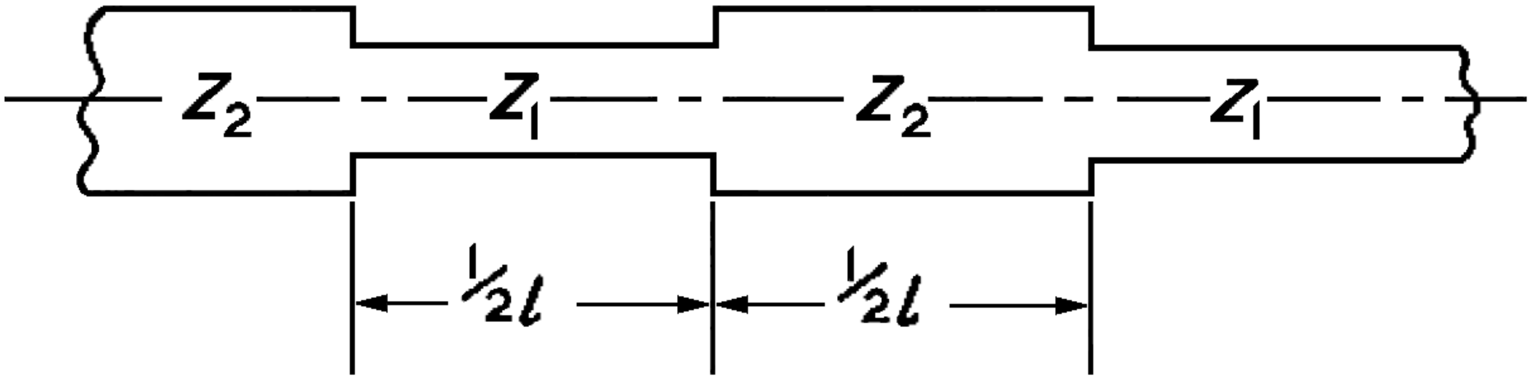
- **Shunt stubs work fine if space allows and losses are low**
- **Series stubs are problematic**
  - Common mode current causes radiation loss
- **Shorted stubs are preferred over open stubs**
  - Zero impedance is an easier boundary condition to enforce than zero admittance due to field fringing
  - You can't stop current with an open circuit. The current merely switches from conduction to displacement current and keeps on going
- **Networks often have greater match bandwidths when made of lumped elements rather than stubs**
- **Impedance function of stubs is periodic in frequency**
- **If a network is made only of lossless transmission line elements, all of which have commensurate lengths, i.e. lengths are multiples of a common length l, then the network response is periodic in frequency with period c/l**

Alternated-Line Match

Alternated-Line Match

- **Special case of matching using cascaded transmission line sections of differing impedances**
- **Invented by Peter Bramham at CERN**
  - B. [sic] Bramham, "A Convenient Transformer for Matching Coaxial Lines," *Electronic Engineering* (London), pp. 42-44, Jan. 1961
- **Known by several names, not all are correct**
  - Non-synchronous match or transformer (Matthaei, Young, Jones, 1964)
  - Series section match or transformer (incorrect, OD5CG, QST, July 1978)
  - Sixth-wave match or transformer
  - Twelfth-wave match or transformer (AA7FV, QST, June 1997)
  - Synchronous match or transformer (incorrect, N0AX, QST, Oct. 2009)
- **Requires just two sections of two kinds of line – one with low Z_0 and the other with high Z_0**

Bramham's Alternated-Line Match



- For Z_1 and Z_2 real (resistances)

$$\frac{l}{2} = \frac{\lambda}{2\pi}\arctan\sqrt{\frac{1}{1+\frac{Z_1}{Z_2}+\frac{Z_2}{Z_1}}}$$

- If $Z_1 / Z_2 \approx 1$, then

$$l \to \frac{\lambda}{6} \quad \text{and} \quad \frac{l}{2} \to \frac{\lambda}{12}$$

- The alternated-line technique can be generalized to match any two complex impedances given lines of two different characteristic impedances, one low and one high
- Similar to series L and shunt C moves using lumped elements

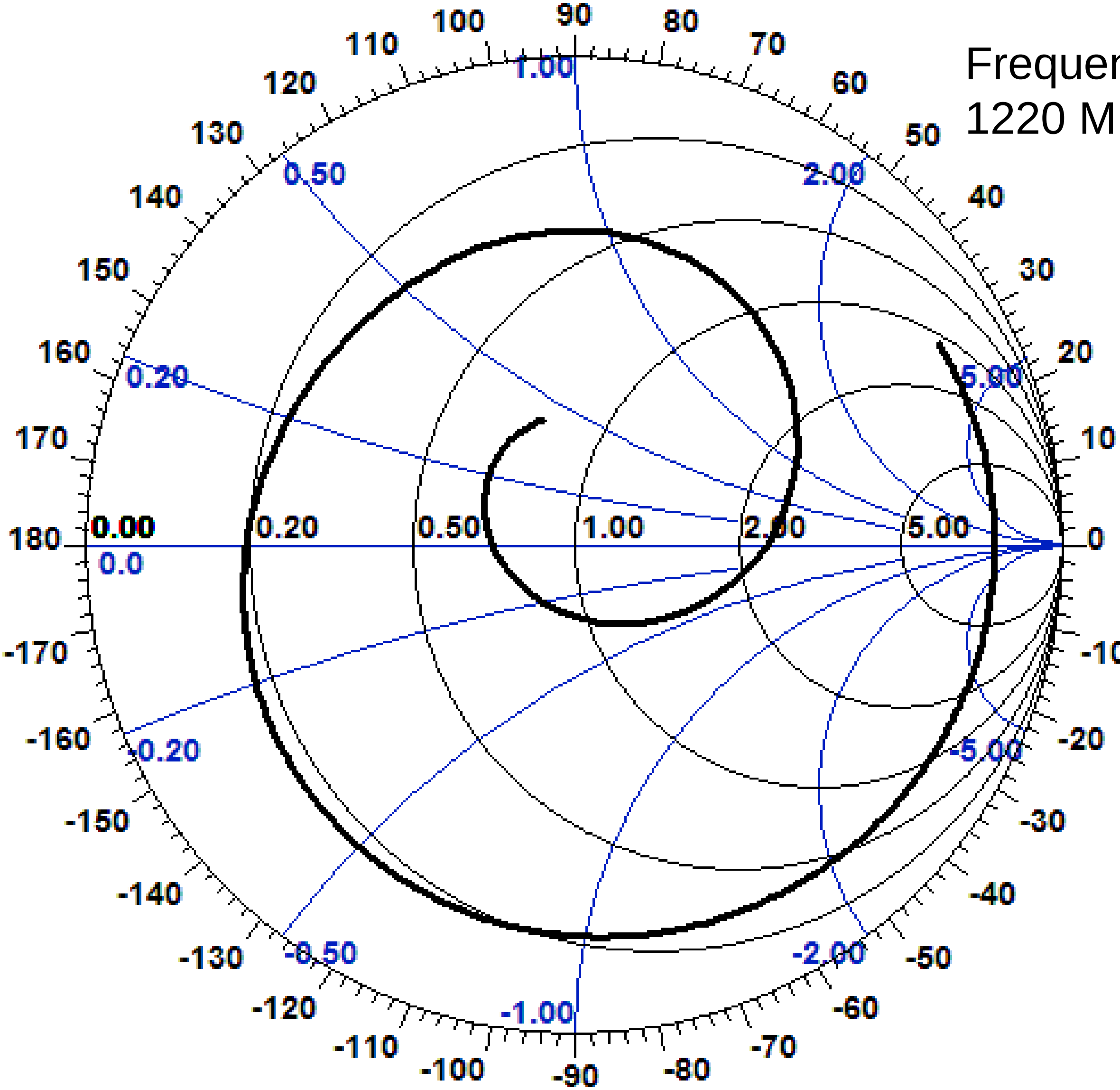
Multi-Frequency Match Networks

Matching at Two Frequencies

Example 1: GPS L1 and L2 bands

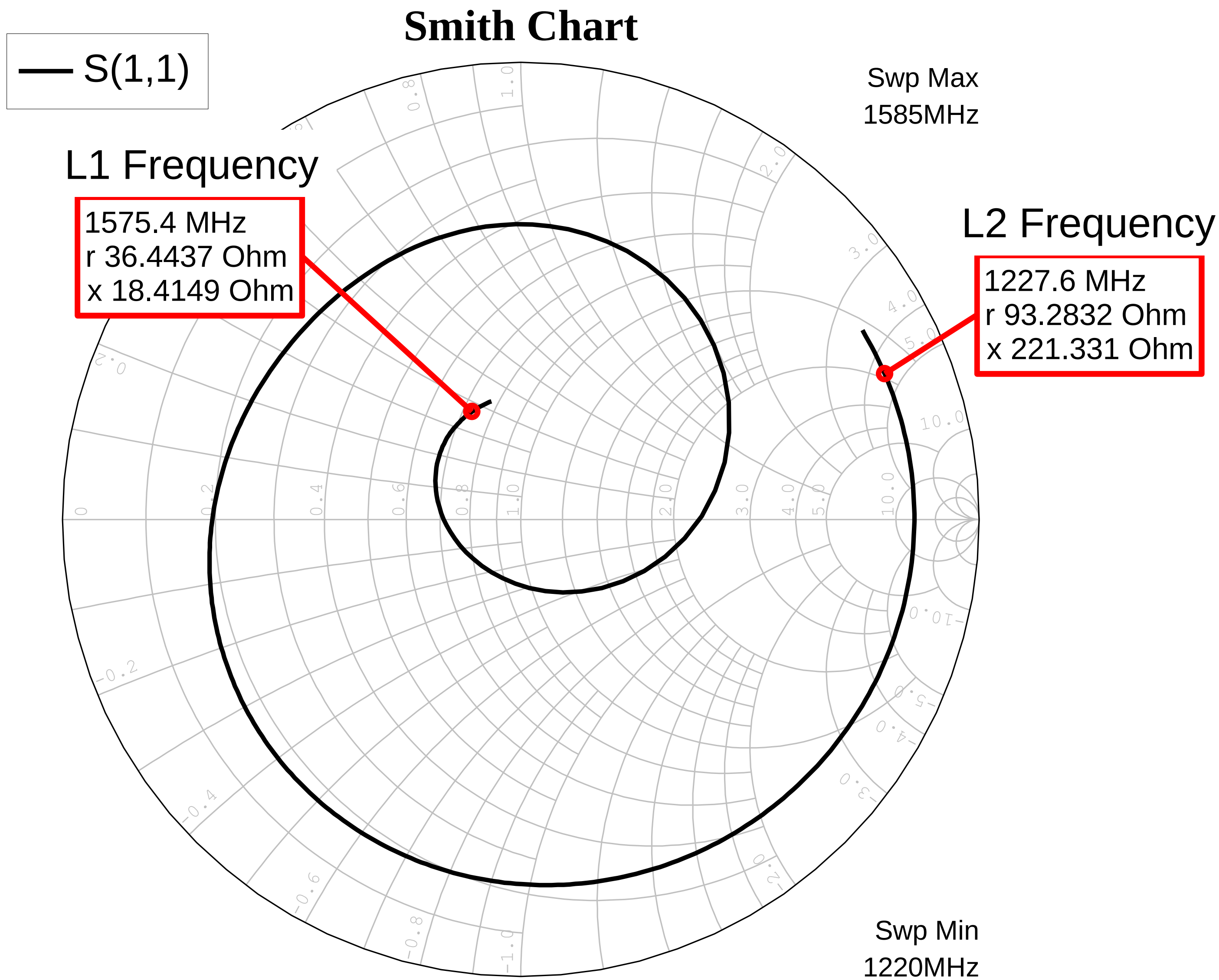
Example 2: Cellular 900-MHz and PCS 1900-MHz bands

Example 1: GPS Antenna Before Matching on Two Bands Shown in Serenade SV

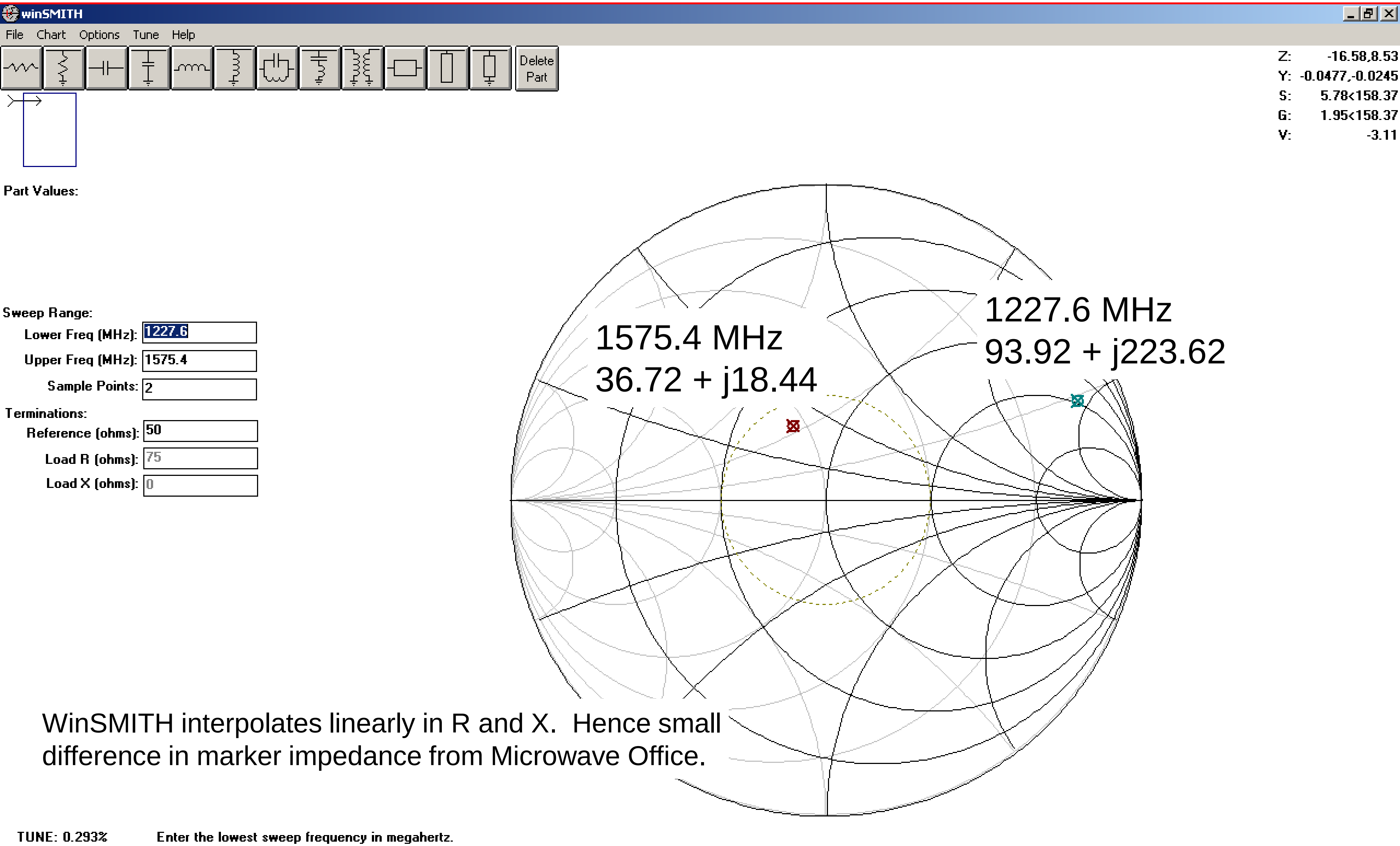


Frequency sweep
1220 MHz to 1585 MHz

Example 1: GPS Antenna Before Matching on Two Bands L1 and L2 Frequencies, Shown in Microwave Office

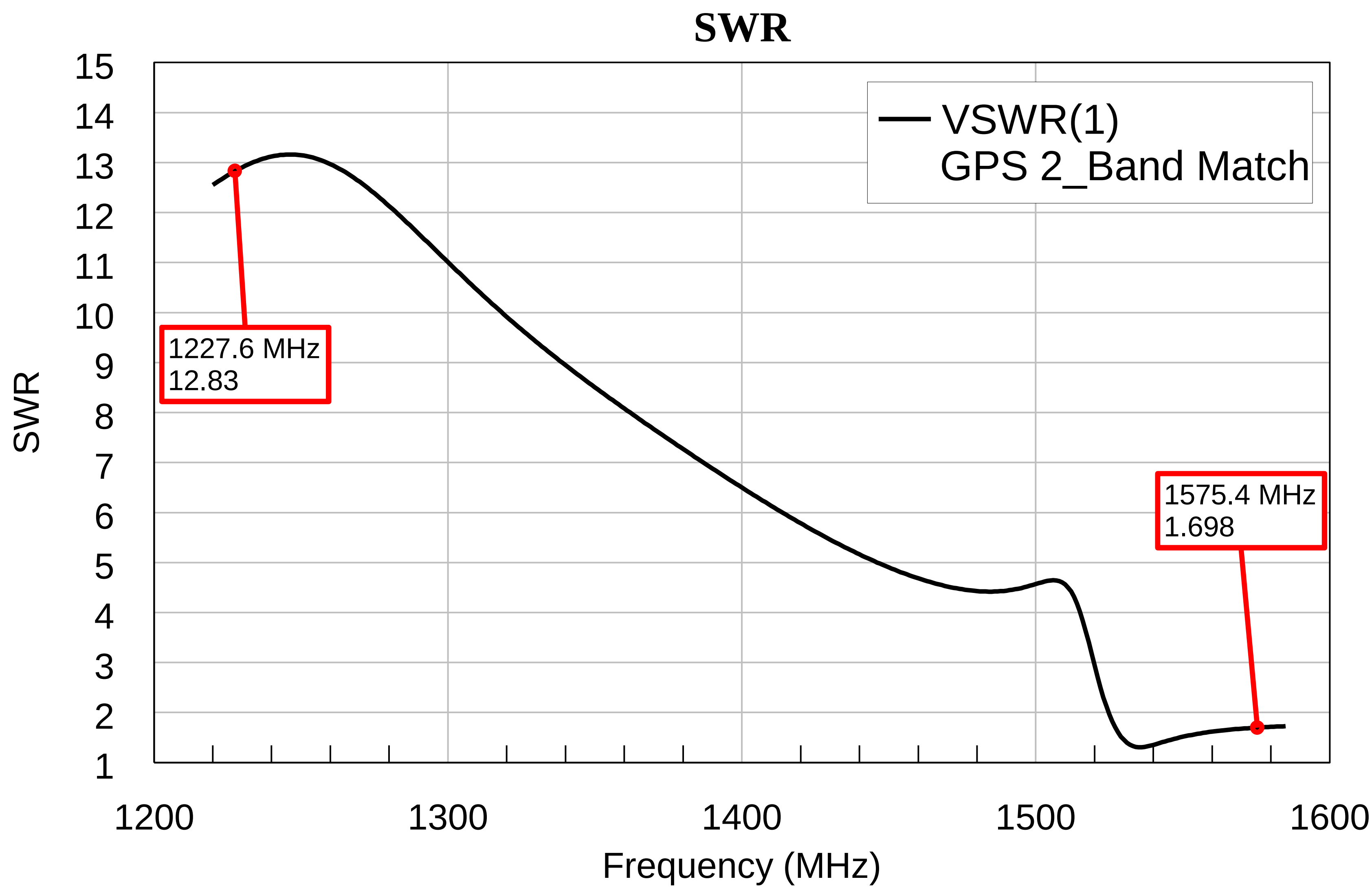


Example 1: GPS Antenna Matching on Two Bands Shown in WinSMITH

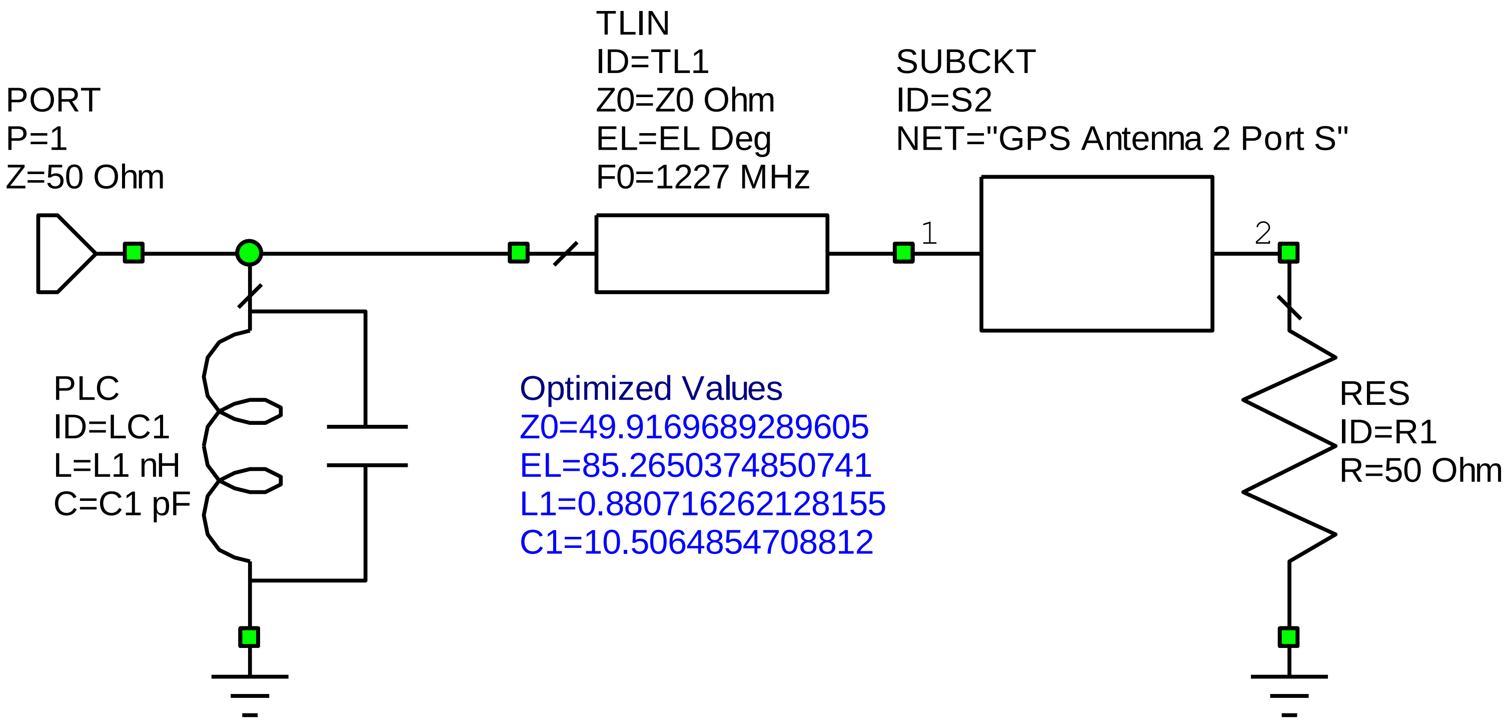


WinSMITH interpolates linearly in R and X. Hence small difference in marker impedance from Microwave Office.

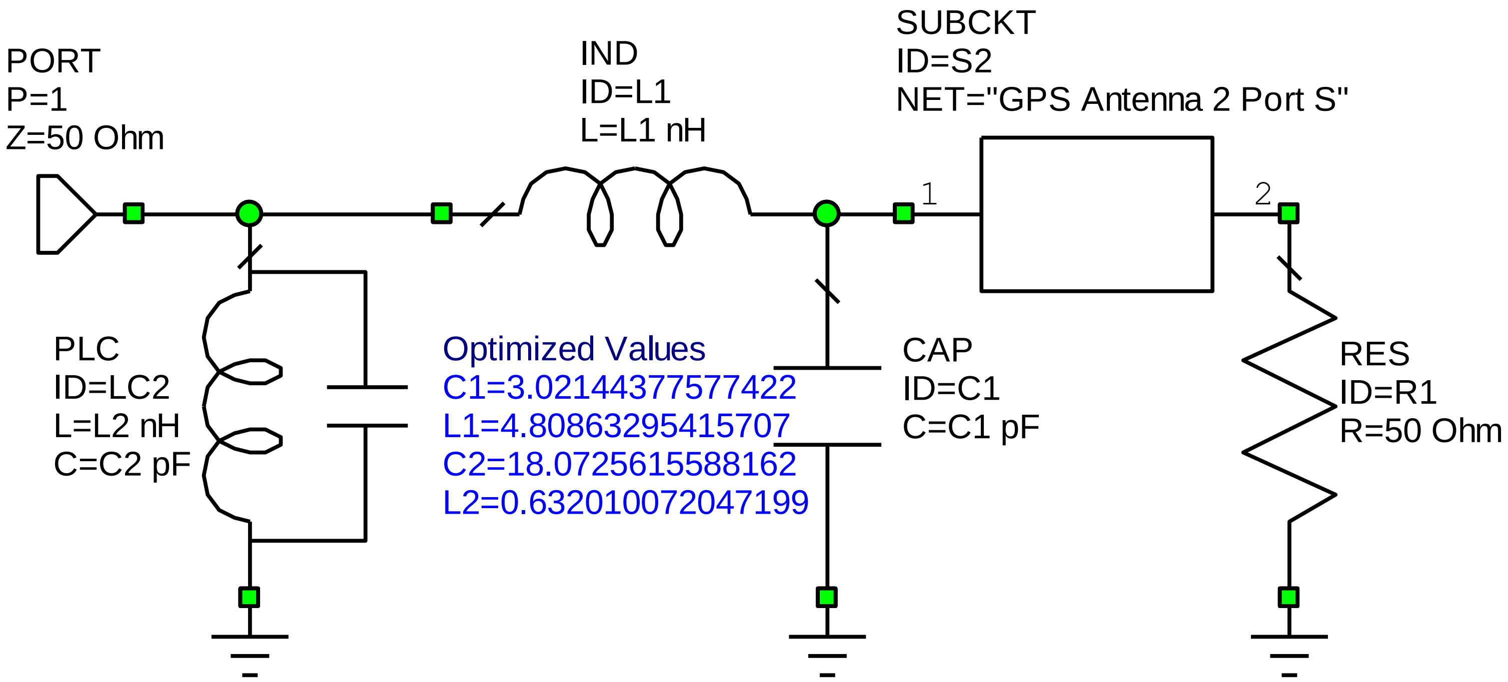
SWR Before Matching



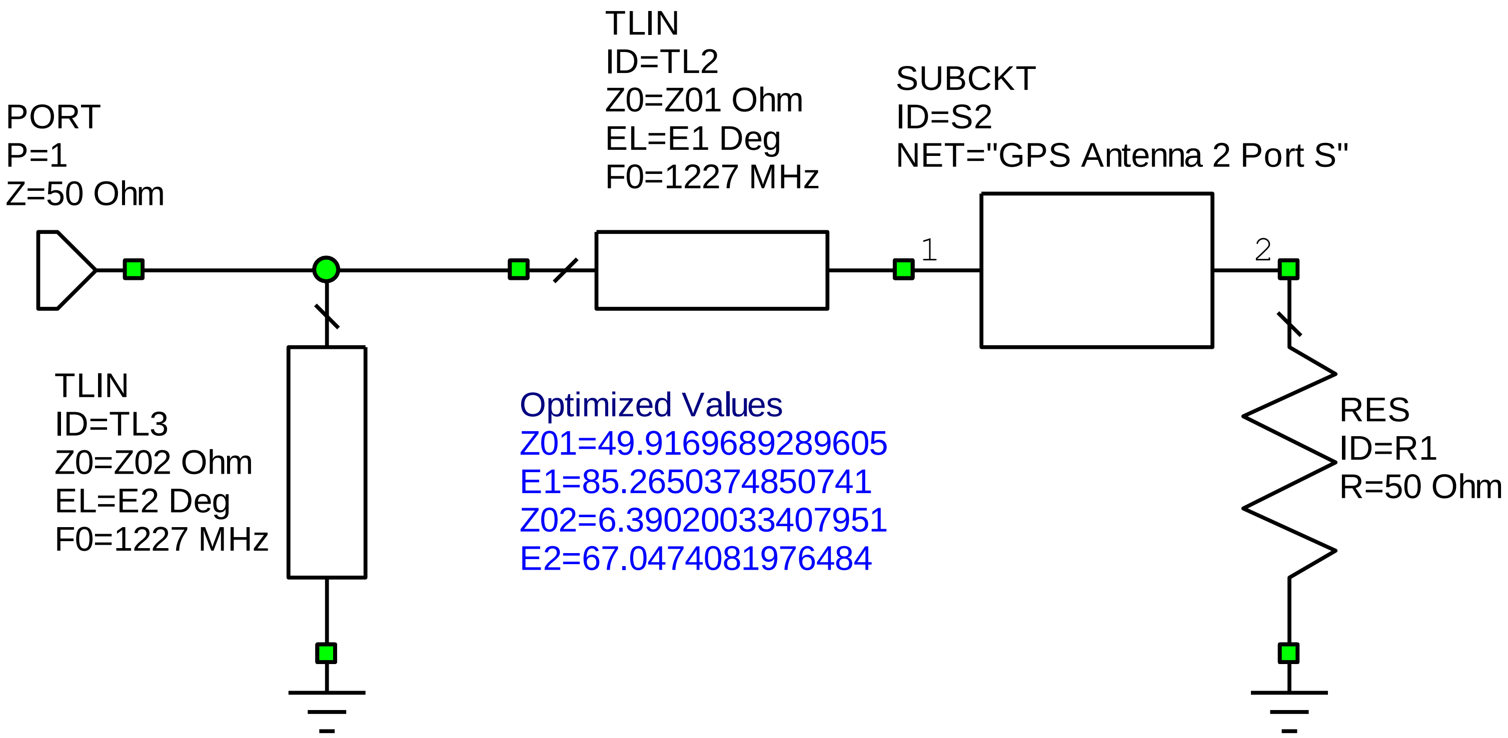
GPS Dual-Band Matching Network No. 1



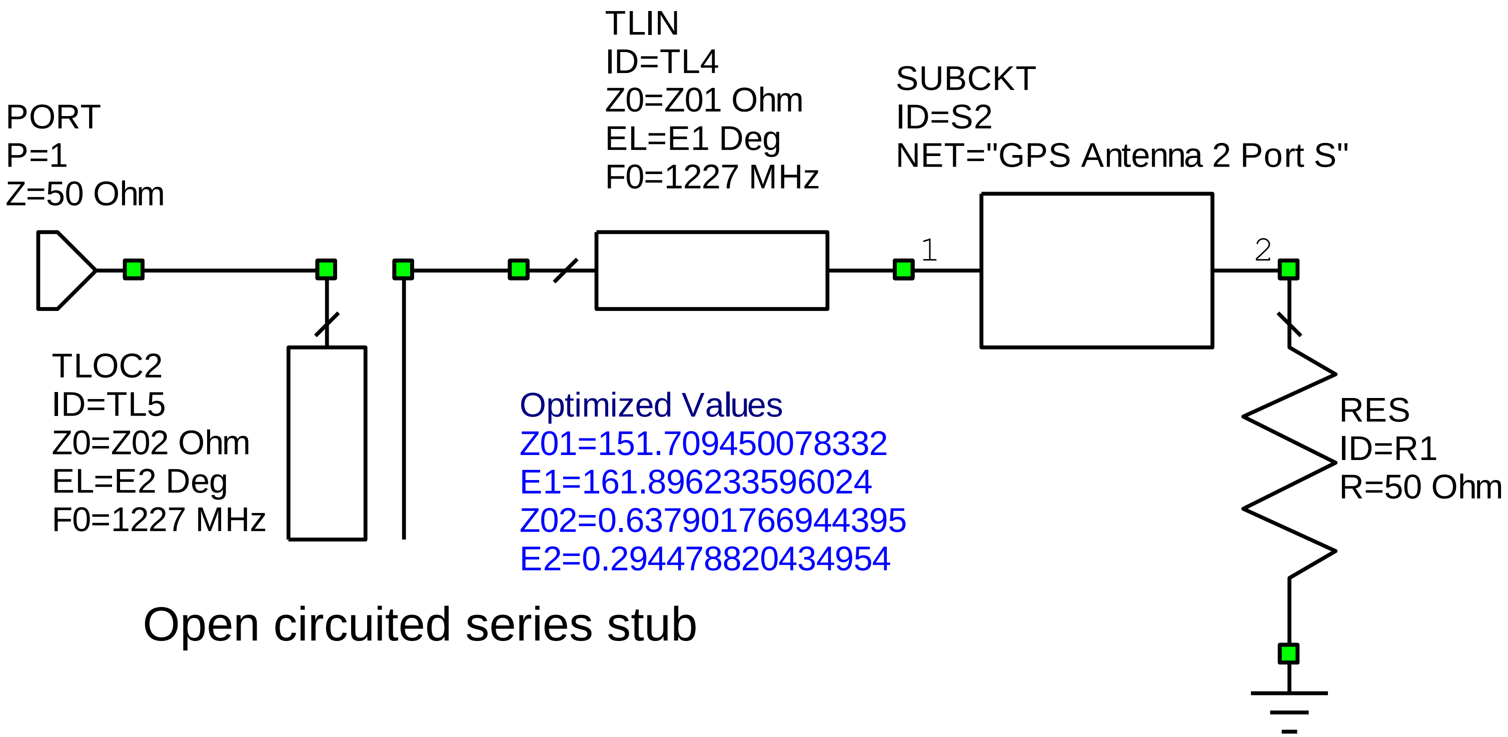
GPS Dual-Band Matching Network No. 2



GPS Dual-Band Matching Network No. 3

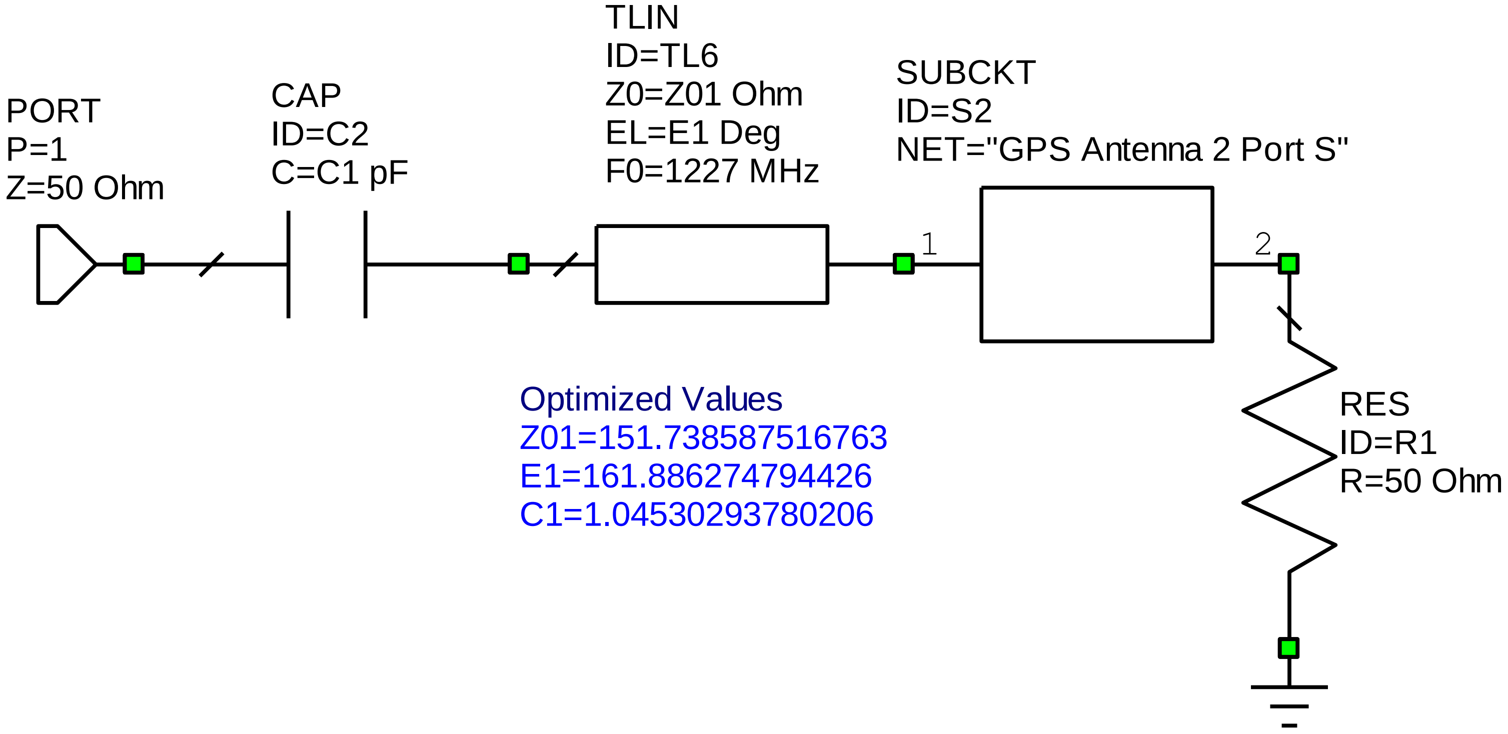


GPS Dual-Band Matching Network No. 4

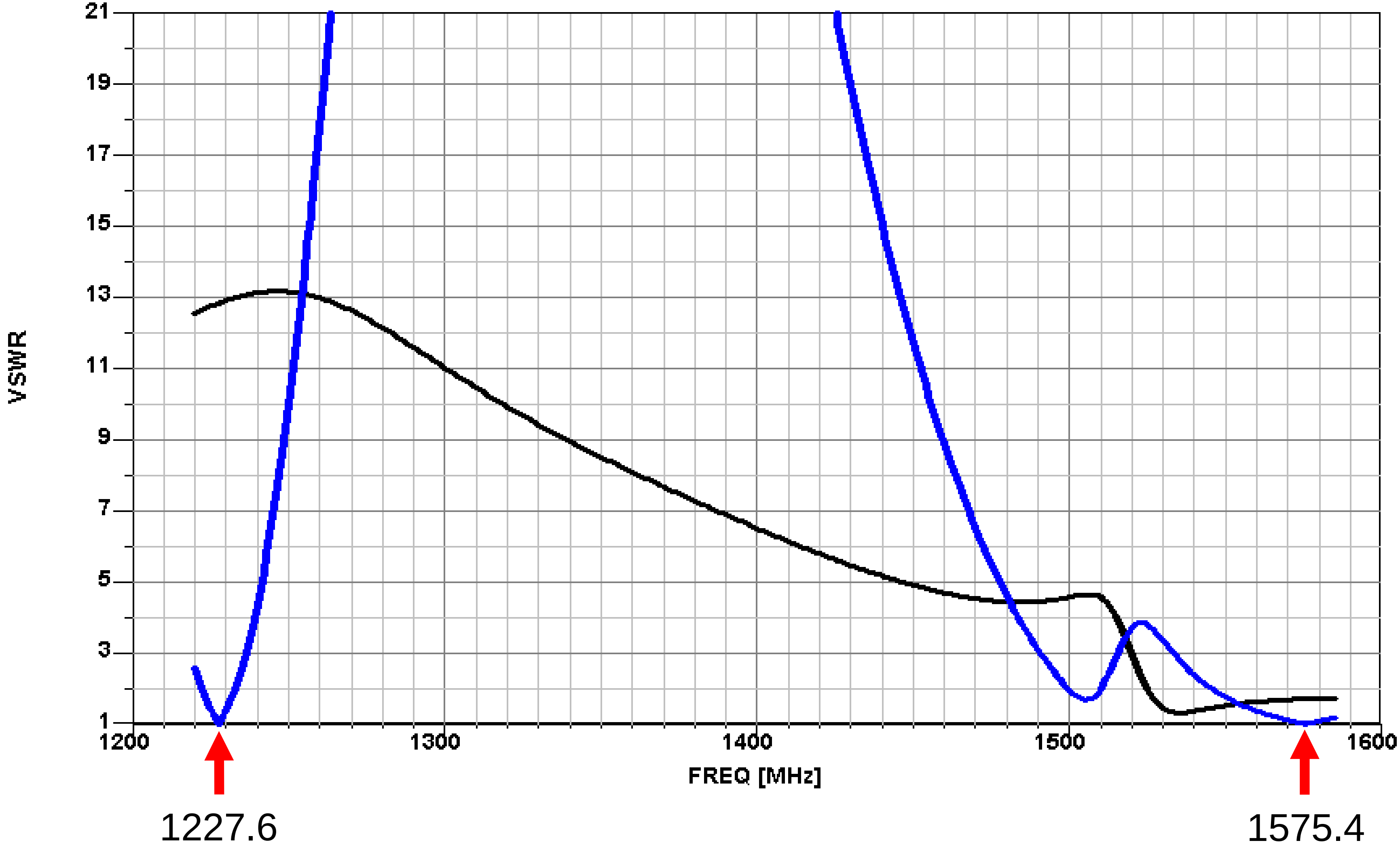


Open circuited series stub

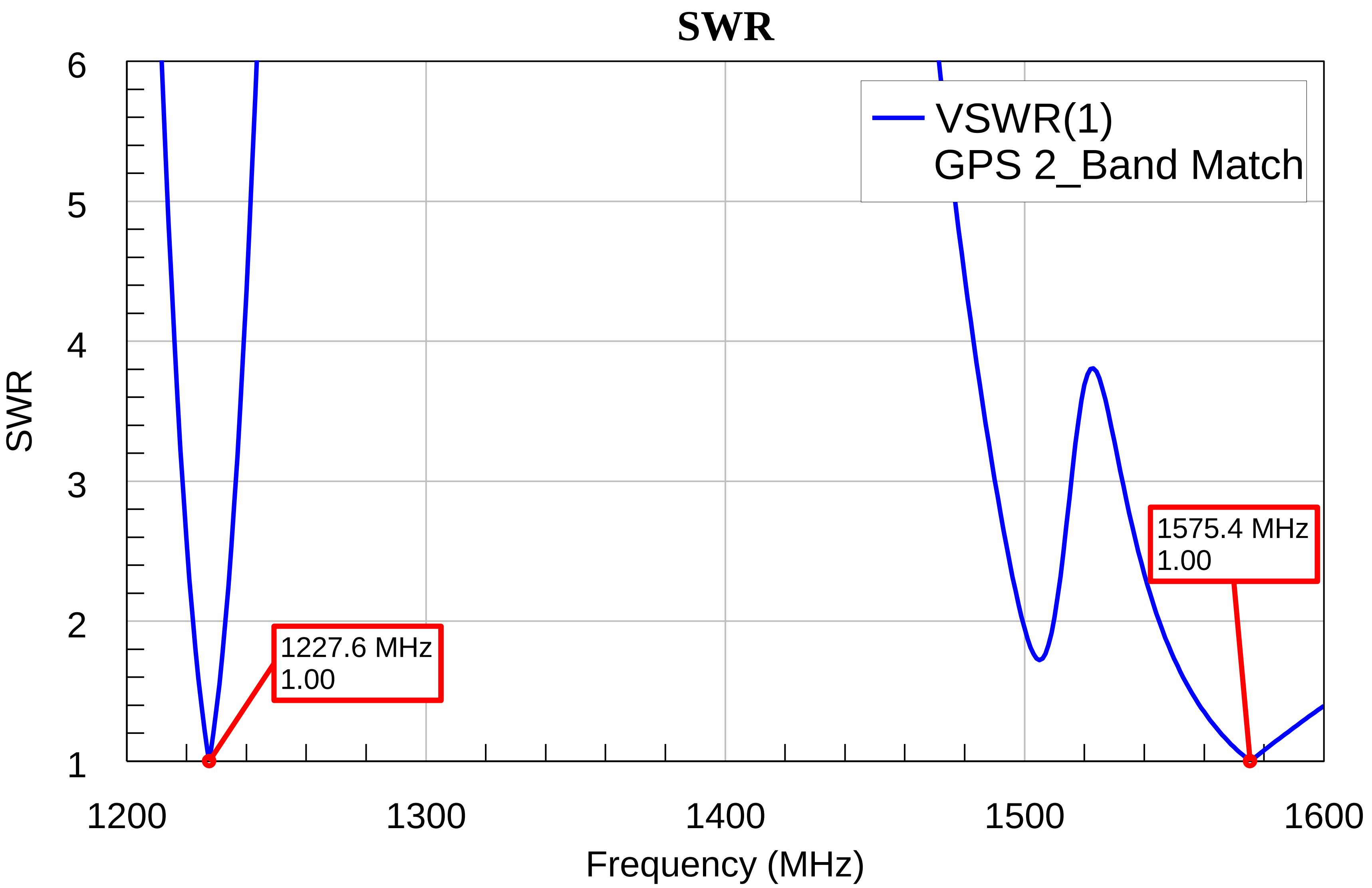
GPS Dual-Band Matching Network No. 5



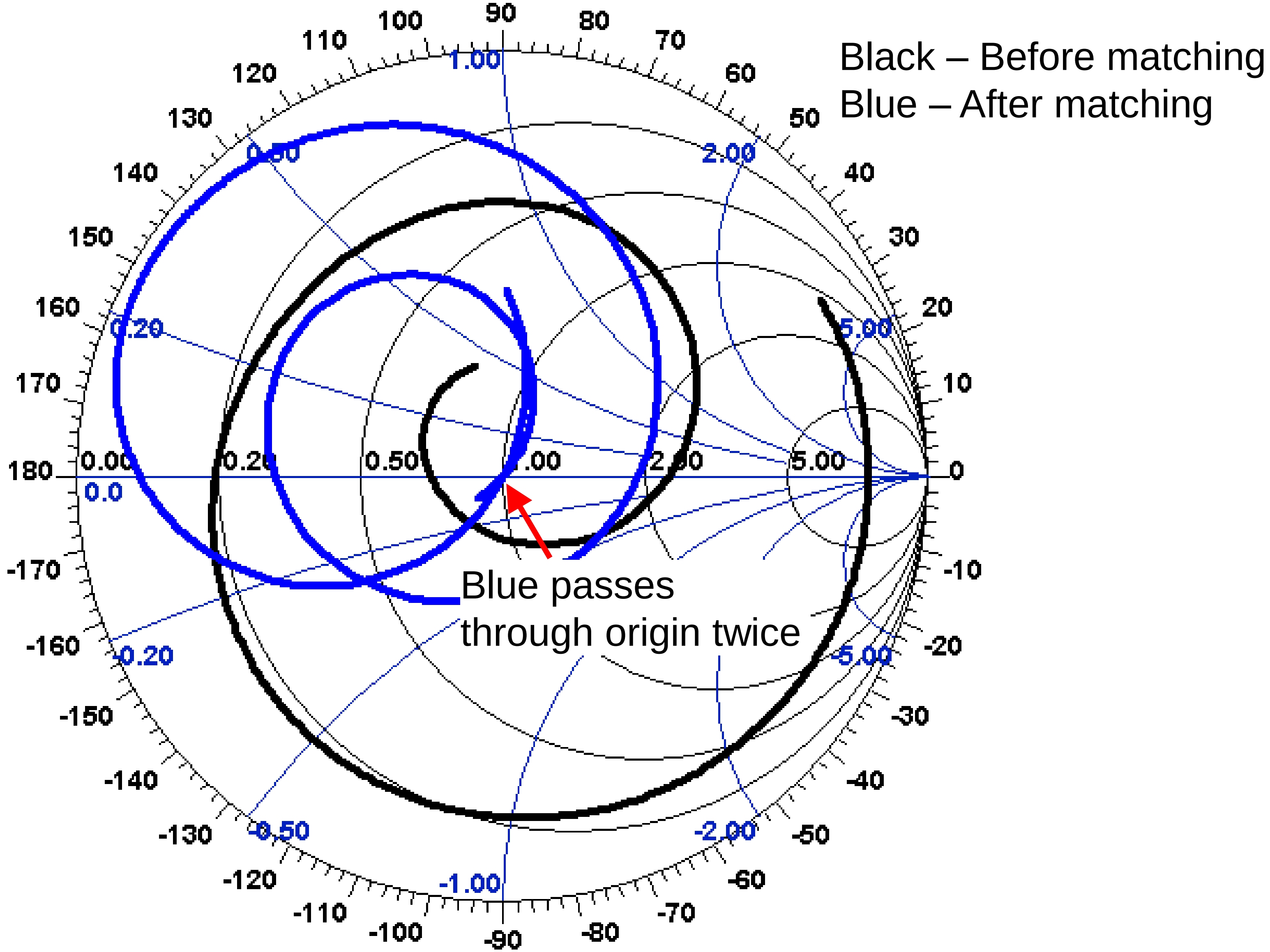
SWR Before and After Match Shown in Ansoft Serenade SV



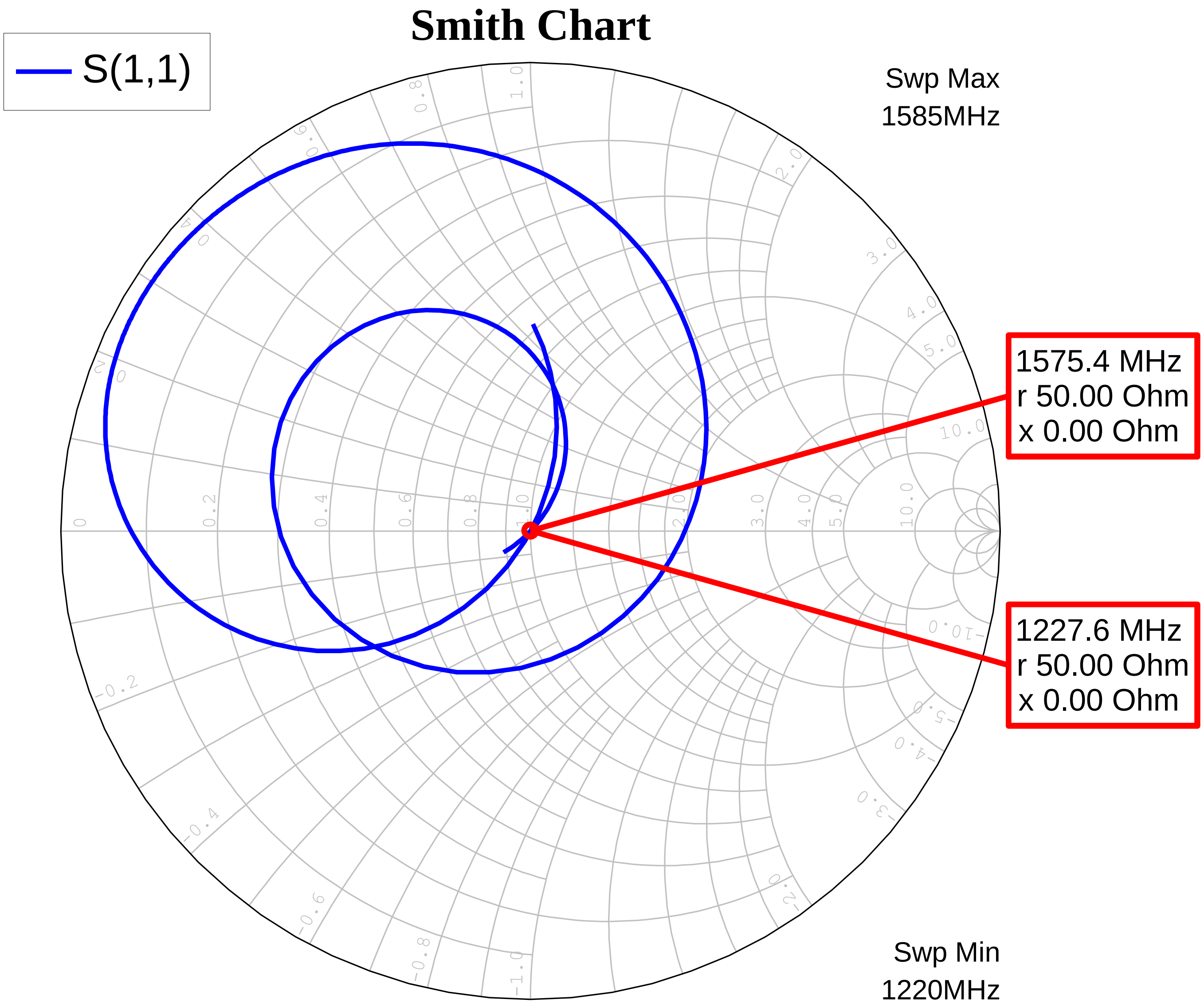
SWR After Match
Shown in Microwave Office



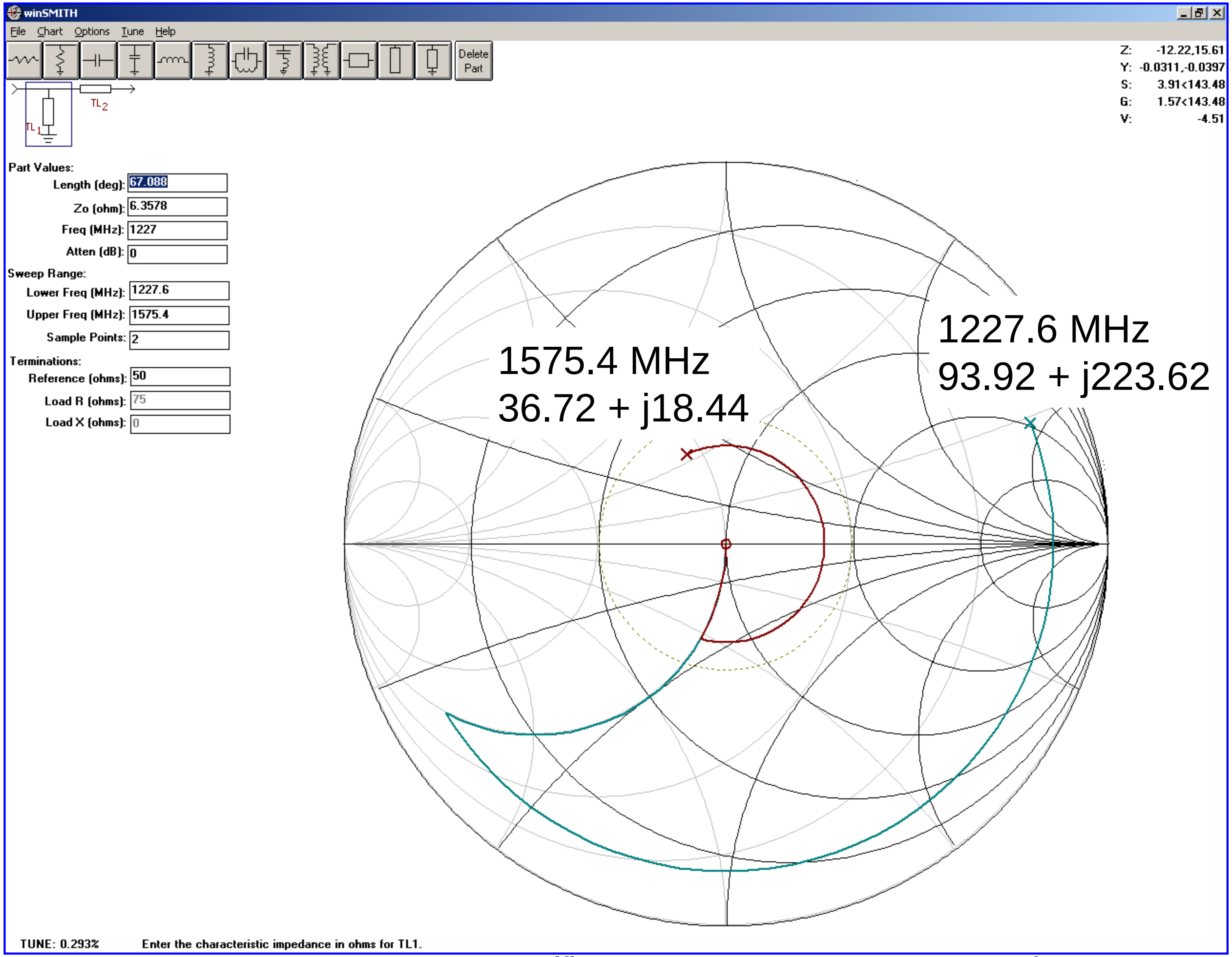
Match Result (Blue)
Shown in Ansoft Serenade SV



Match Result
Shown in Microwave Office



GPS Antenna Dual-Band Matching Network No. 1 Done in WinSMITH

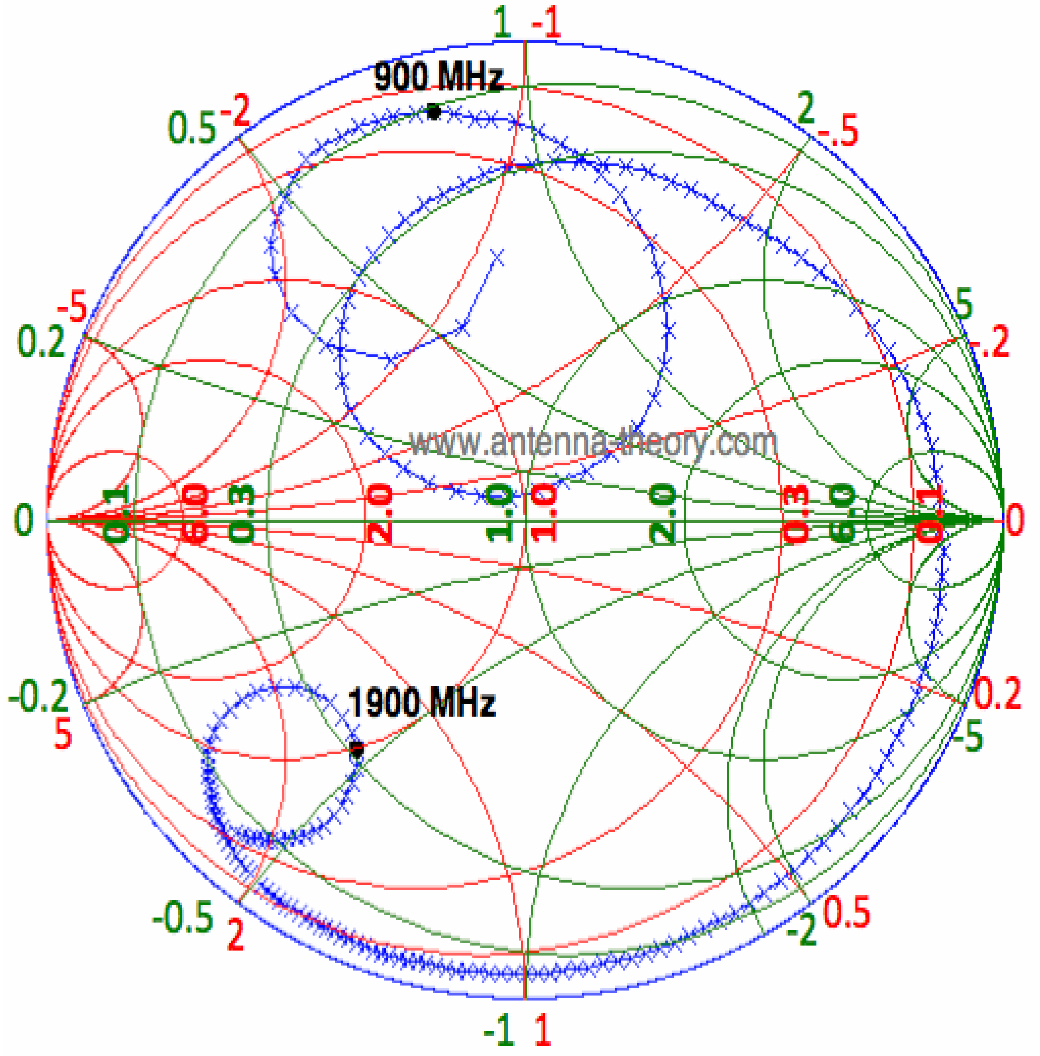


Design Procedure for 2-Frequency Matching

- **Step 1: Move both points onto the unit resistance or conductance circle with the low frequency point clockwise from the high frequency point**
  - There are many ways to do this first step. For example, one can use an L-network to move one point to center and then use a transmission line segment to rotate the other point onto a unit resistance or unit conductance circle.
- **Step 2: Add a series or shunt LC resonator**
  - For points on the unit resistance circle, add a series LC resonator in series
  - For points on the unit conductance circle, add a parallel LC resonator in shunt
- **Step 3: Adjust *L* and *C* to move both points to center of Smith chart**
- **Two frequency points can be brought to center by using a single LC resonator stage**

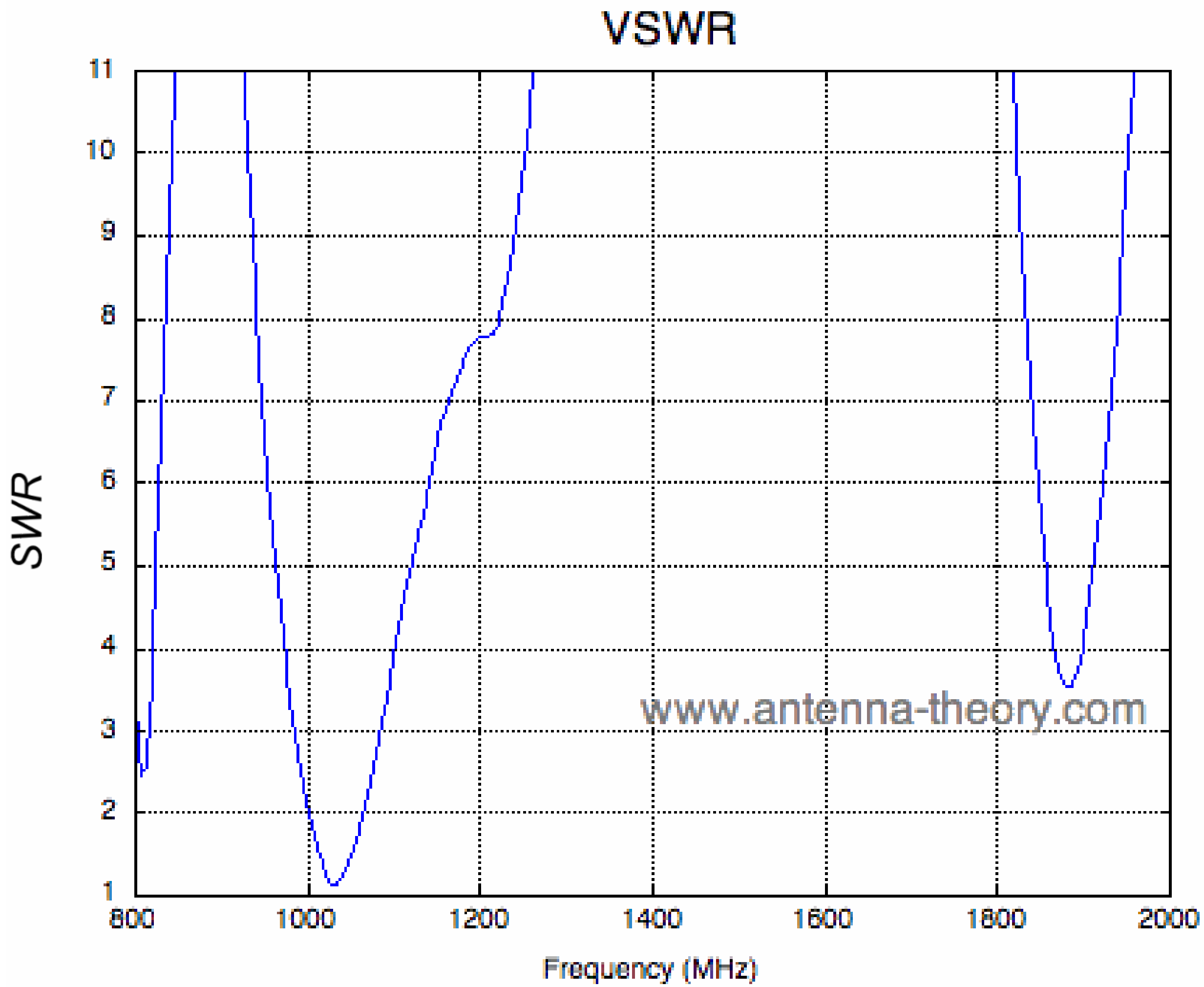
Do the above steps while viewing a Smith chart and using tuning sliders. When the points are close to center, use an optimizer to fine tune the parts values.

Example 2: Cellular and PCS Dual-Band Matching at 900 MHz and 1900 MHz



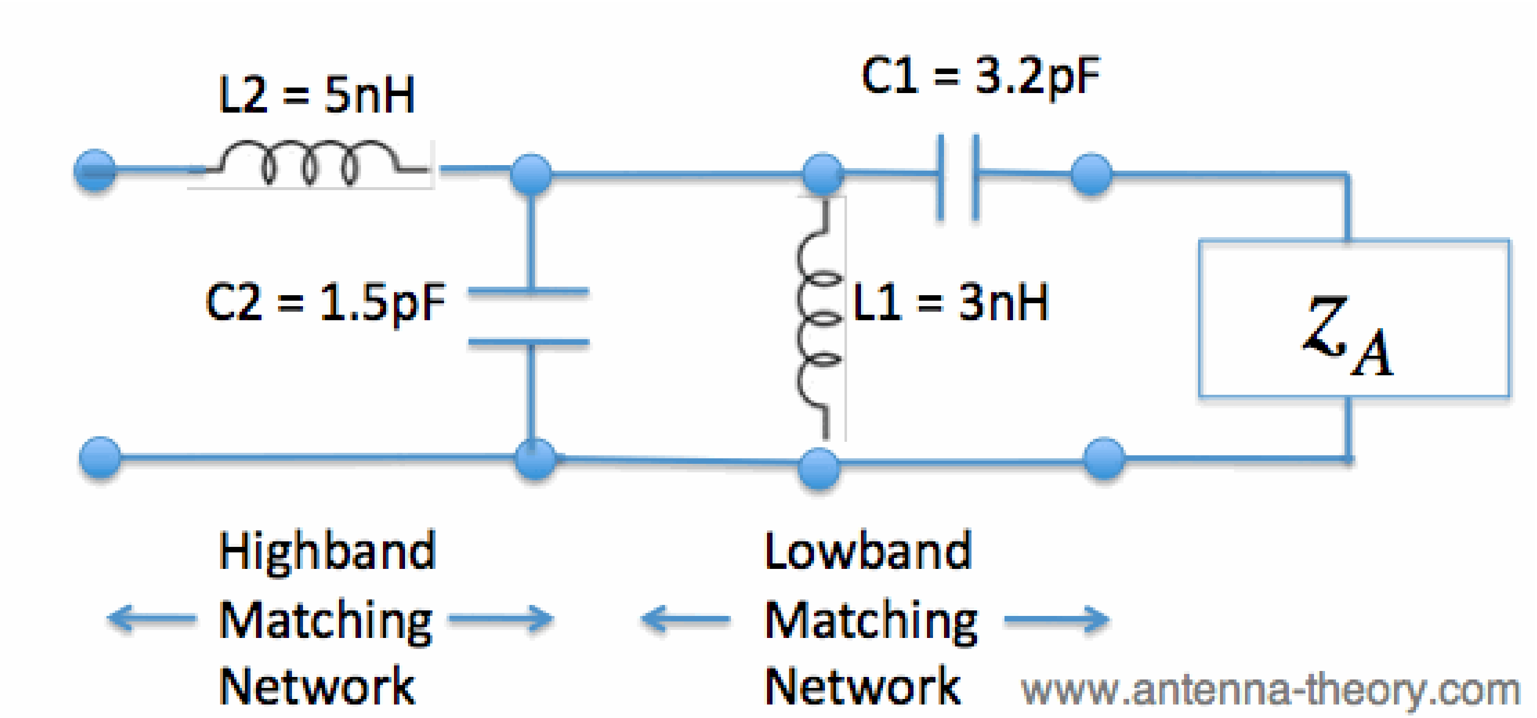
P.J. Bevelacqua, “Dual-Band Impedance Matching,” online at http://www.antenna-theory.com/tutorial/smith/smithchartC.php

SWR Before Matching



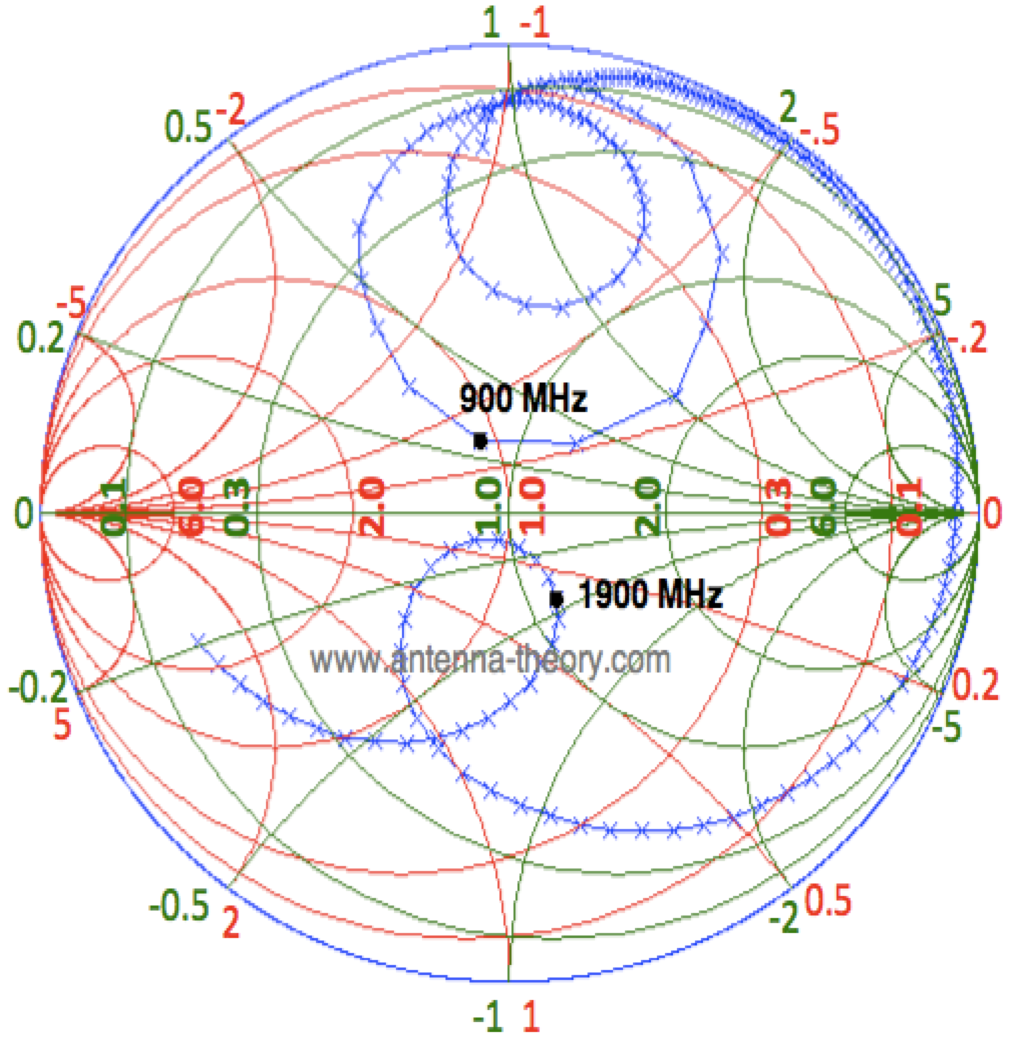
P.J. Bevelacqua, "Dual-Band Impedance Matching," online at http://www.antenna-theory.com/tutorial/smith/smithchartC.php

Bevelacqua's Solution – A 2-Band, 4-Element Matching Network



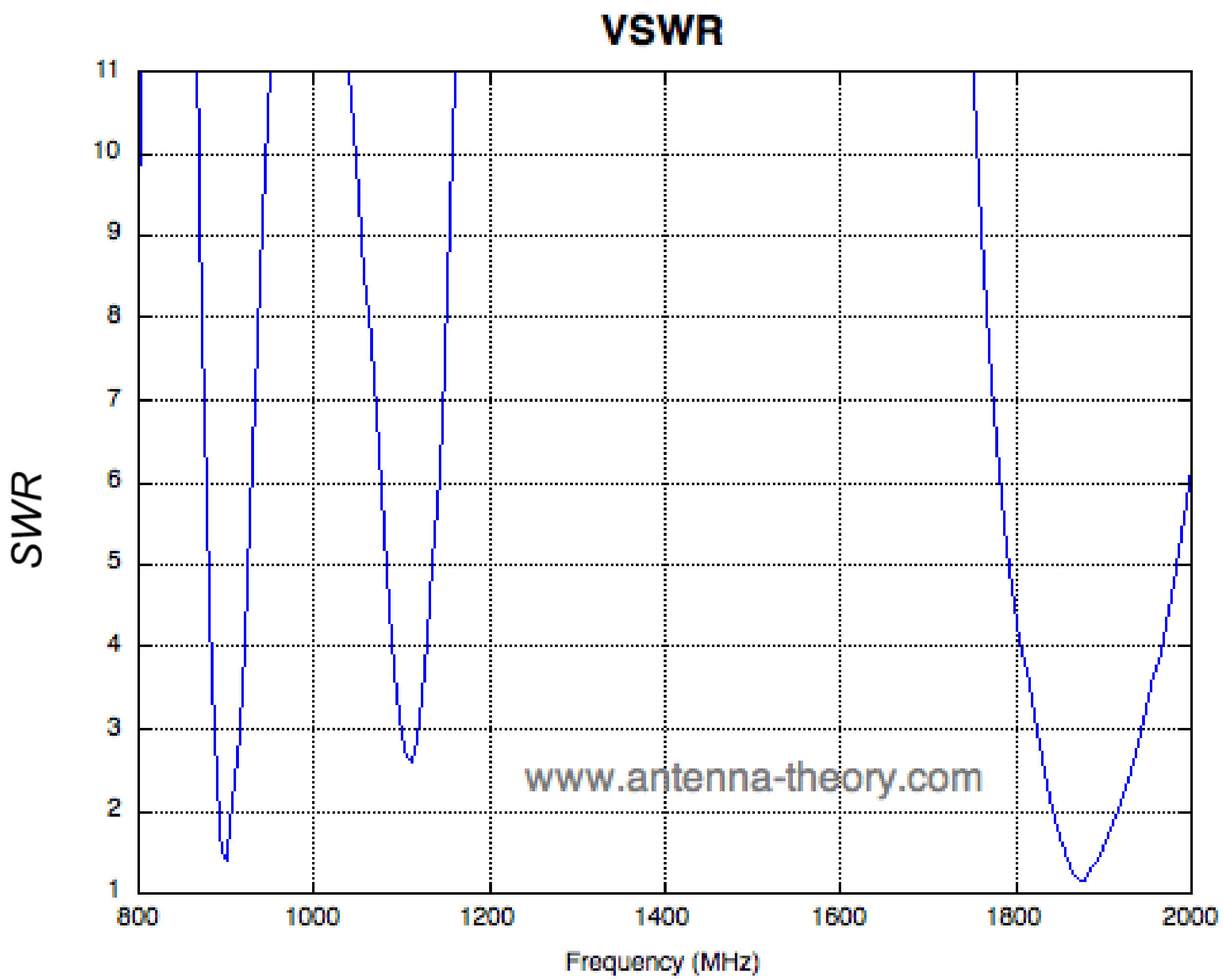
P.J. Bevelacqua, "Dual-Band Impedance Matching," online at http://www.antenna-theory.com/tutorial/smith/smithchartC.php

Result After Matching



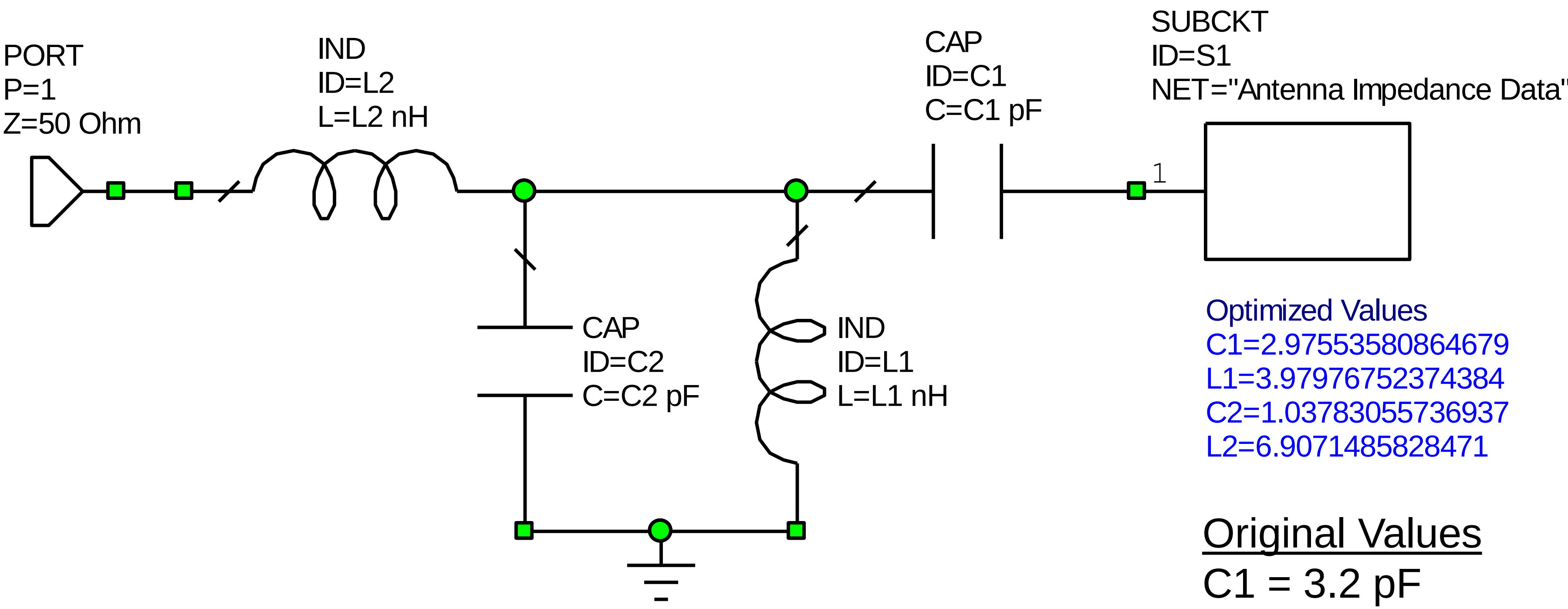
P.J. Bevelacqua, "Dual-Band Impedance Matching," online at http://www.antenna-theory.com/tutorial/smith/smithchartC.php

SWR After Matching



P.J. Bevelacqua, "Dual-Band Impedance Matching," online at http://www.antenna-theory.com/tutorial/smith/smithchartC.php

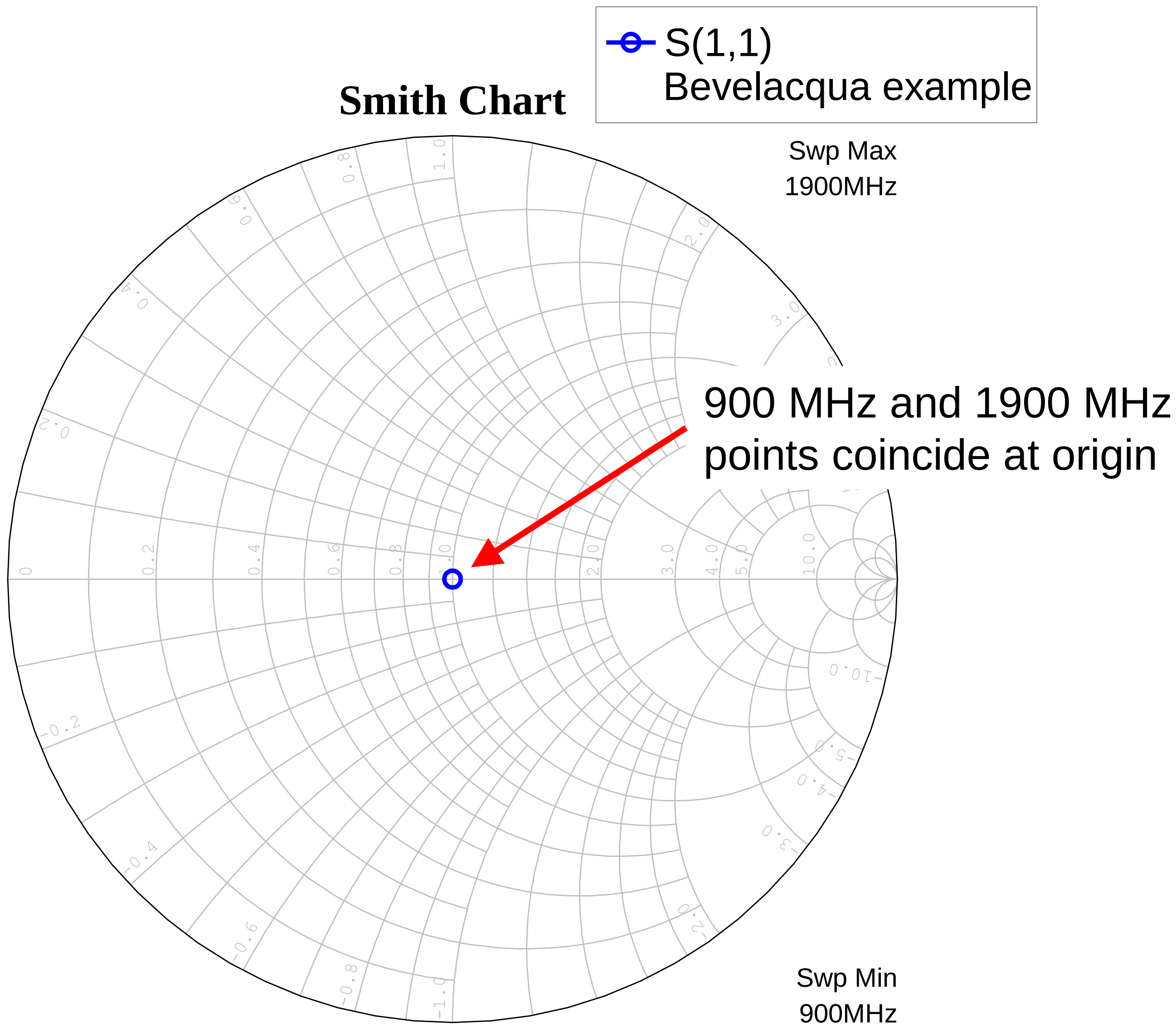
Missing Final Step – Fine Tuning!



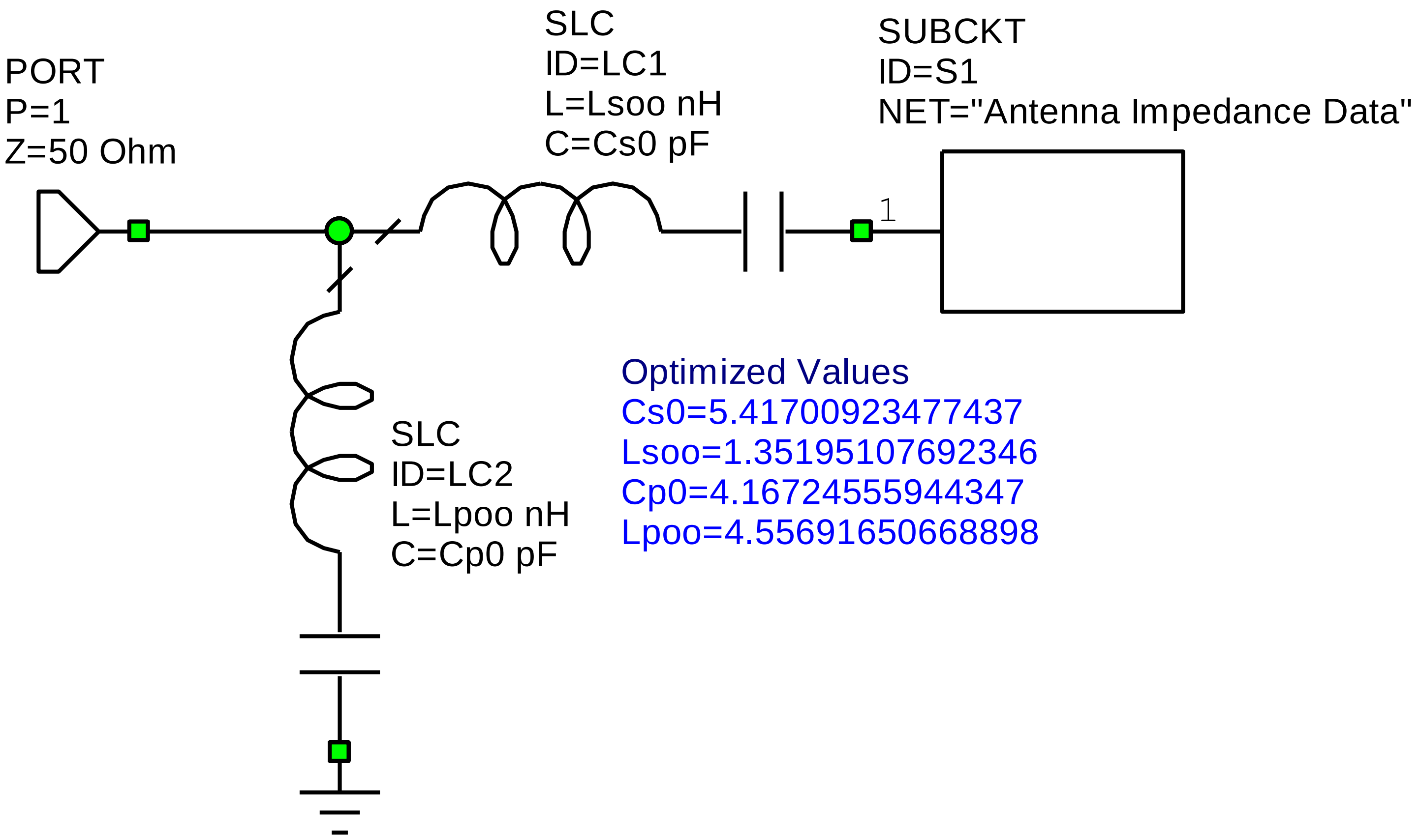
Optimized Values
C1=2.97553580864679
L1=3.97976752374384
C2=1.03783055736937
L2=6.9071485828471

Original Values
C1 = 3.2 pF
L1 = 3 nH
C2 = 1.5 pF
L2 = 5 nH

A Perfect 2-Frequency Match Shown in Microwave Office

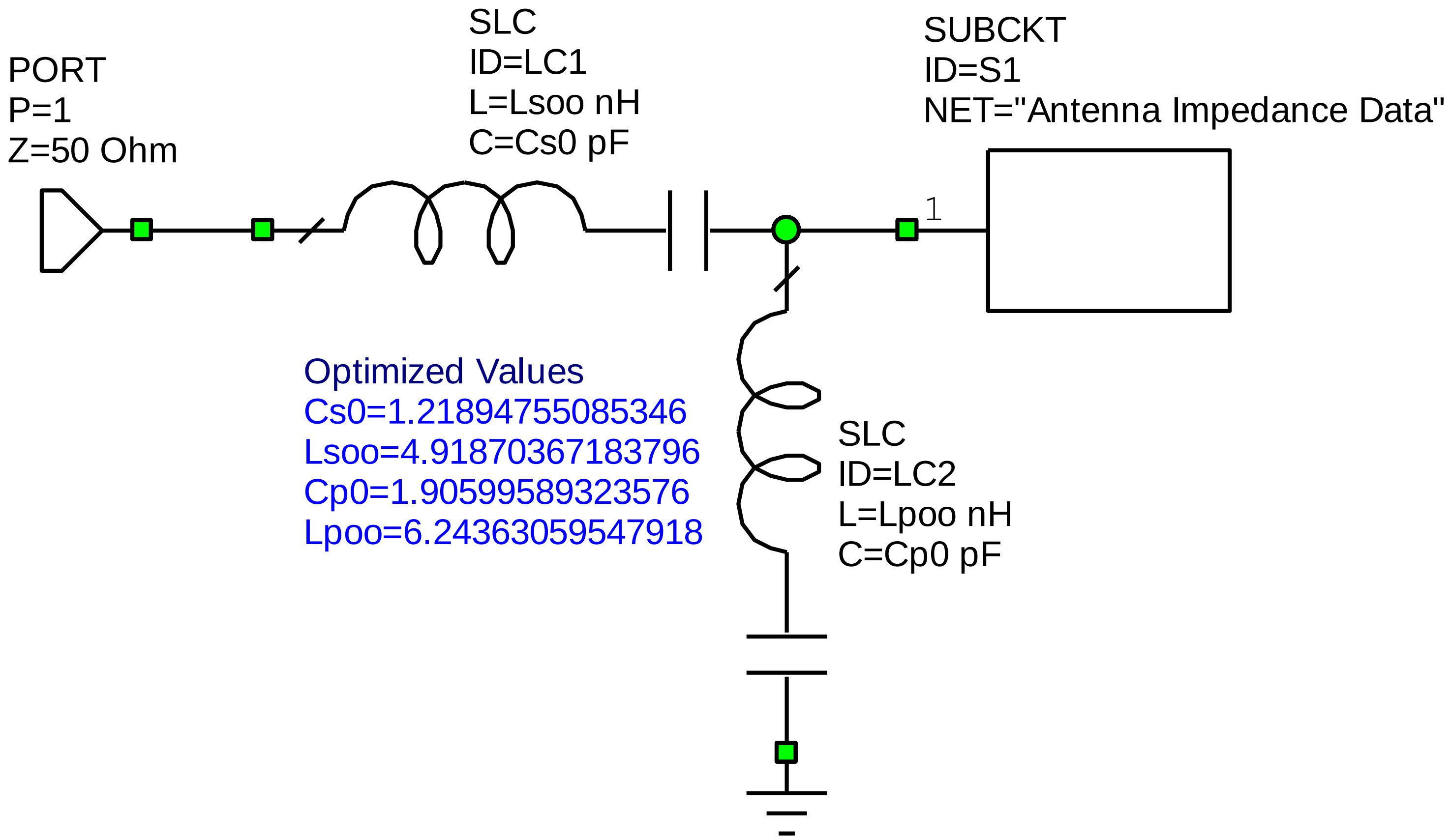


Author’s Alternate Matching Network No. 1



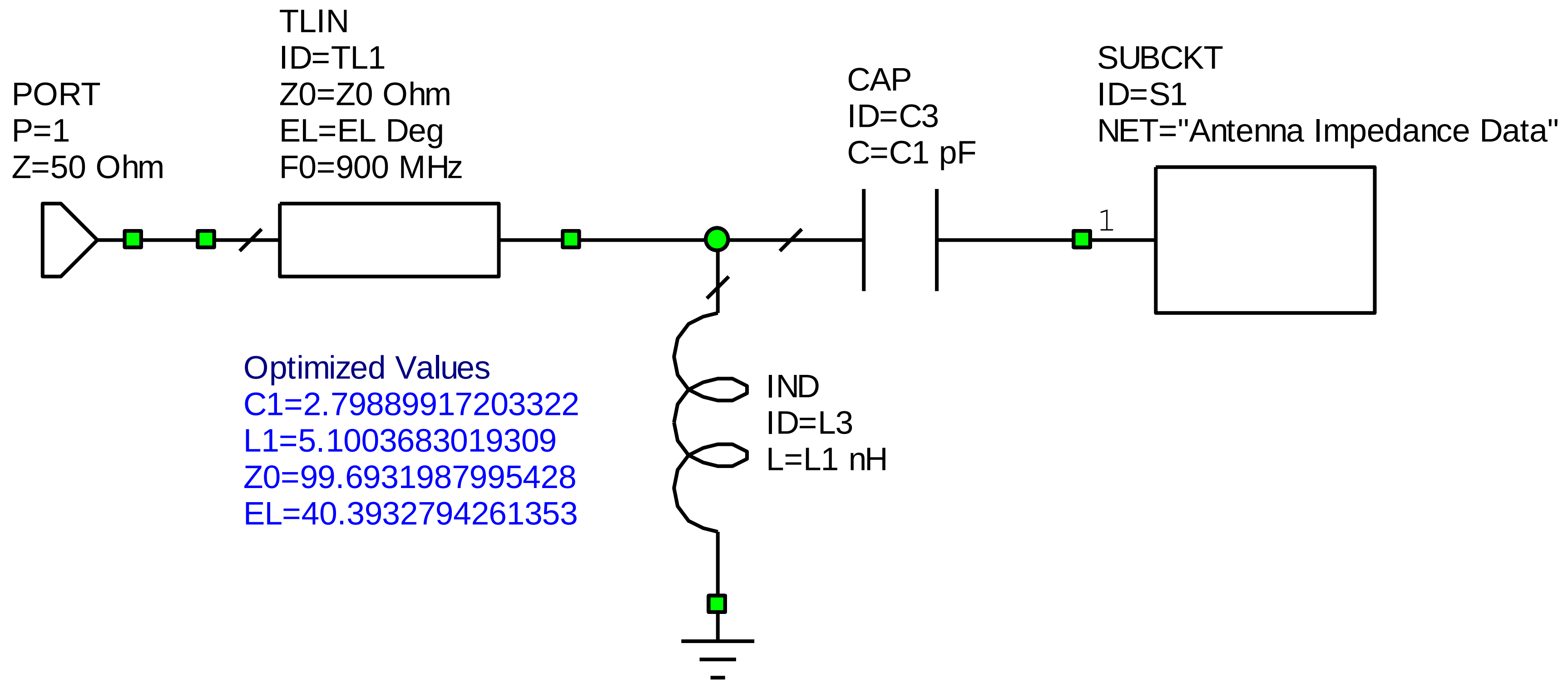
Series-Shunt L Topology Using 4 Lumped Elements

Author's Alternate Matching Network No. 2

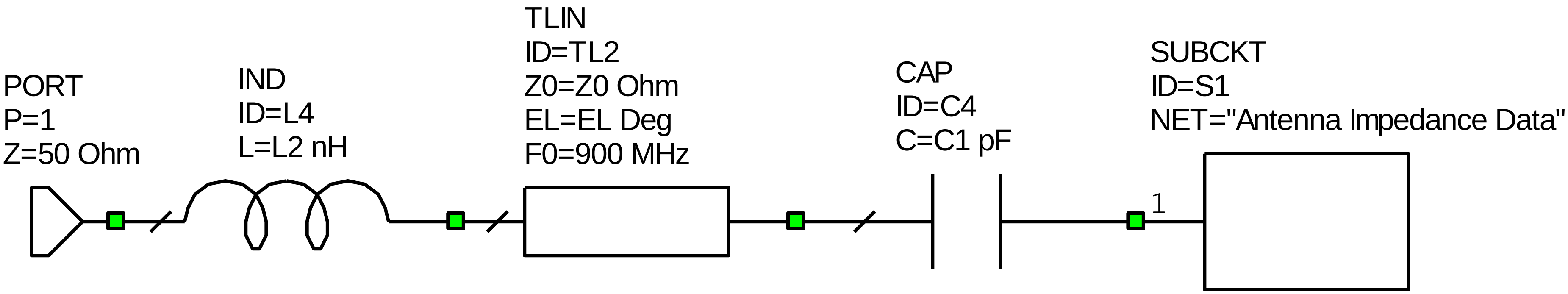


Shunt-Series L Topology Using 4 Lumped Elements

Author's Alternate Matching Network No. 3

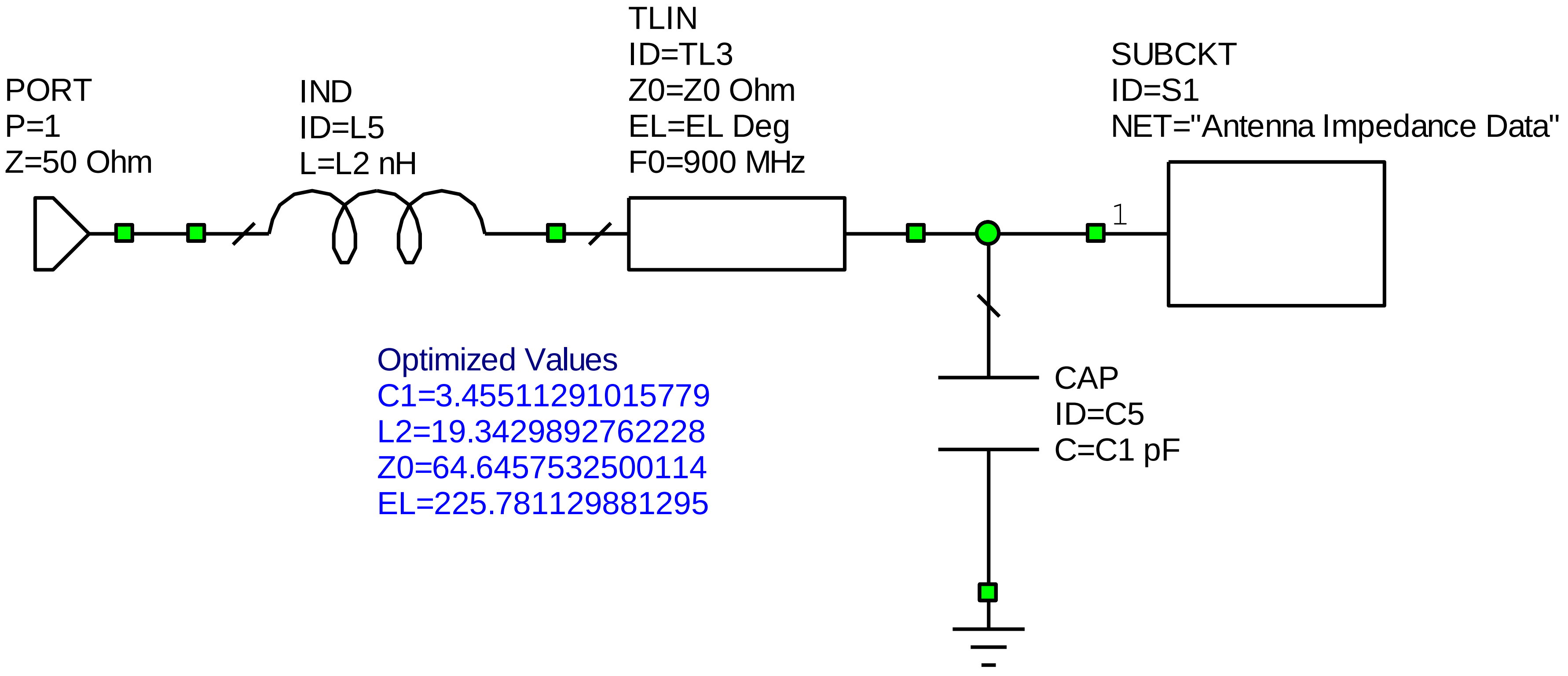


Author’s Alternate Matching Network No. 4

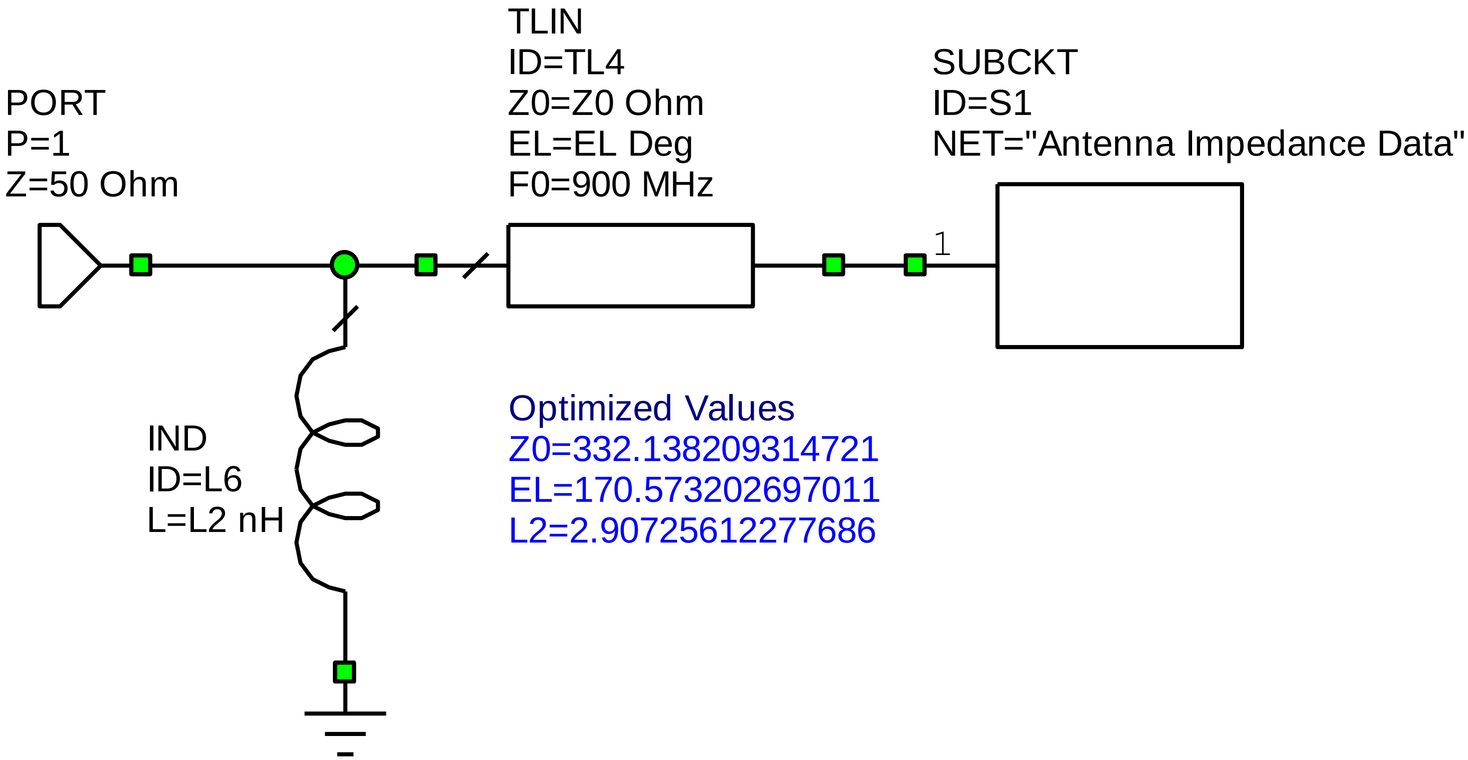


Optimized Values
C1=3.00320074469912
Z0=4.73187364081775
EL=169.410323034274
L2=6.88846379694707

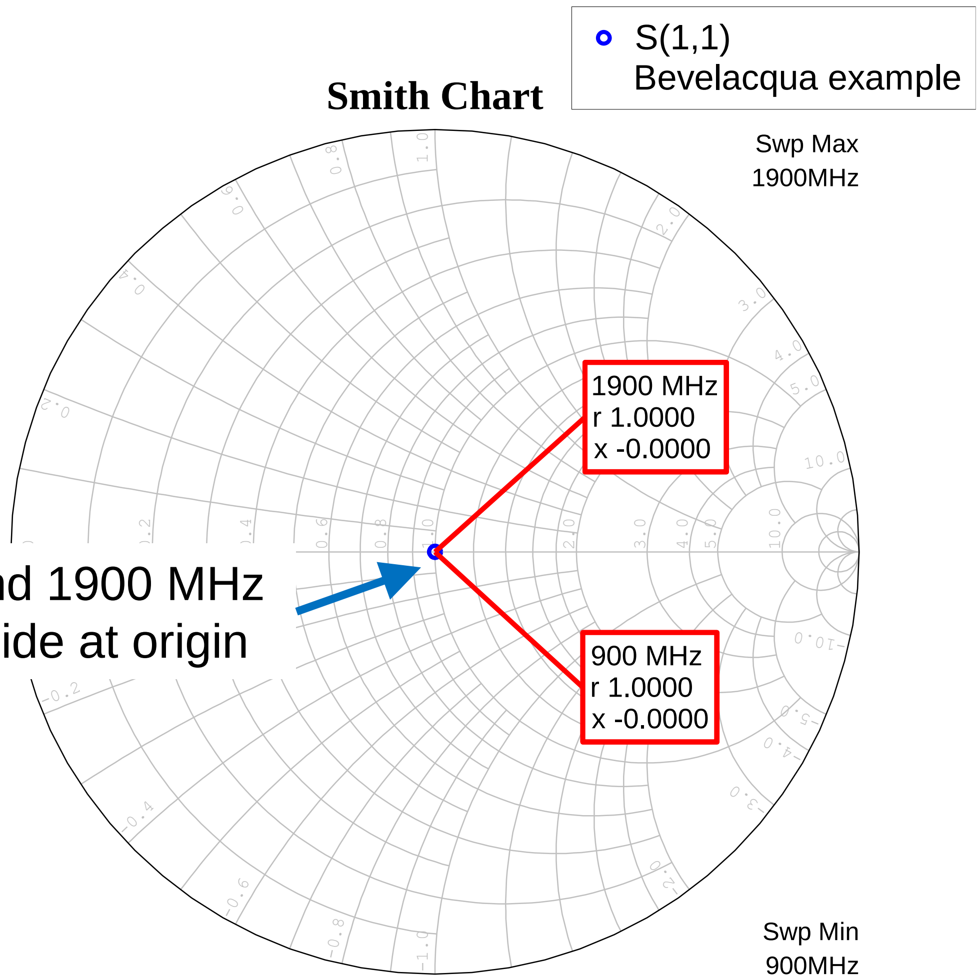
Author’s Alternate Matching Network No. 5



Author’s Alternate Matching Network No. 6



All 7 Networks Achieve Perfect 2-Frequency Match



900 MHz and 1900 MHz points coincide at origin

Choice among networks can depend on losses, ease of practical implementation, sensitivity to parts tolerances, statistical yield, or other criteria.

Matching at Three Frequencies

Example 3: KM5KG 3-frequency match (*QEX*, 2001)

Example 4: Author's 3-band match for 75, 60, and 40 meters

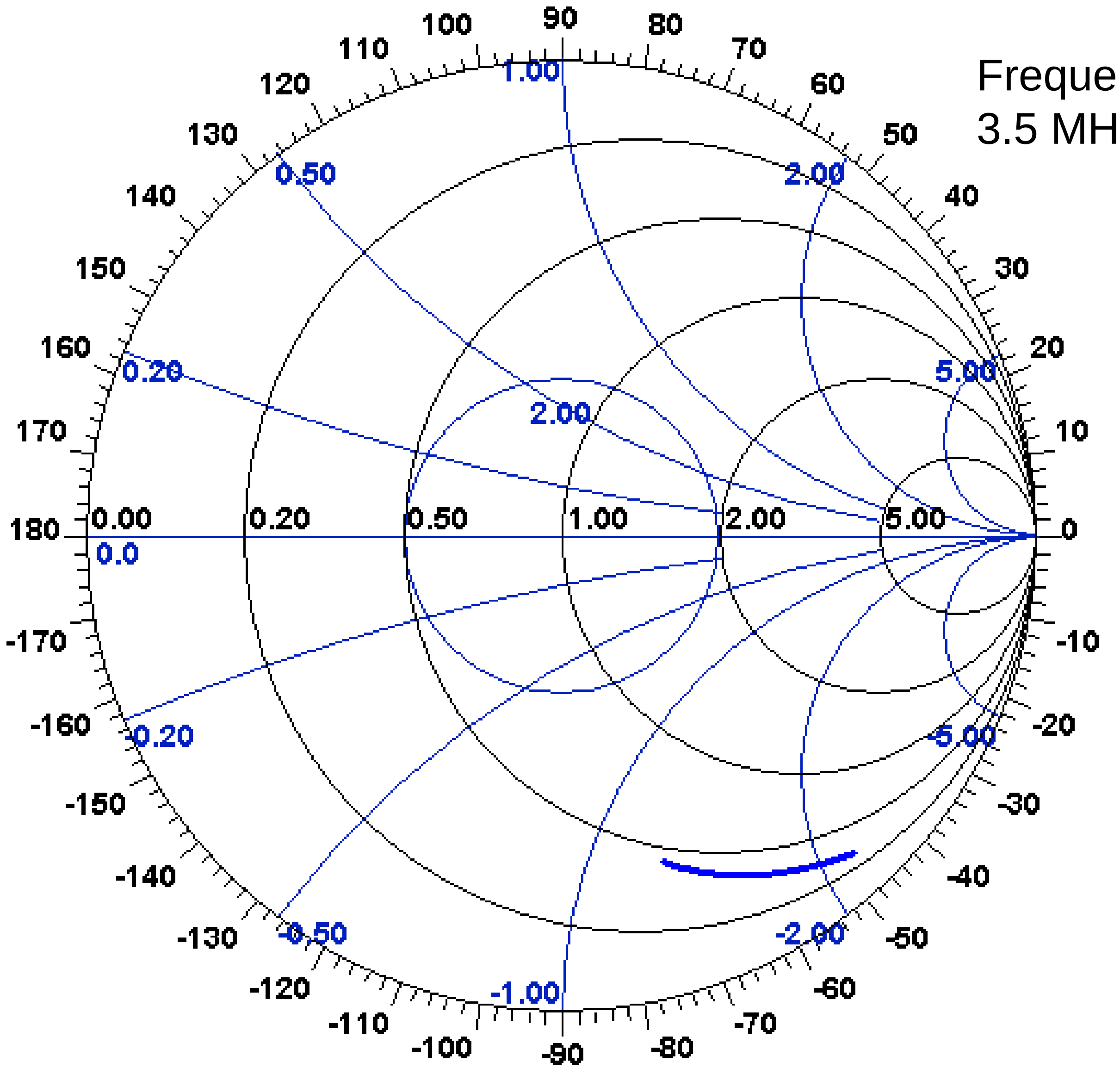
Example 5: Author's 3-band match for 75, 40, and 20 meters

Example 3: KM5KG's 3-Frequency Match for 80-Meter Band
QEX, Sept /Oct. 2001

- Illustrates pitfalls of attempting wideband matching at fewer discrete frequencies than the order of the network
- Antenna: An electrically-short vertical monopole
- KM5KG matched three frequencies in 80-meter band

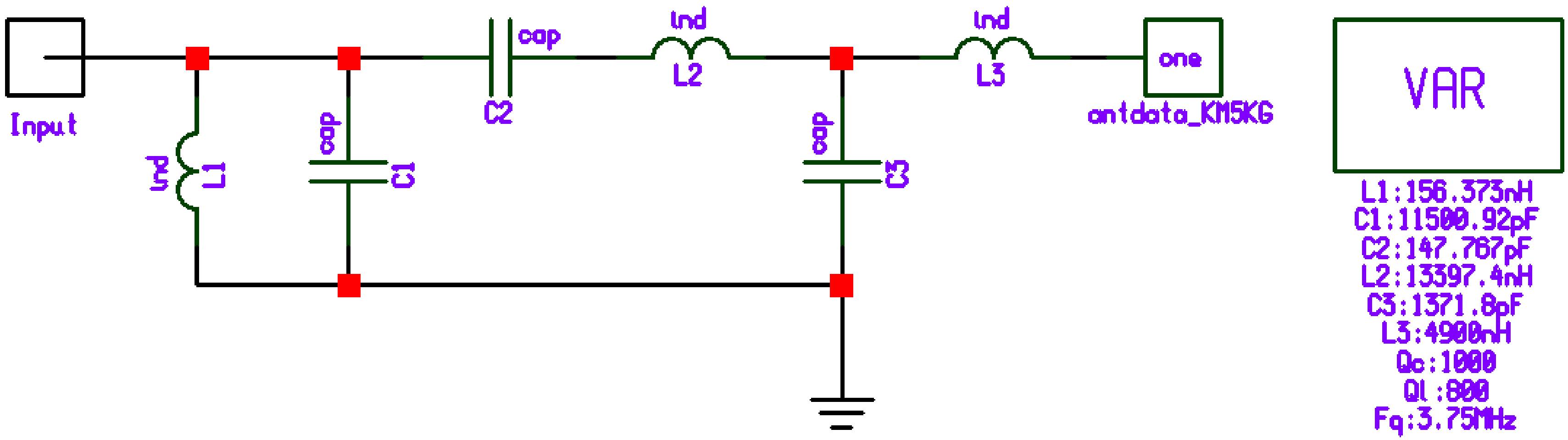
Frequency (MHz)	Real (ohms)	Imaginary (ohms)
3.500	15.9	-112.0
3.750	19.0	-86.6
4.000	22.6	-62.6

Impedance of KM5KG's Vertical Monopole



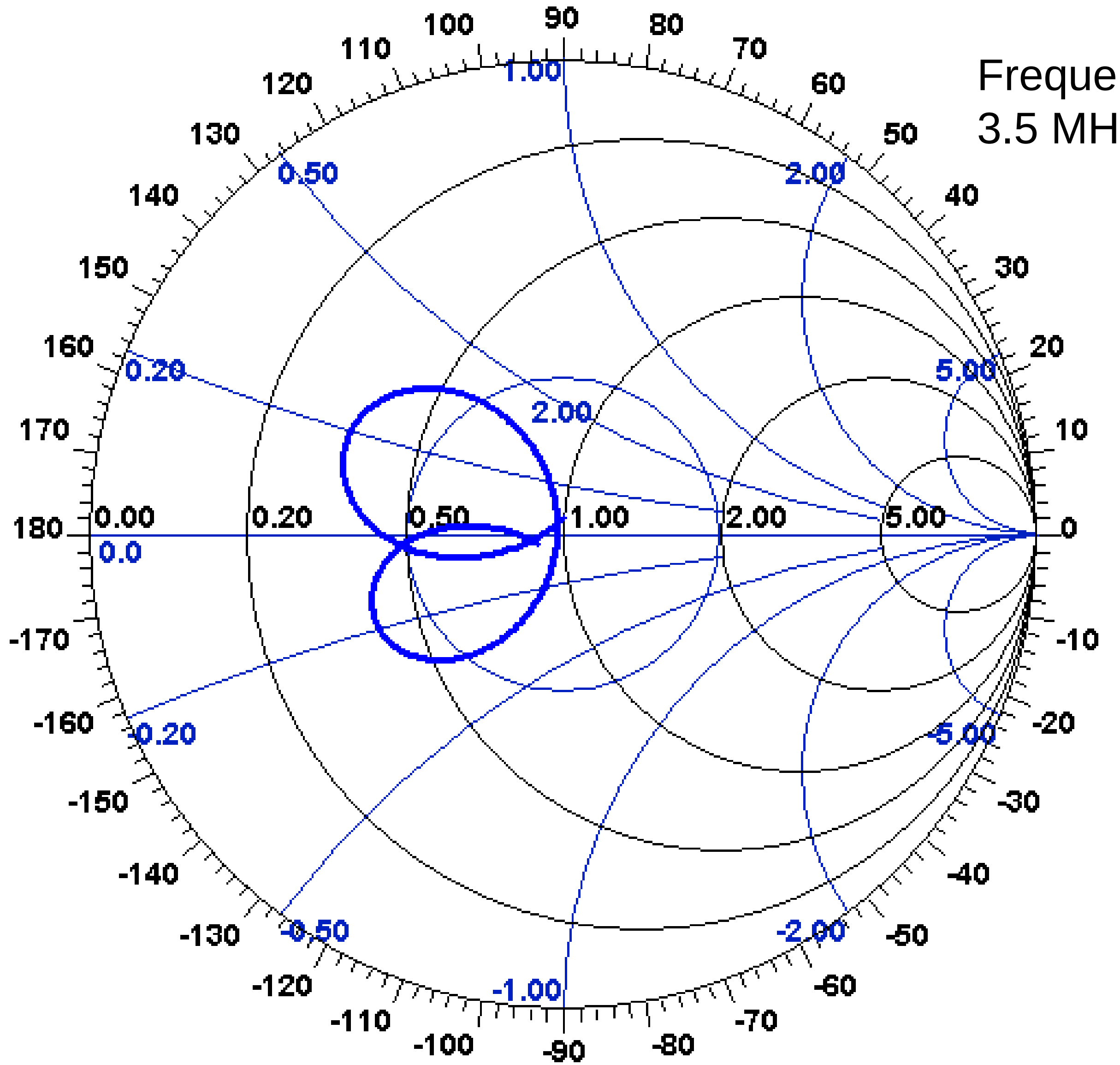
Frequency sweep
3.5 MHz to 4 MHz

KM5KG's Solution: A 3-Frequency, 6-Element Match Network



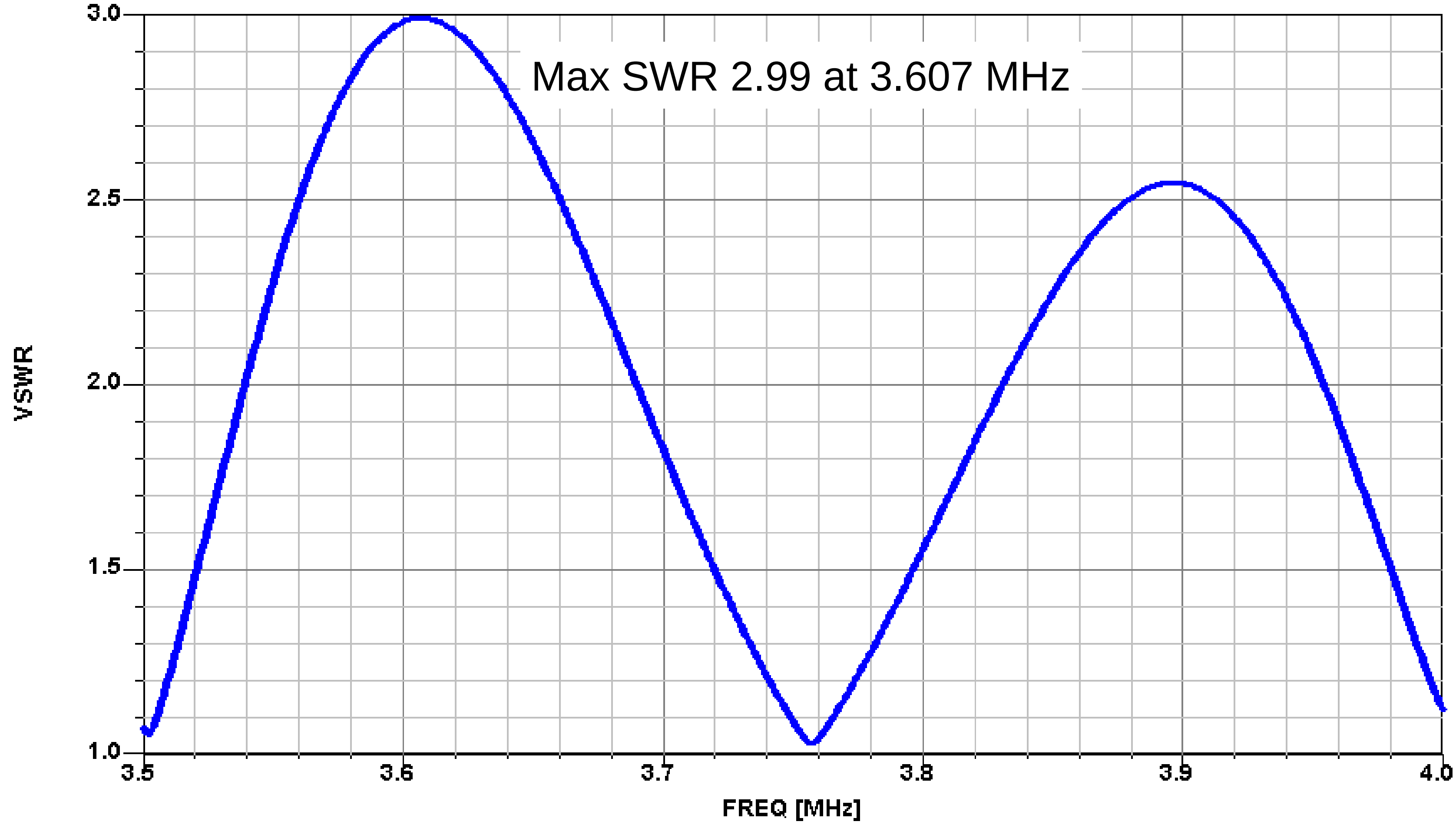
- **Component losses included**
  - Capacitors: $Q_C = 1{,}000$
  - Inductors: $Q_L =$ 800
- **Components calculated for match at 3.5, 3.75, and 4 MHz**
- **KM5KG assumed that matching at 3 frequencies (band edges and midpoint) would result in a continuous frequency match across the 80-meter band**
- **Network topology is correct for Wheeler "triple-tuned" broadband match, but components were calculated for 3-frequency match instead**

Matching Result – Not Published in *QEX*



Frequency sweep
3.5 MHz to 4 MHz

A 3-Frequency Match Is Not a Broadband Match!

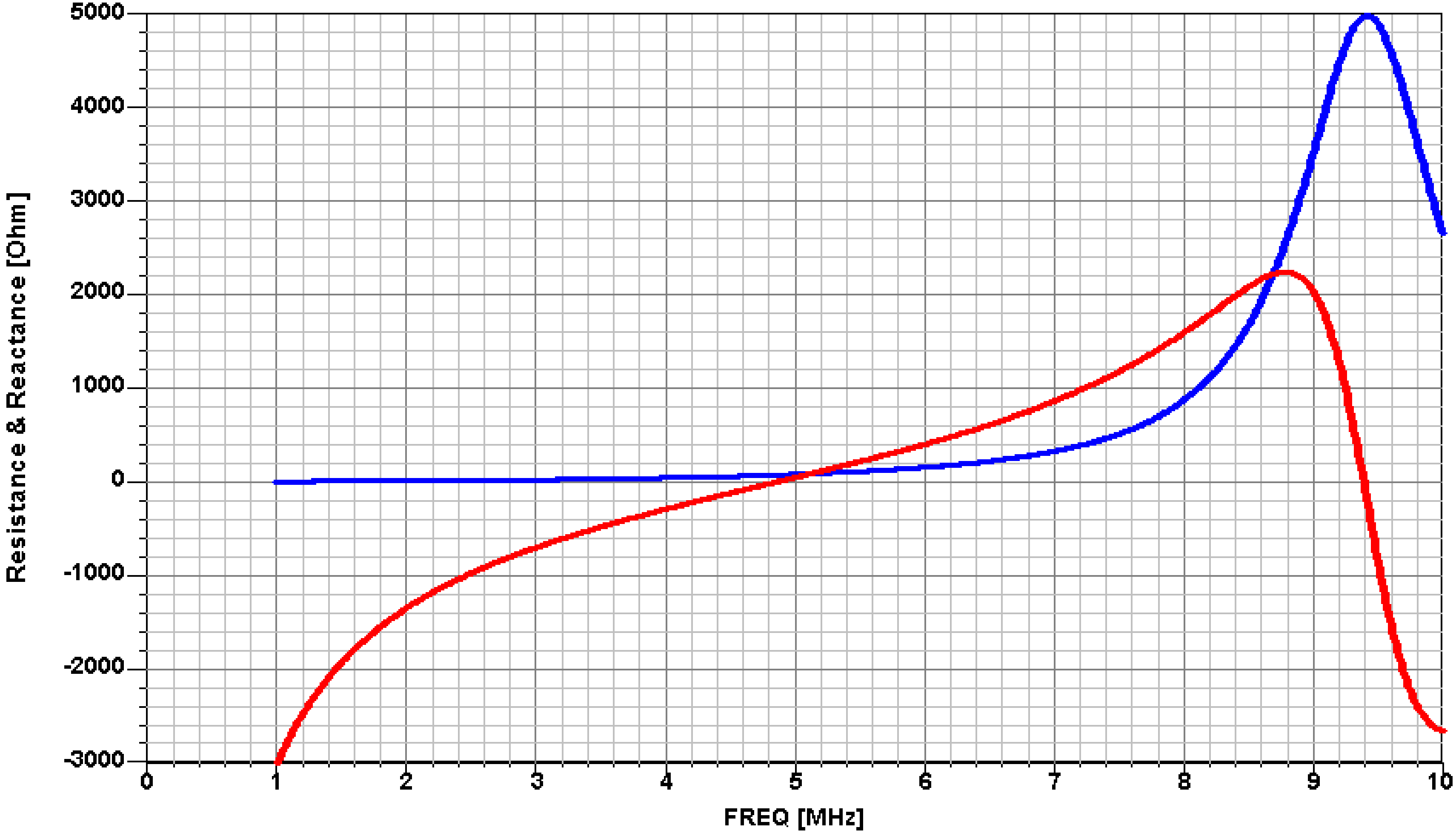


This example illustrates the Fano bound. Multi-frequency matching is the wrong approach to broadband continuous frequency matching.

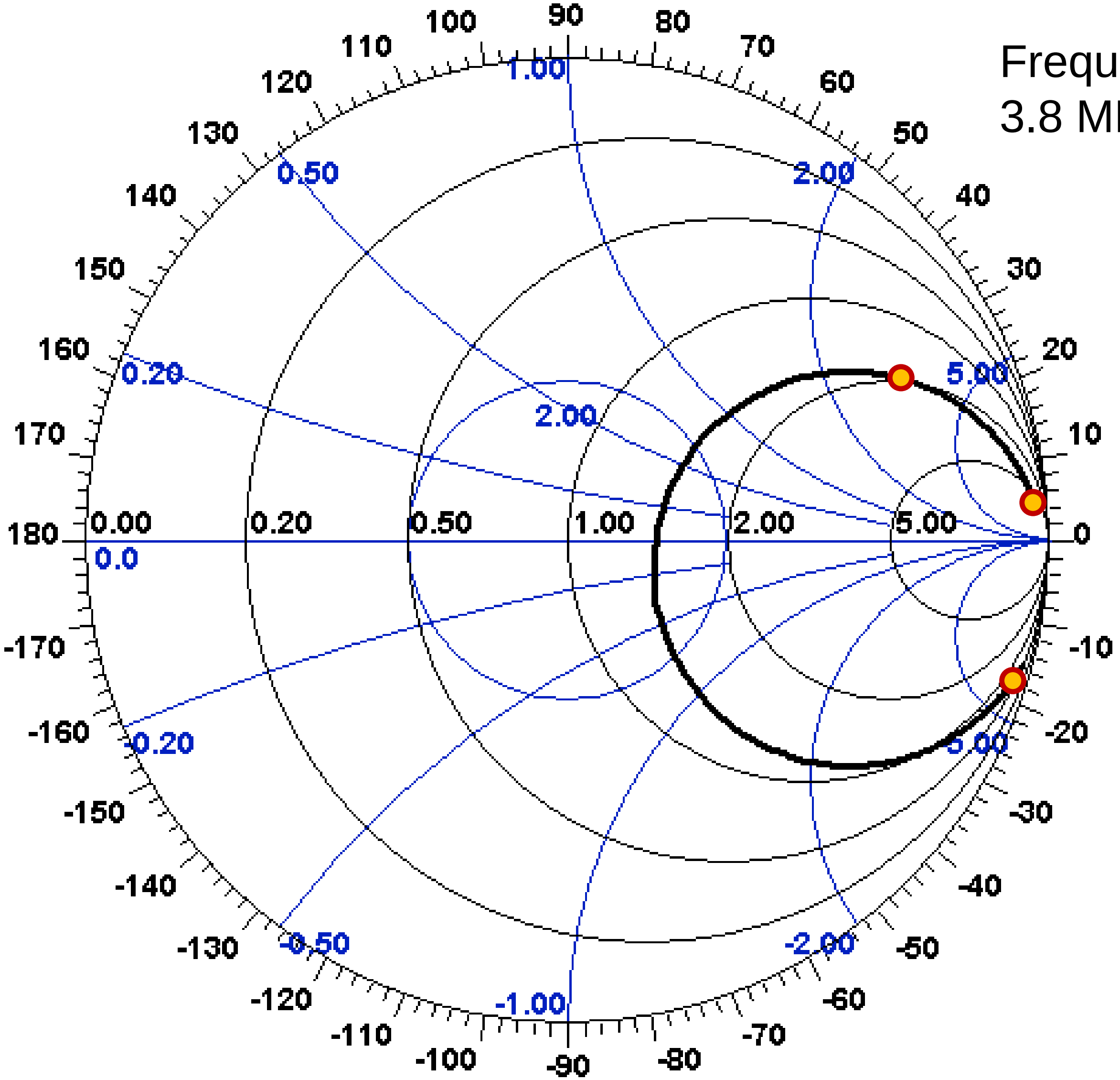
Example 4: Matching a 98.4-ft. Dipole on Three Bands

- Ward Silver (N0AX) proposed to design an antenna for the 75, 60, and 40-meter bands
- Author's "lazy man" solution: Take an existing antenna and design a feedpoint match network for the three bands
- Antenna: 98.4-foot wire dipole antenna resonant at 4.868 MHz
- Frequencies: 3.9, 5.35, and 7.2 MHz
- Design goal: SWR = 1 at these three frequencies

Dipole Impedance

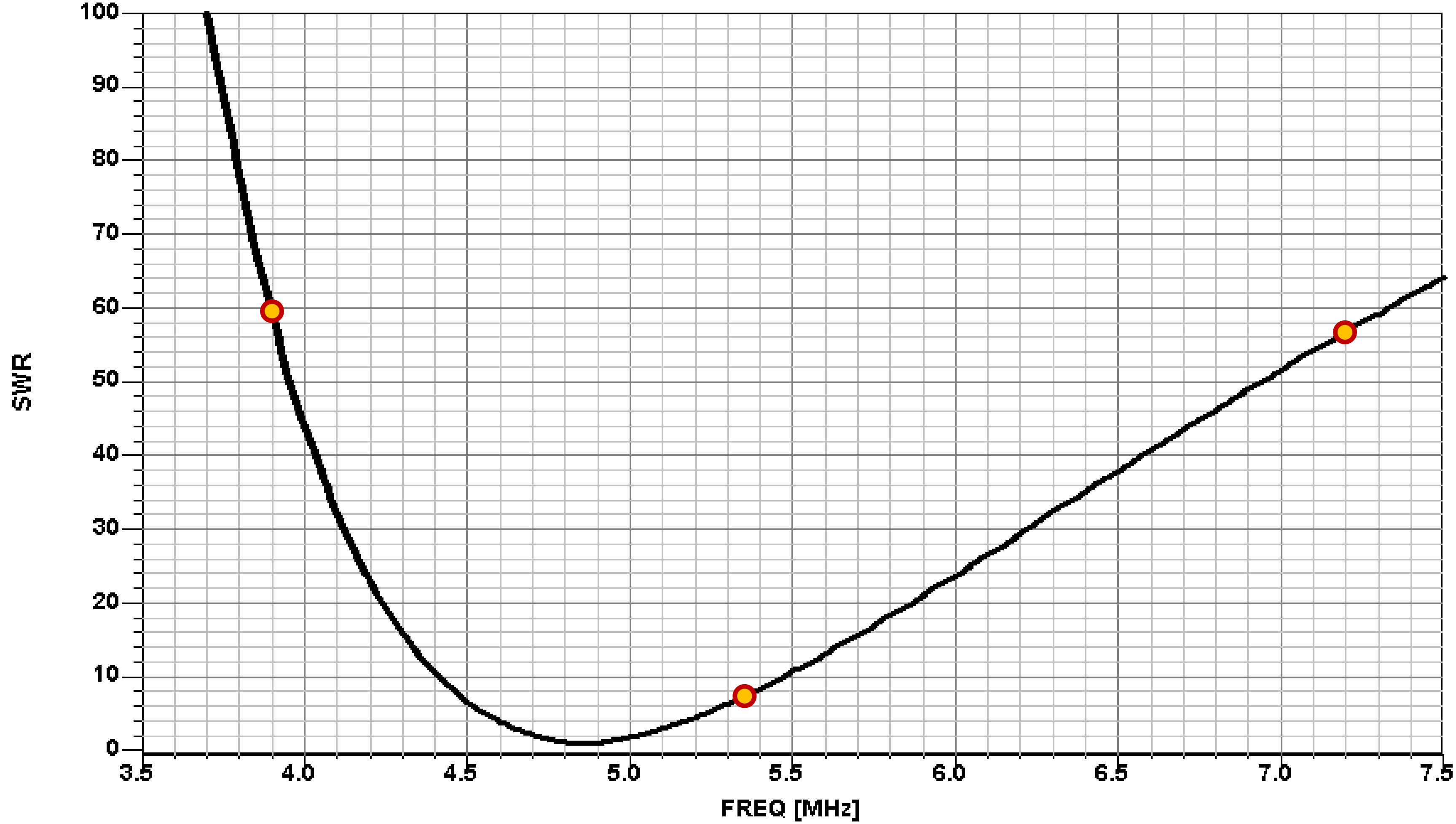


Dipole Impedance on the Smith Chart

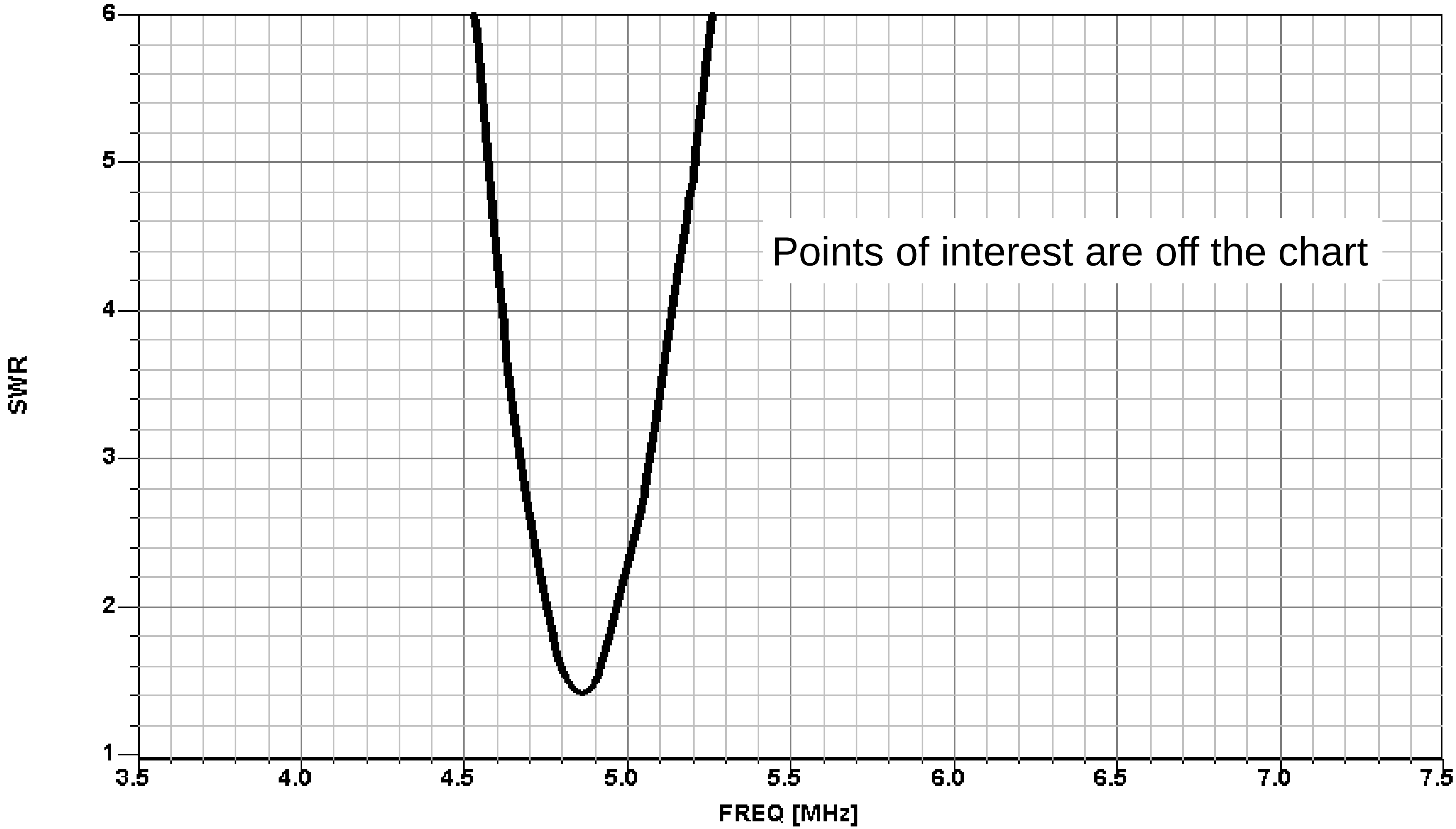


Frequency sweep
3.8 MHz to 7.3 MHz

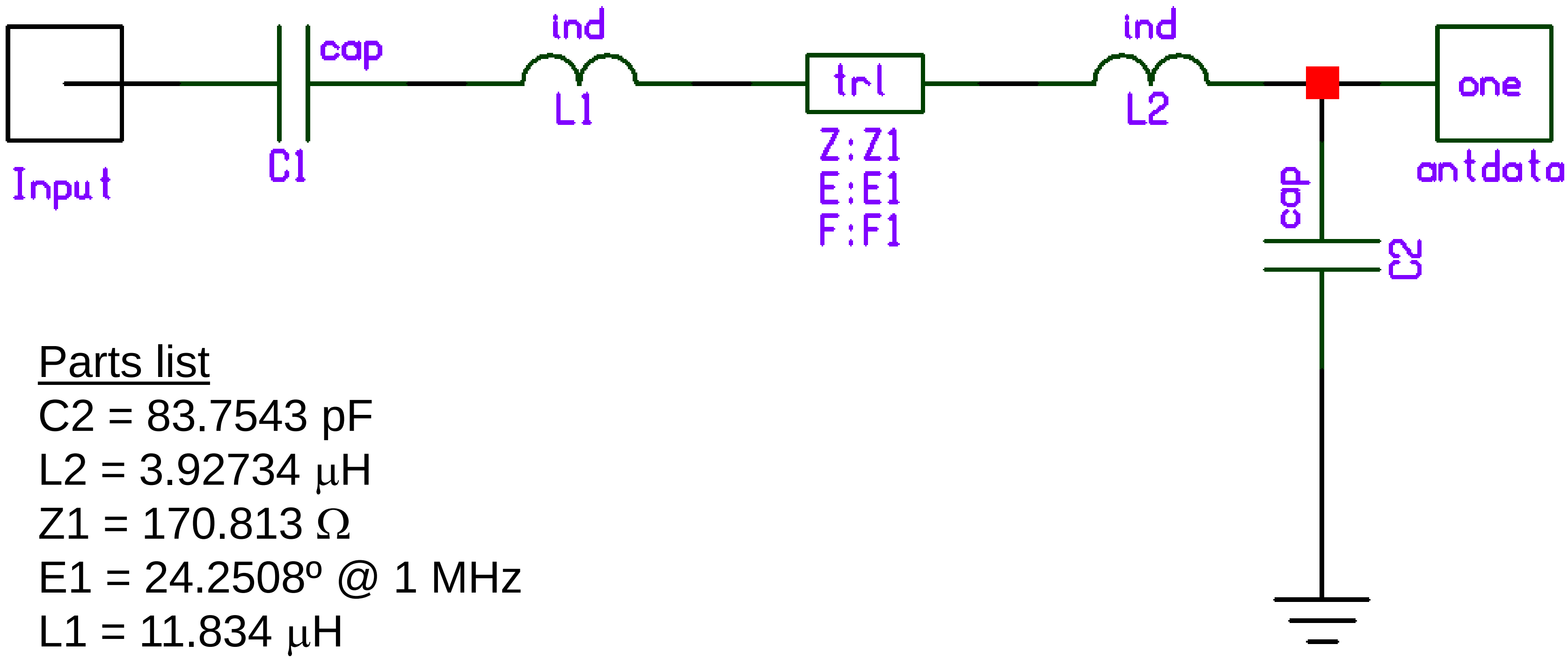
Dipole SWR Before Matching



Zoom View Dipole SWR Before Matching



5-Element, 3-Band Matching Network Shown in Serenade SV



Parts list

C2 = 83.7543 pF

L2 = 3.92734 μH

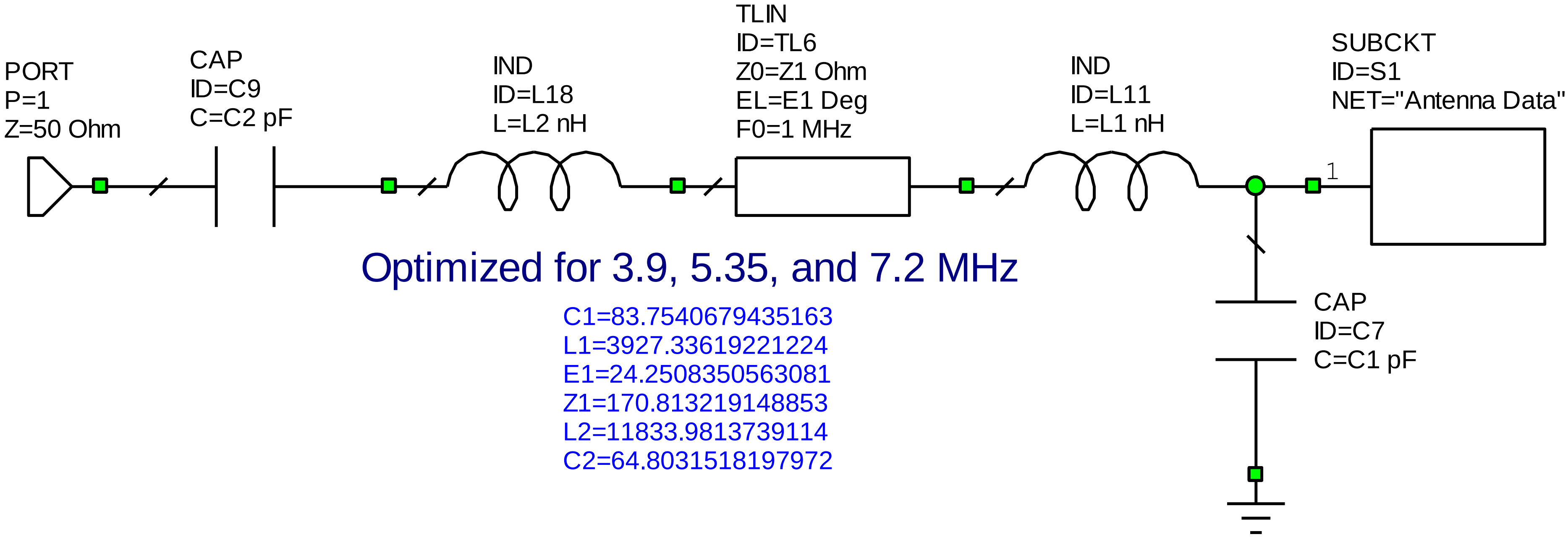
Z1 = 170.813 Ω

E1 = 24.2508º @ 1 MHz

L1 = 11.834 μH

C1 = 64.8033 pF

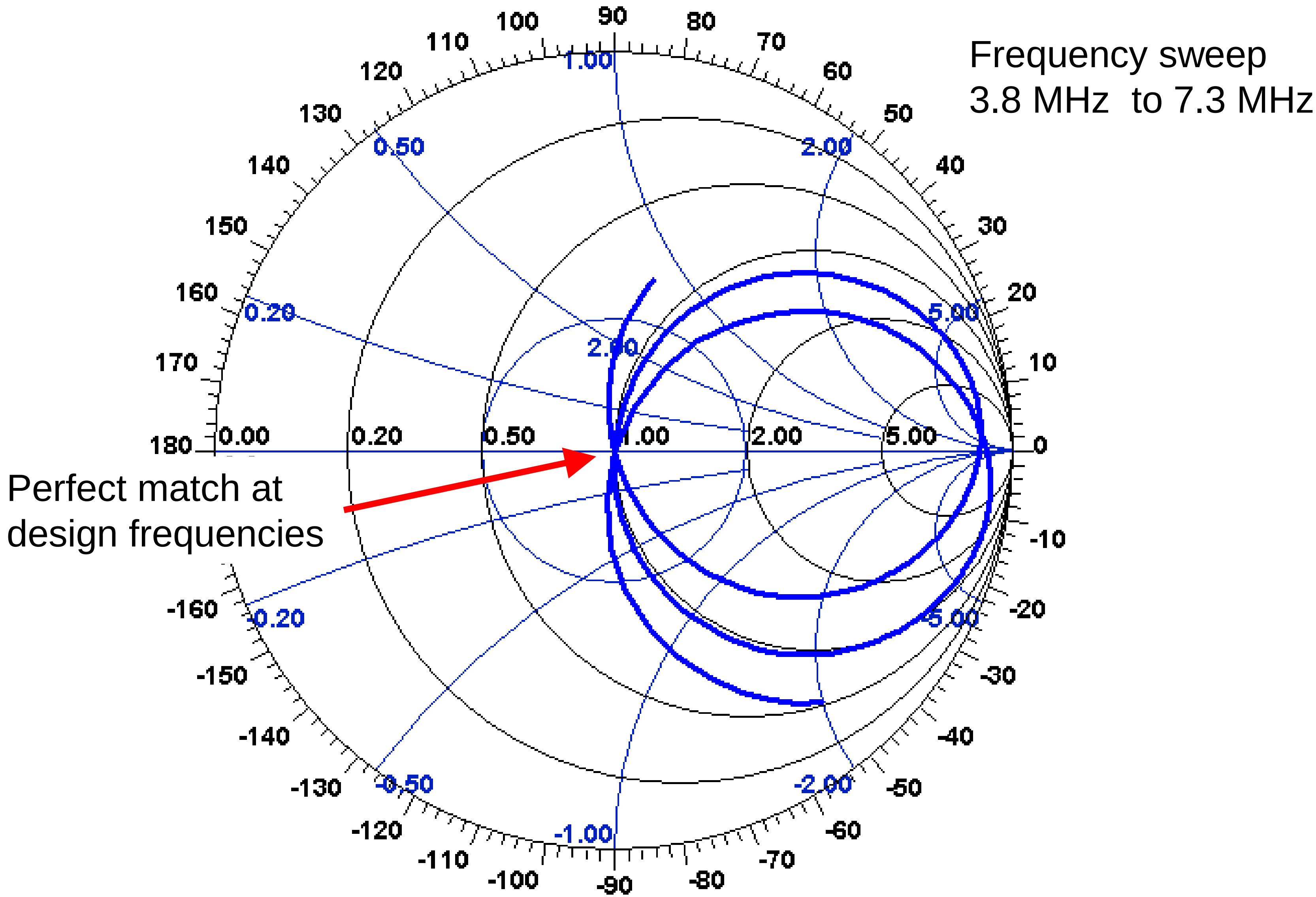
Matching Network Shown in Microwave Office



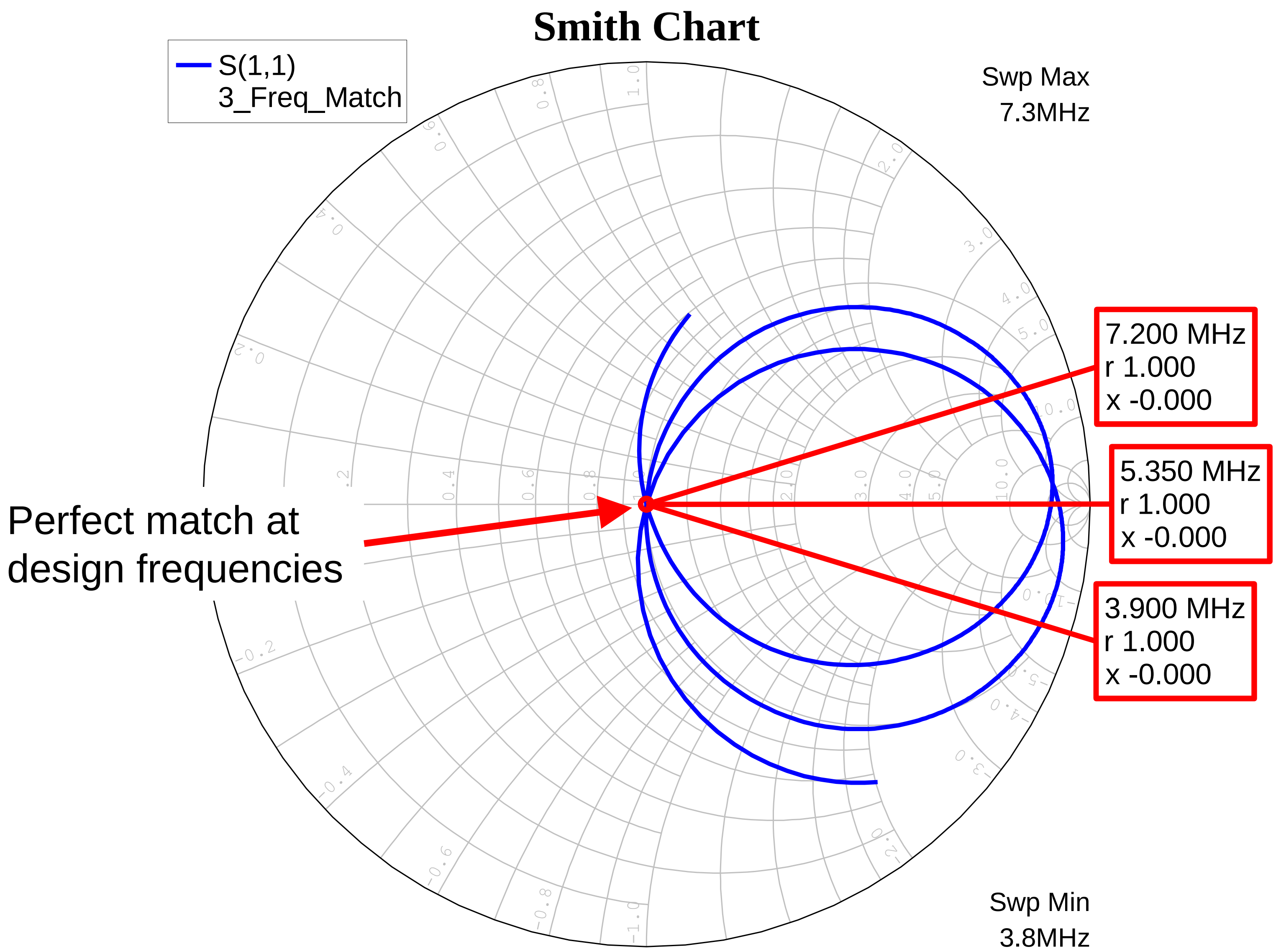
Optimized for 3.9, 5.35, and 7.2 MHz

C1=83.7540679435163
L1=3927.33619221224
E1=24.2508350563081
Z1=170.813219148853
L2=11833.9813739114
C2=64.8031518197972

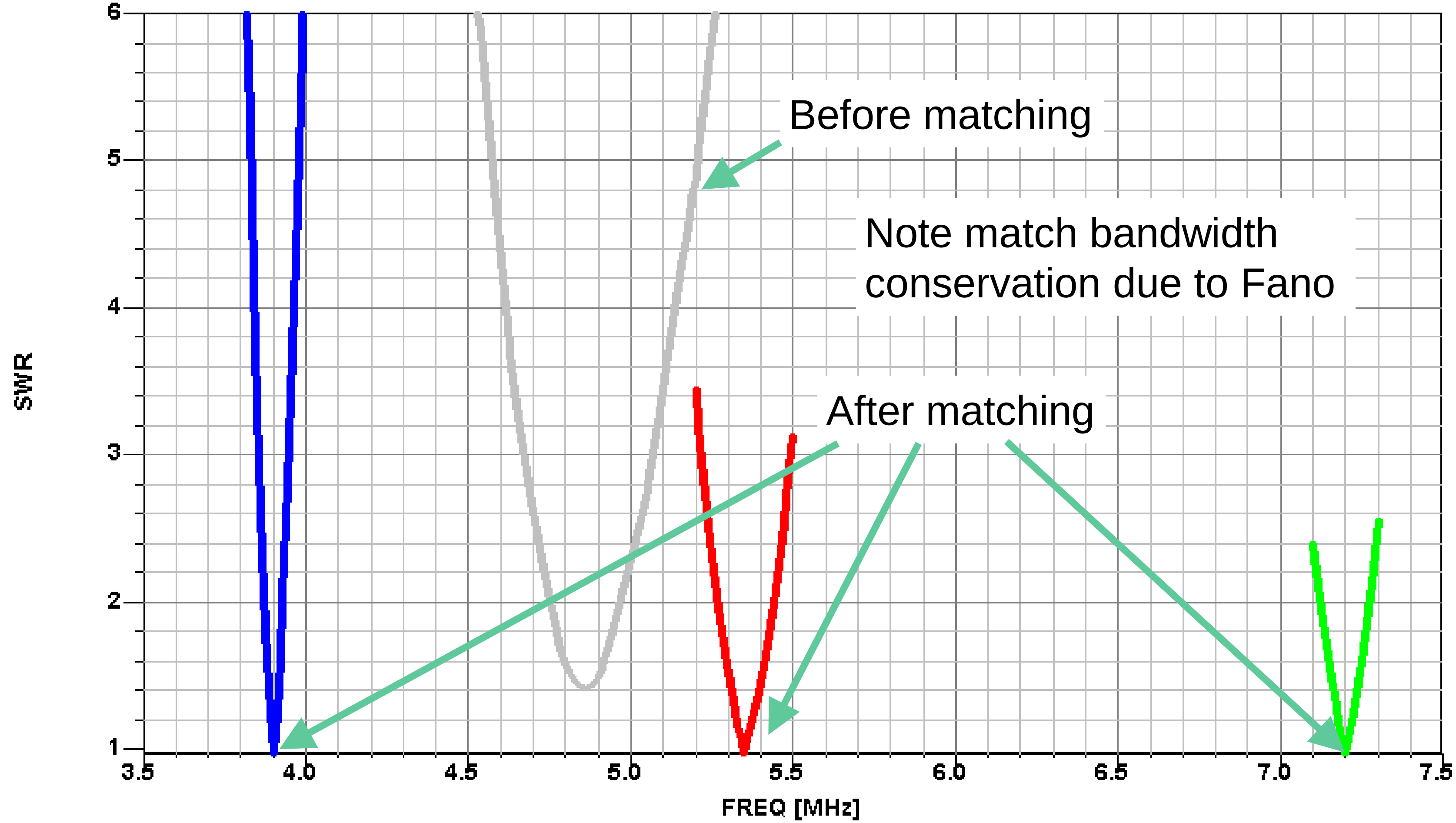
Impedance After Matching, Shown in Serenade SV



Impedance After Matching, Shown in Microwave Office

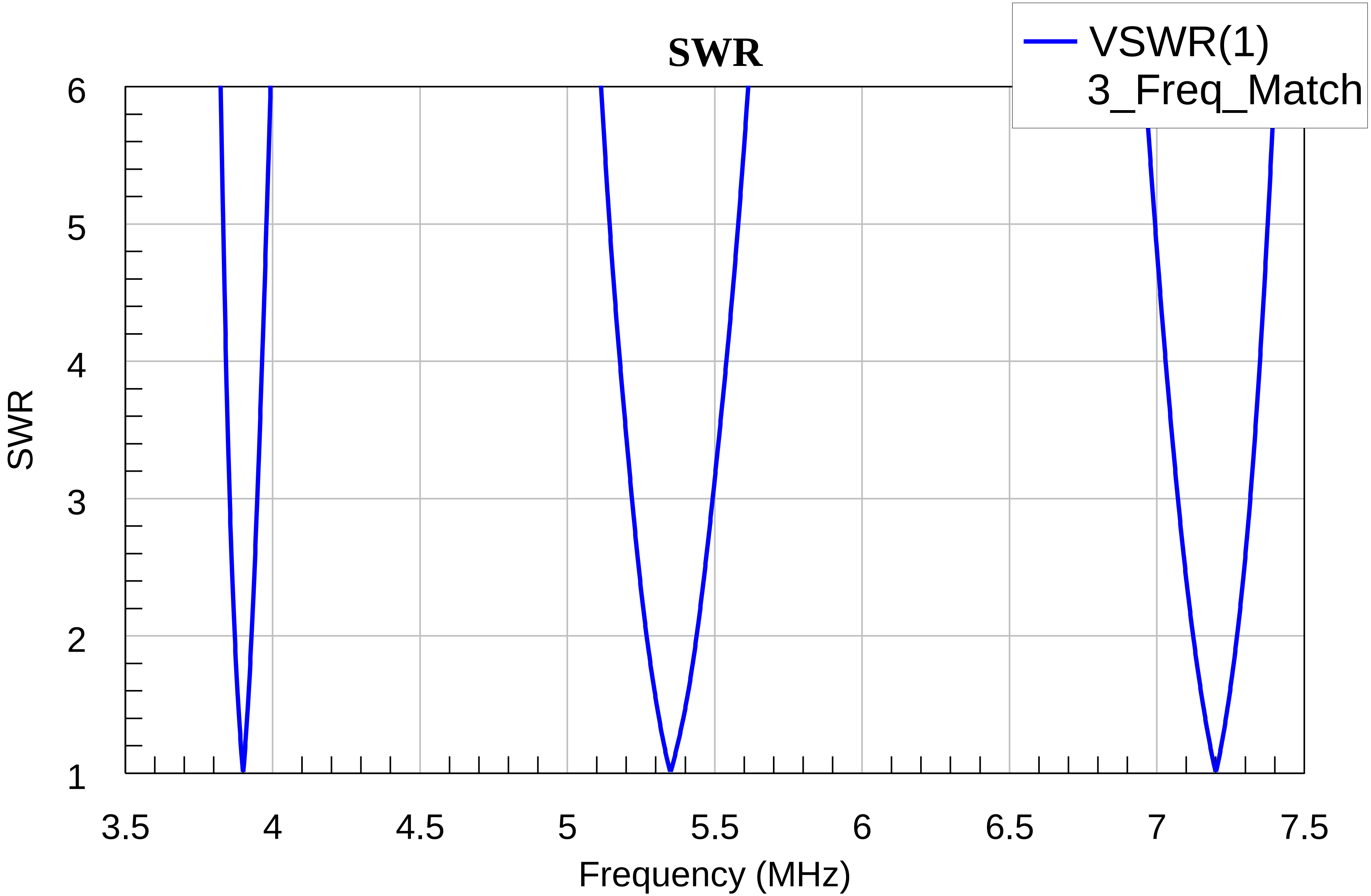


SWR After Matching, Shown in Serenade SV



The total match bandwidth for all frequencies is limited by the Fano bound. By matching more frequencies, the match bandwidth at each frequency shrinks.

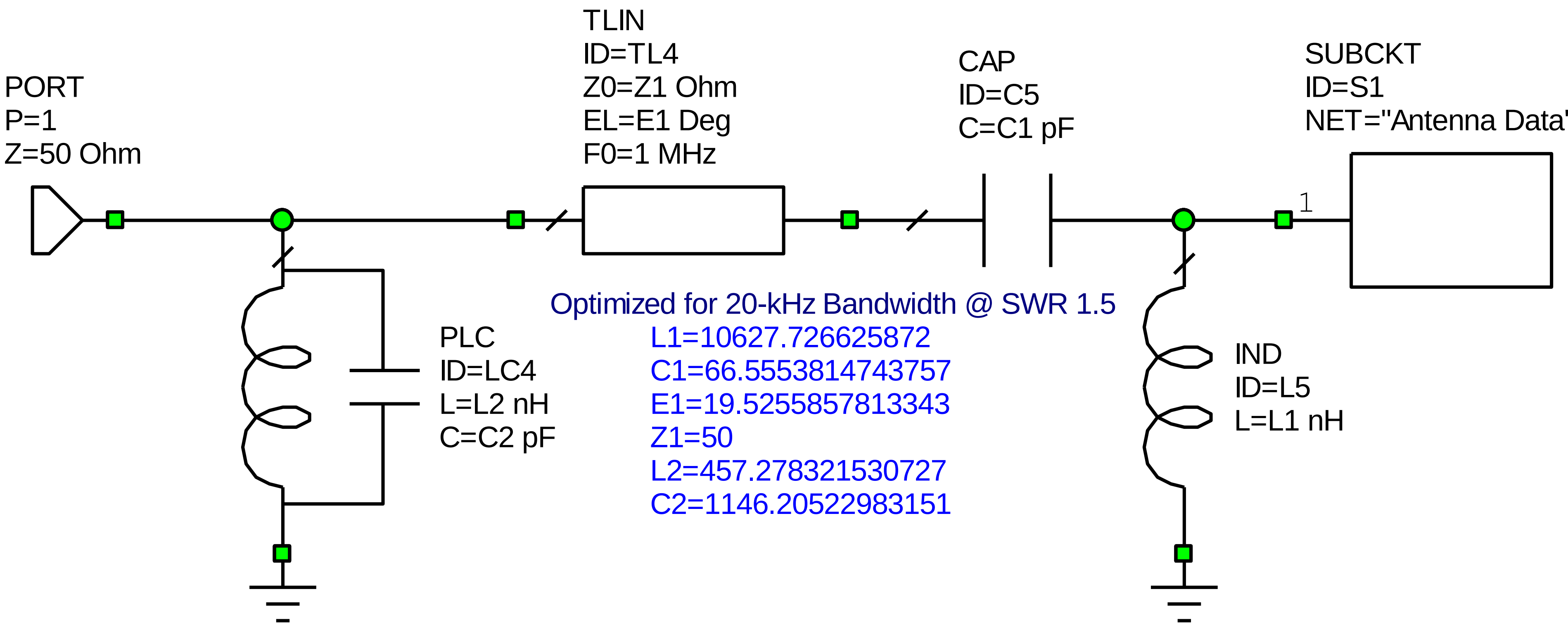
SWR After Matching, Shown in Microwave Office



Example 5: Matching 98.4-ft. Dipole on 75, 40, and 20 Meter Bands

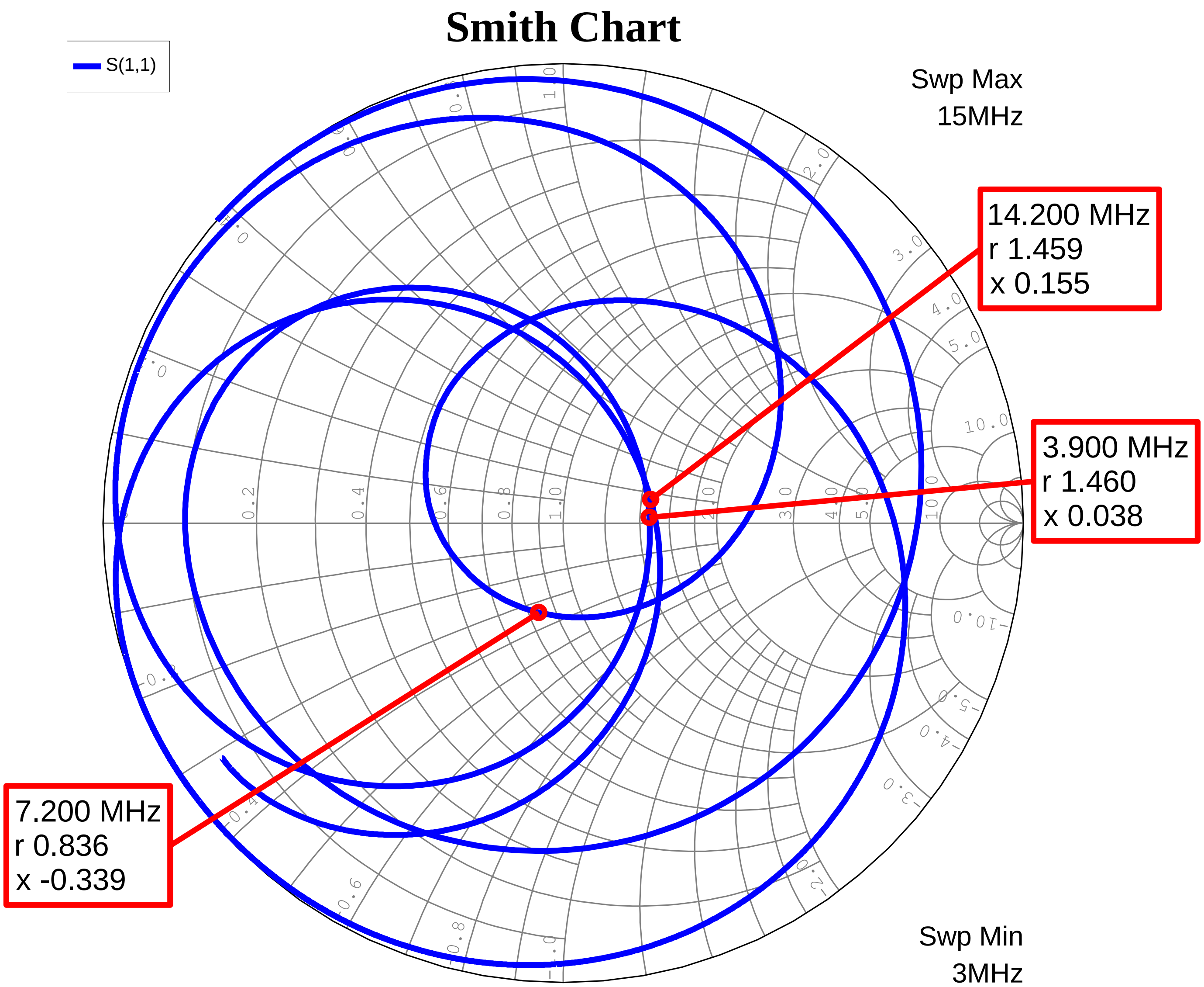
- Same as Example 4 except bands are 75, 40, and 20 meters
- Antenna: 98.4-foot wire dipole antenna resonant at 4.868 MHz
- Frequencies: 3.9, 7.2, and 14.2 MHz
- Design goal: SWR < 1.5 across 20-kHz bands centered on the design frequencies

3-Band Match Network for 3.9, 7.2, and 14.2 MHz Optimized for 20-kHz Match Bandwidths at SWR 1.5

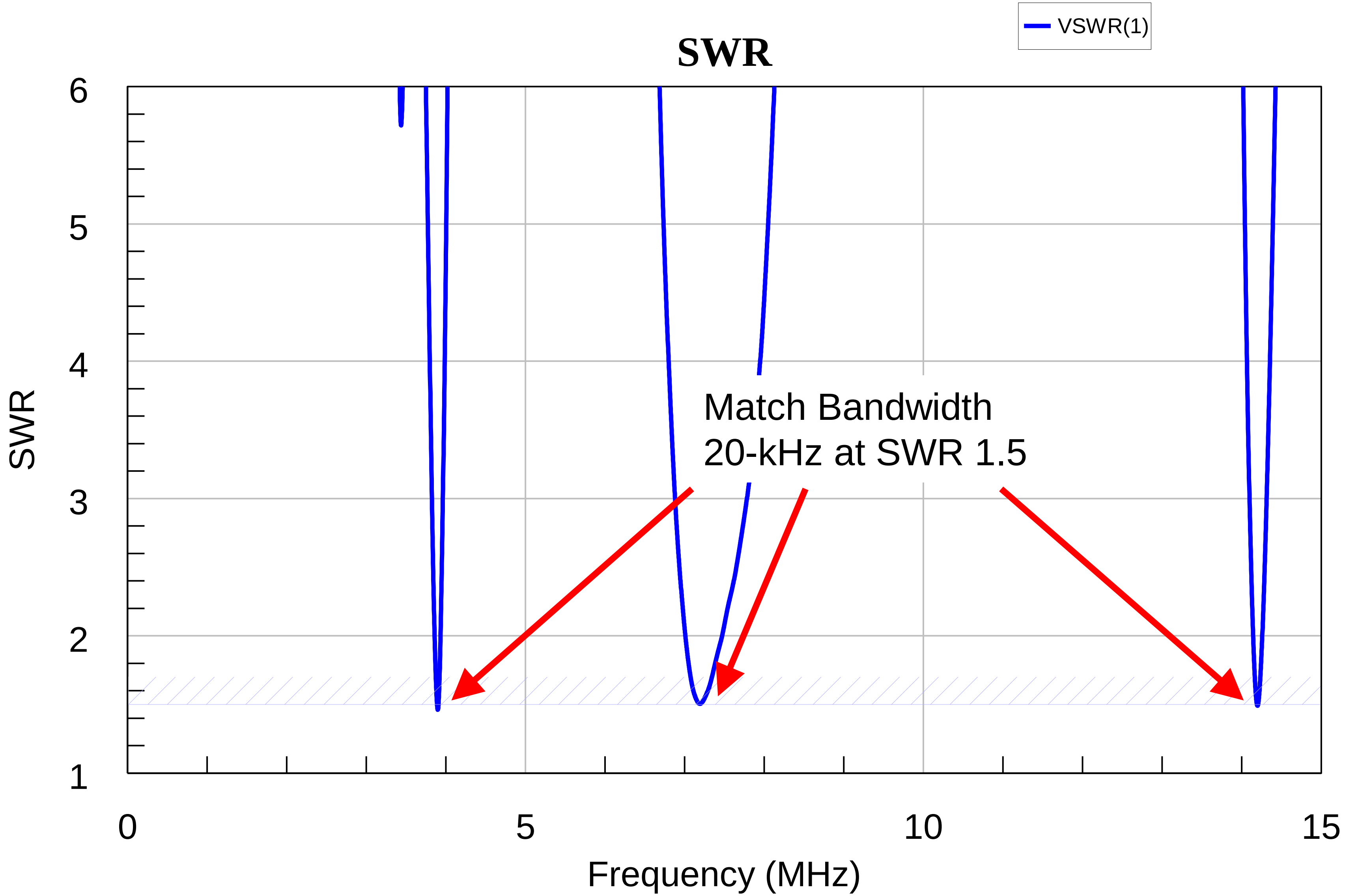


Optimized Match Network for 75, 40, and 20 meters

Match Result



SWR



VSWR(1)
SWR
6
5
4
3
2
1
SWR
0
5
10
15
Frequency (MHz)
Match Bandwidth
20-kHz at SWR 1.5

Design Procedure for 3-Frequency Matching

- **Step 1: Use an L-network to move one point to the center of the Smith chart**
- **Step 2: Add a Z_0-ohm transmission line segment to rotate another point to the unit resistance (r = 1) circle or the unit conductance (g = 1) circle such that**
  - On the r = 1 circle, the low-frequency point is clockwise from the high-frequency one
  - On the g = 1 circle, the low-frequency point is clockwise from the high-frequency one
- **Step 3: Add an LC resonator section and adjust *L* and *C* to move the points to the center of the Smith chart**
  - For points on a resistance circle, use a series LC resonator to move the points
  - For points on a conductance circle, use a shunt LC resonator to move the points
  - This step requires that you label and keep track of each point's frequency
- **Step 4: Repeat Steps 2 and 3 for the third point**
- **Three points in the proper order on the r = 1 or g = 1 circle can often be brought to center by using a single resonator stage and optimizing all values**

Do the above steps manually viewing a Smith chart and using tuning sliders. When the points are close to center, use an optimizer to finish fine tuning the parts values.

Matching at More than 3 Frequencies

Subharmonic stubs for adding a low frequency to an existing design

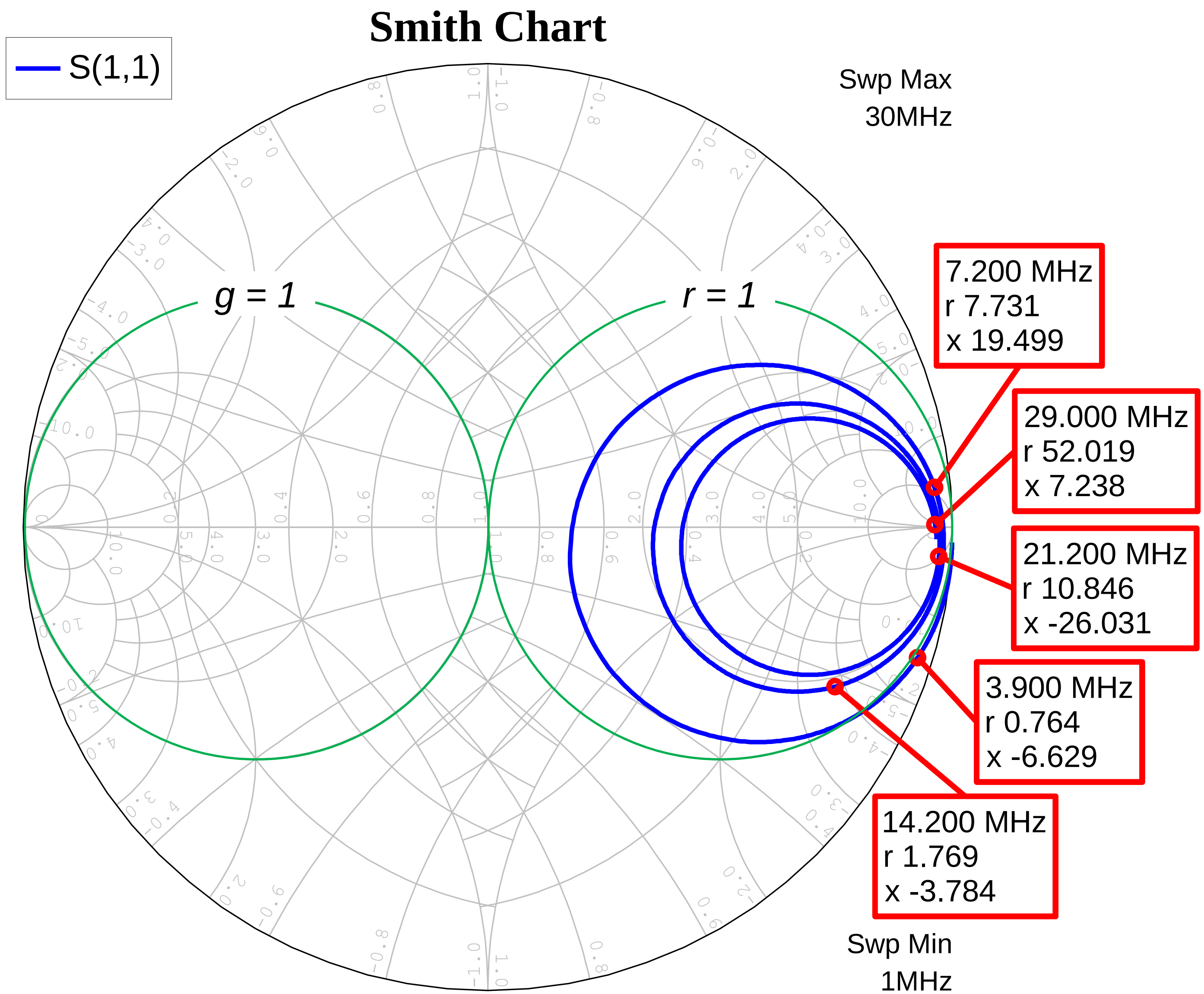
Distributed matching via network loads on a main transmission line

L-network synthesis for matching an arbitrary number of frequencies

Examples 6 and 7: Add Top Band to Existing Match Networks

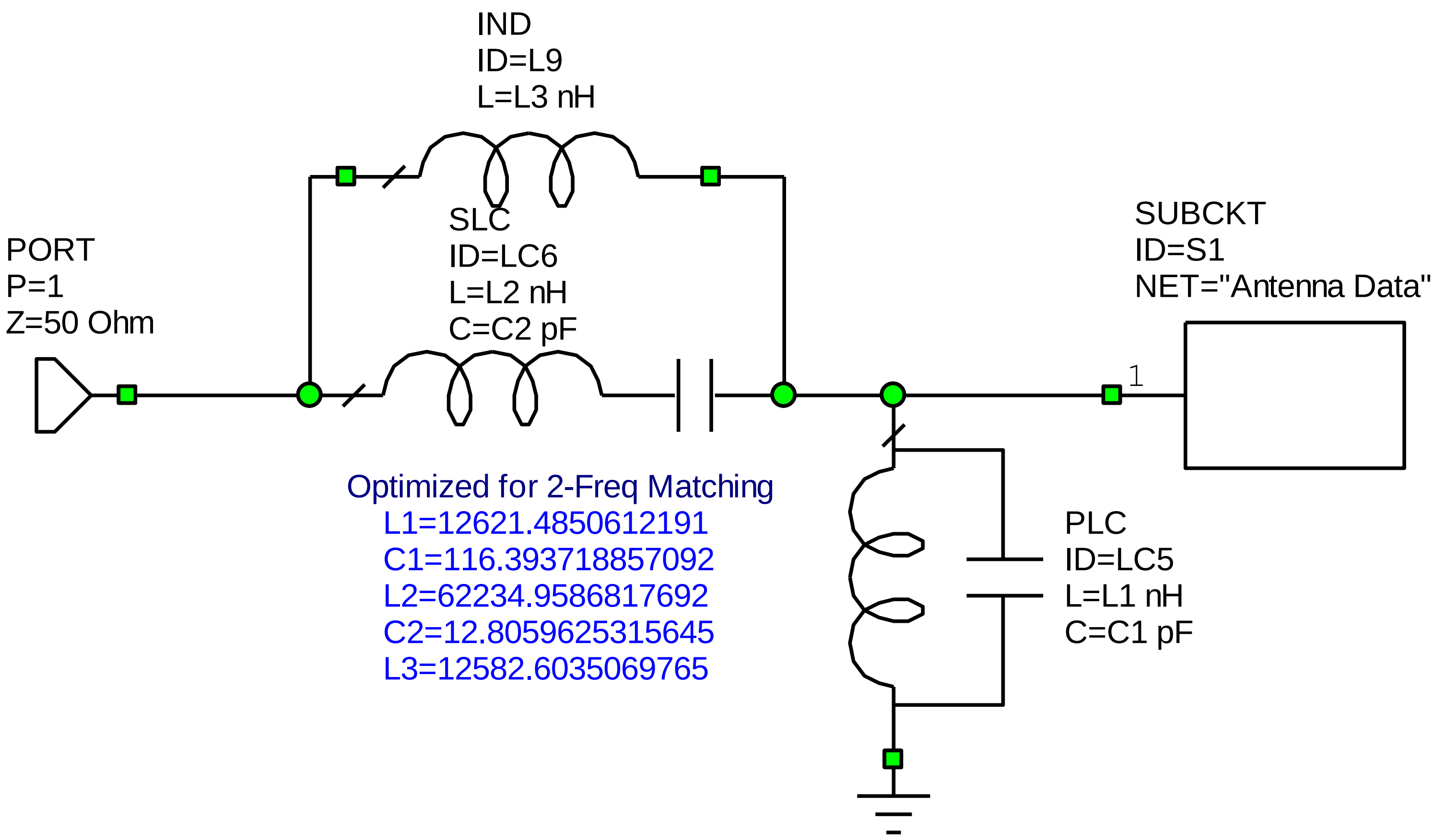
- **Antenna is 98.4 ft. dipole**
- **Objectives**
  - Add-A-Band, while keeping the match on other bands
- **Approach**
  - Example 6: Add 160 meters to a 2-band match network for 75 and 40 meters
  - Example 7: Add 160 meters to 3-band match network of Example 4

Impedance of 98.4 ft. Dipole Antenna

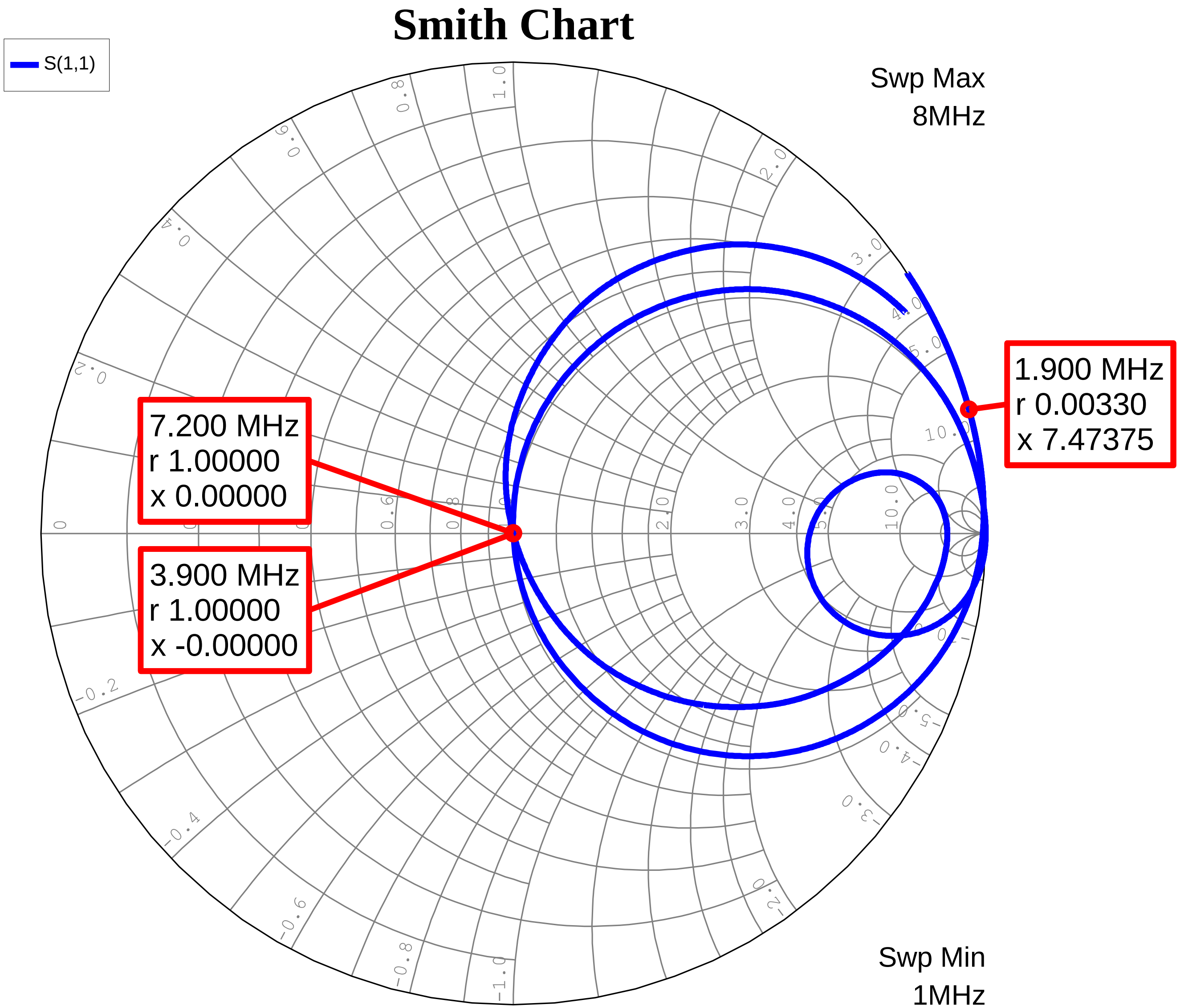


Add-A-Band via an L with Subharmonic Stubs

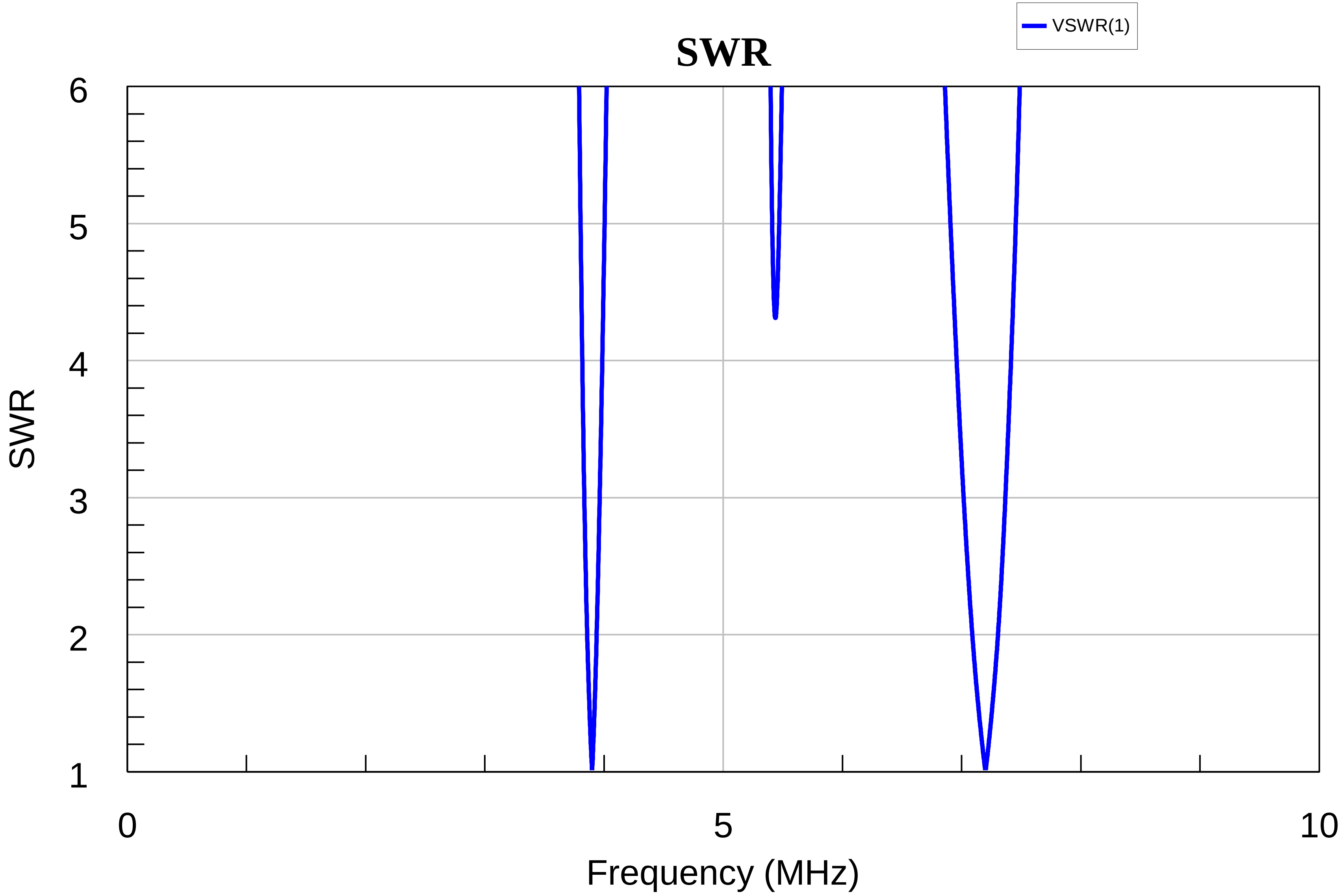
Initial 2-Band Match Network



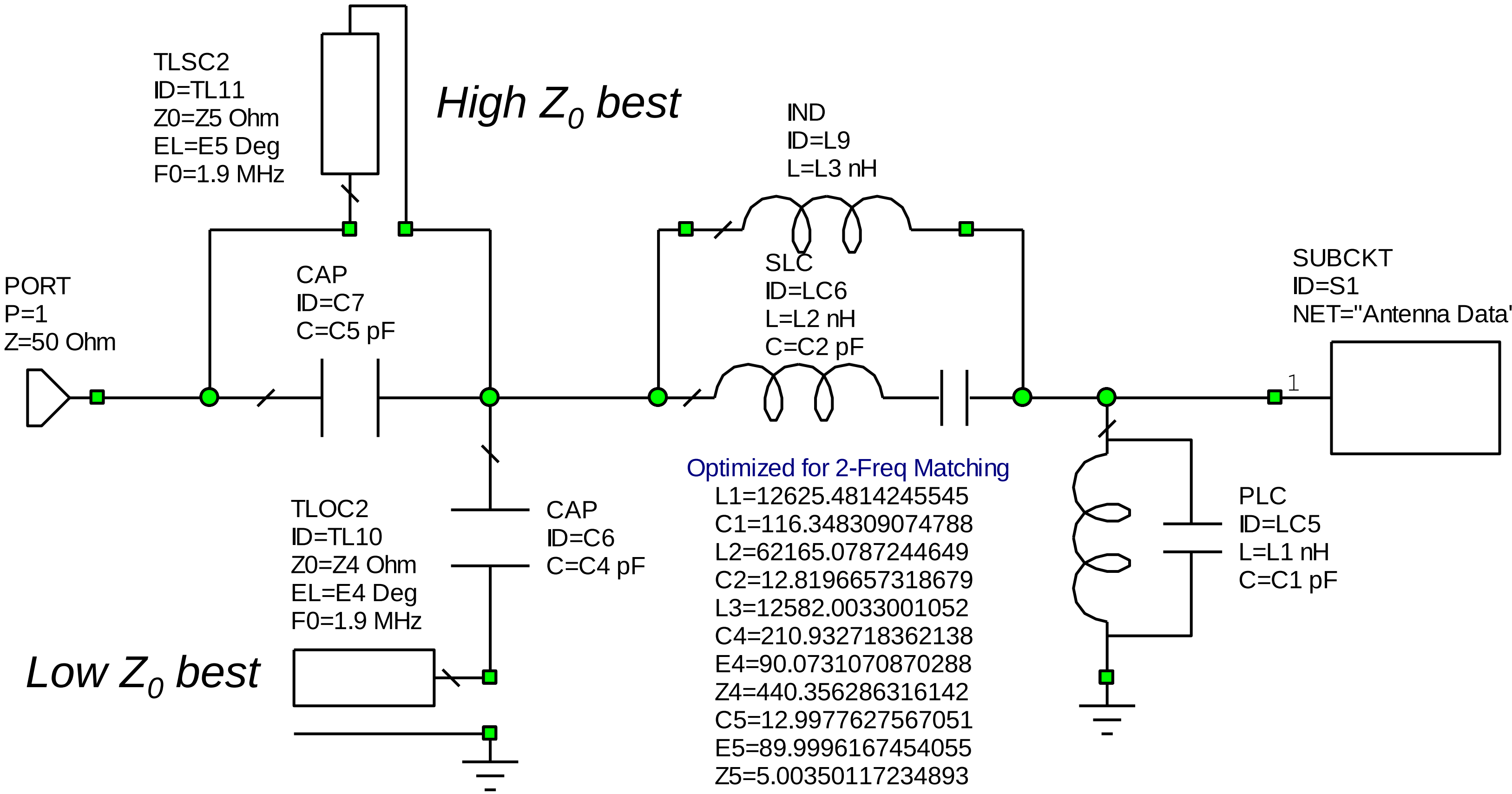
Performance of Initial 2-Band Match Network



SWR of Initial 2-Band Match Network



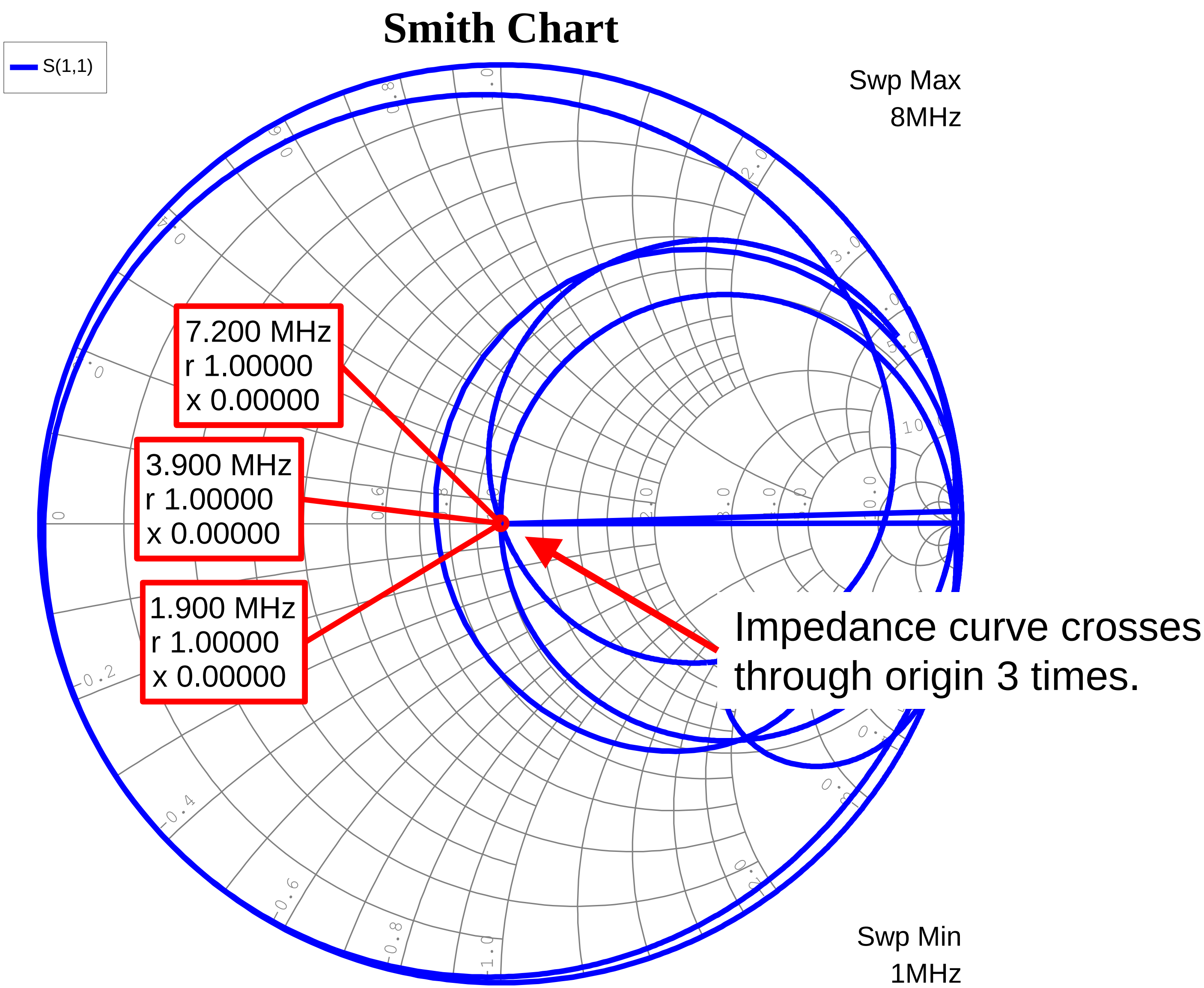
Example 6: Adding 160 to 75 and 40 Meter Match By Using an L Topology with Subharmonic Stubs



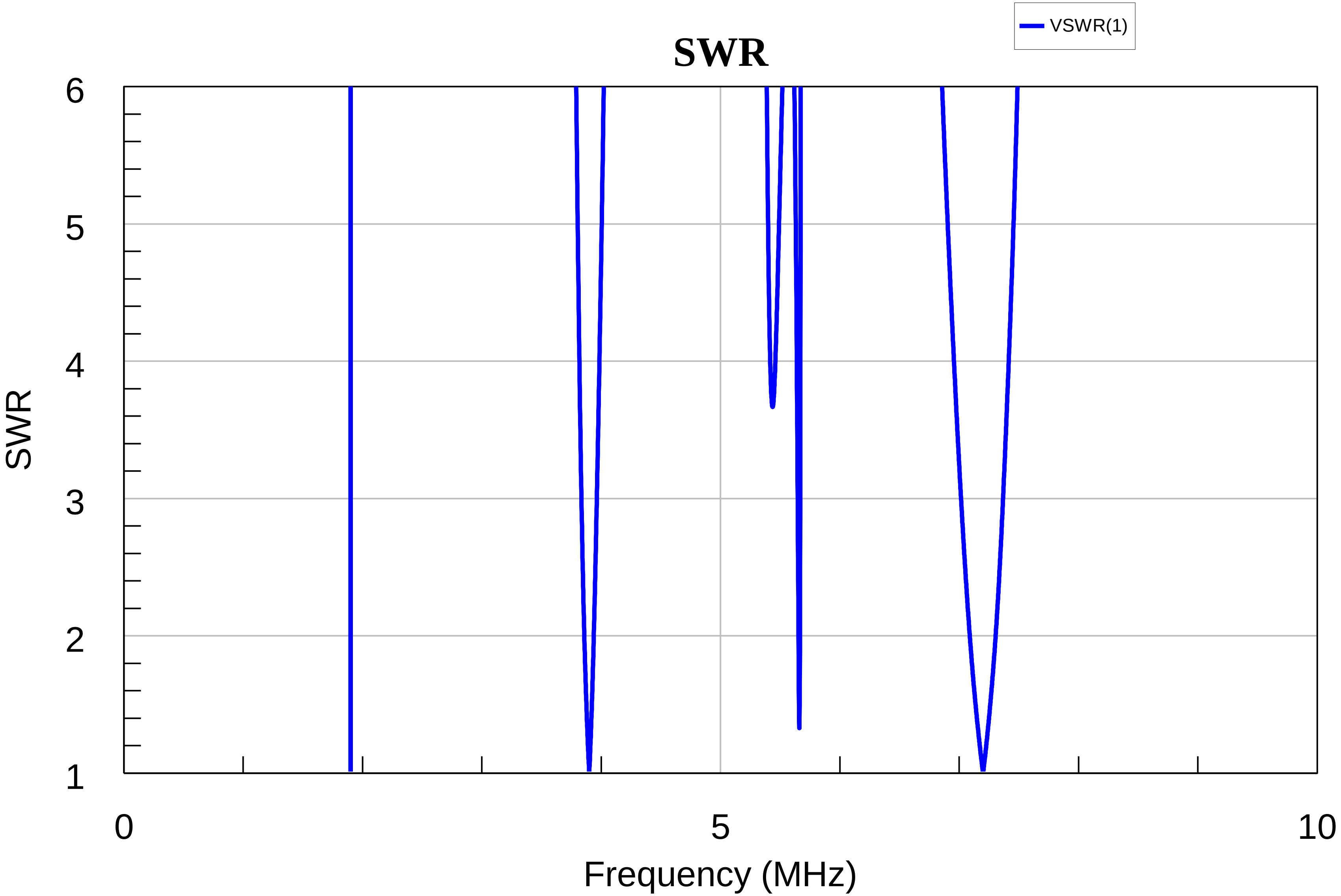
Optimized Match Network for 160, 75 and 40 meters

- An L network is added to match 1.9 MHz
- Quarter-wave stubs bypass the L network on harmonic bands, thereby preserving the match on 75, 40, and 20 meters

Result


Smith Chart
S(1,1)
Swp Max
8MHz
Swp Min
1MHz
7.200 MHz
r 1.00000
x 0.00000
3.900 MHz
r 1.00000
x 0.00000
1.900 MHz
r 1.00000
x 0.00000
Impedance curve crosses
through origin 3 times.

SWR

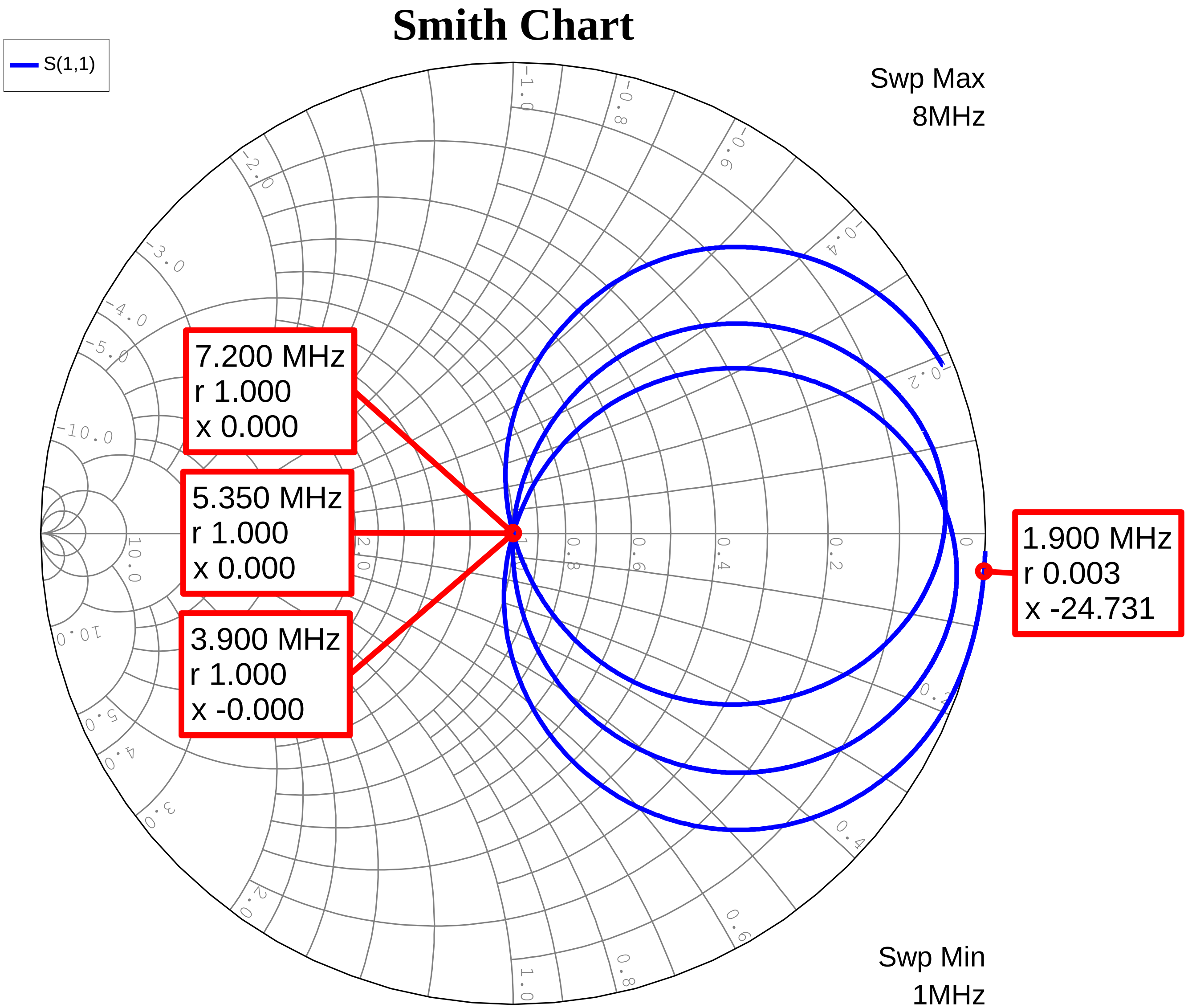


Comments on Subharmonic Stubs for Add-A-Banding

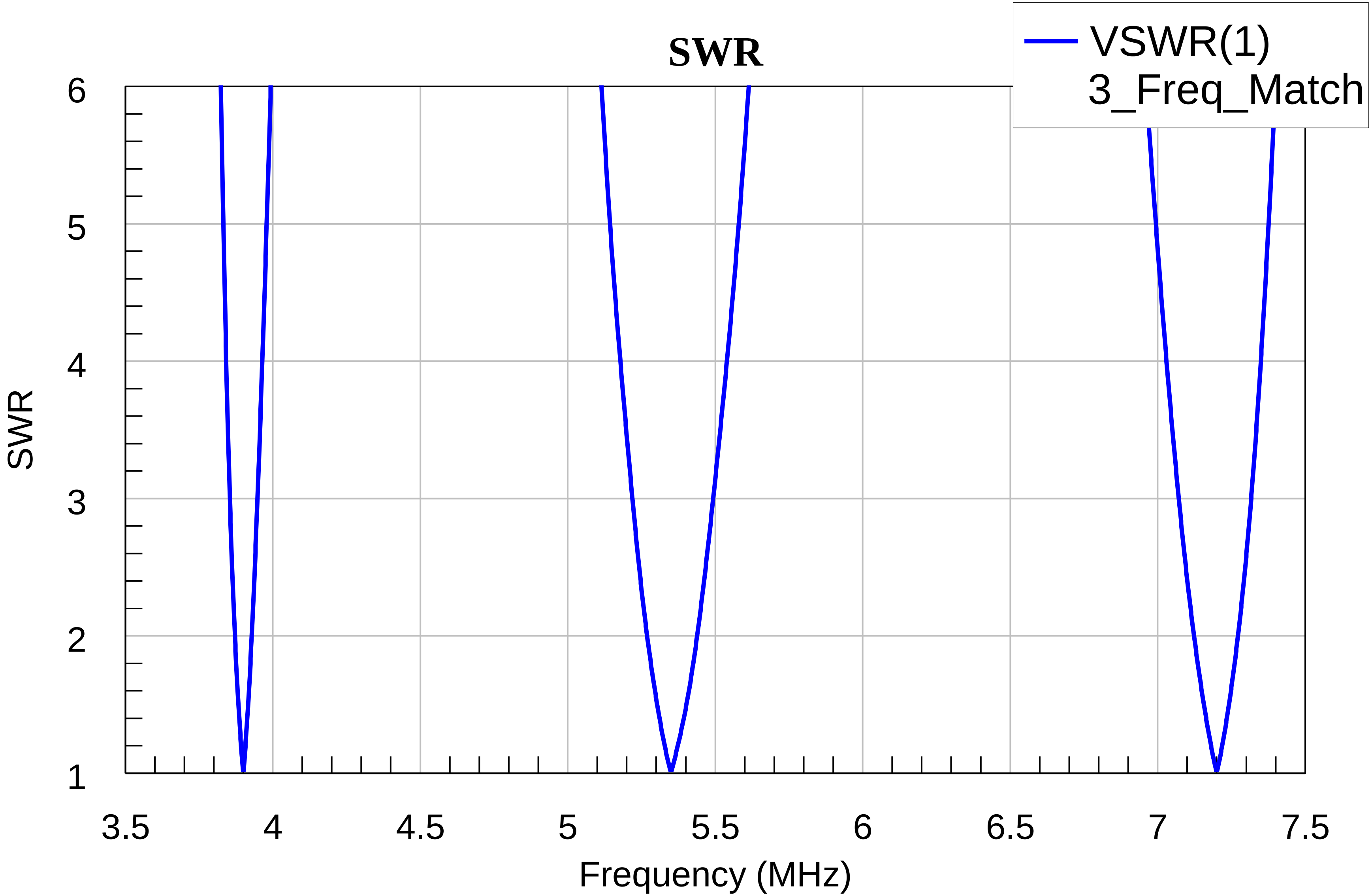
- **Method is simple in concept but limited in utility**
  - Bands must be harmonic, and added band must be the fundamental
- **Requires a lot of transmission line**
  - Two quarter-wave stubs require a half wavelength of line at the fundamental frequency
  - Impractical at HF, but ok at VHF/UHF
- **If impedance of band to be added is near Smith chart boundary, as is the case for electrically small antenna, small match bandwidth should be expected**
  - Added band will have small match bandwidth, and other matched bands may suffer reduced match bandwidths
- **This method uses a lot of circuit elements for what it does**
  - 9 elements having 11 degrees of freedom just to match 3 frequencies!
- **It is generally better to design a match network for all frequencies than to add a decoupled input L network to an existing match network**

Add-A-Band via Line Segment and Shunt Susceptance with Zeros

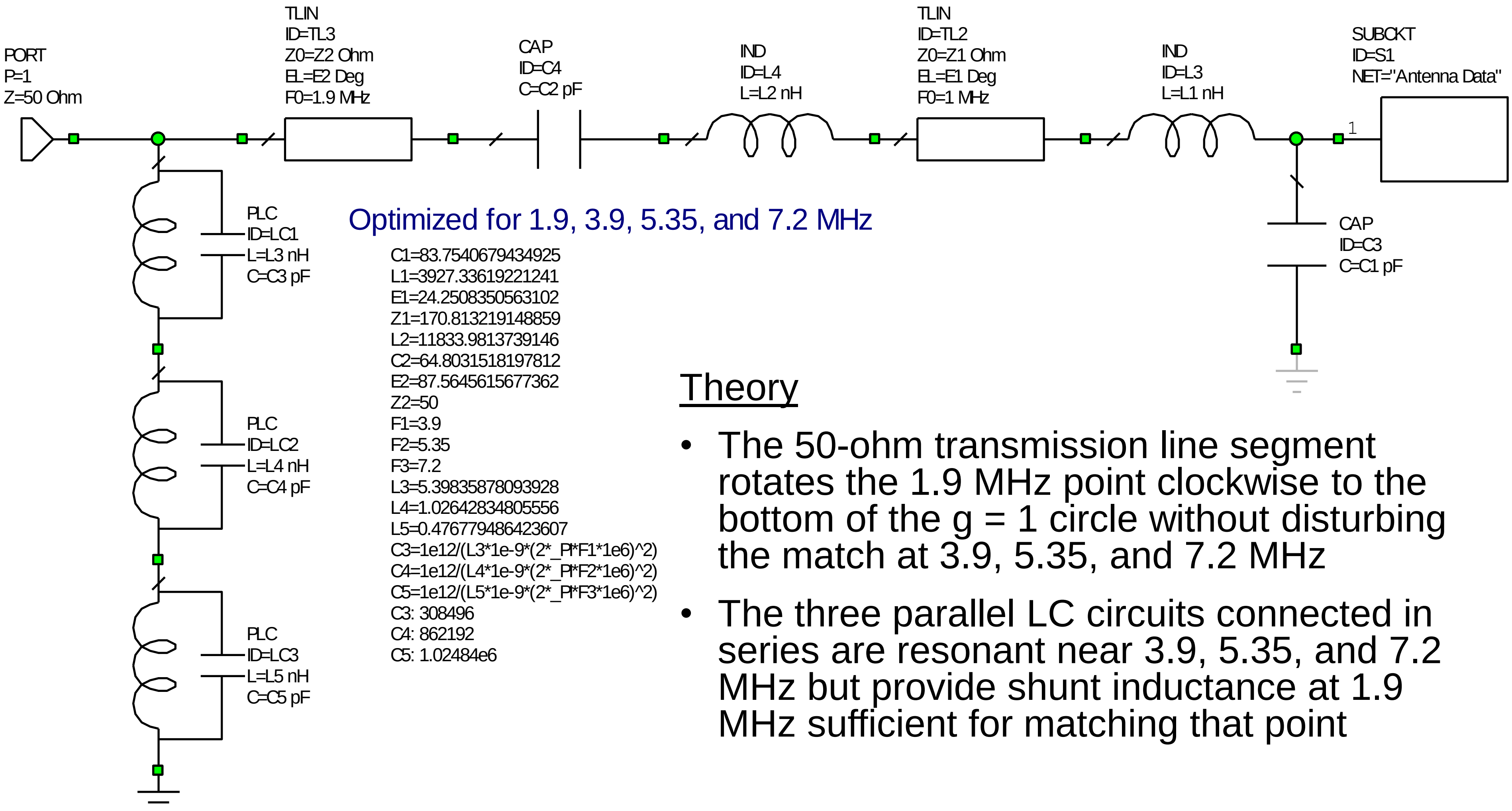
Add-A-Band: Add 160 meters to Example 4



Initial SWR, Shown in Microwave Office



Example 7: Adding 160 to 75, 60, 40 Meter Match Using a Line Segment and Shunt Susceptance with Zeros



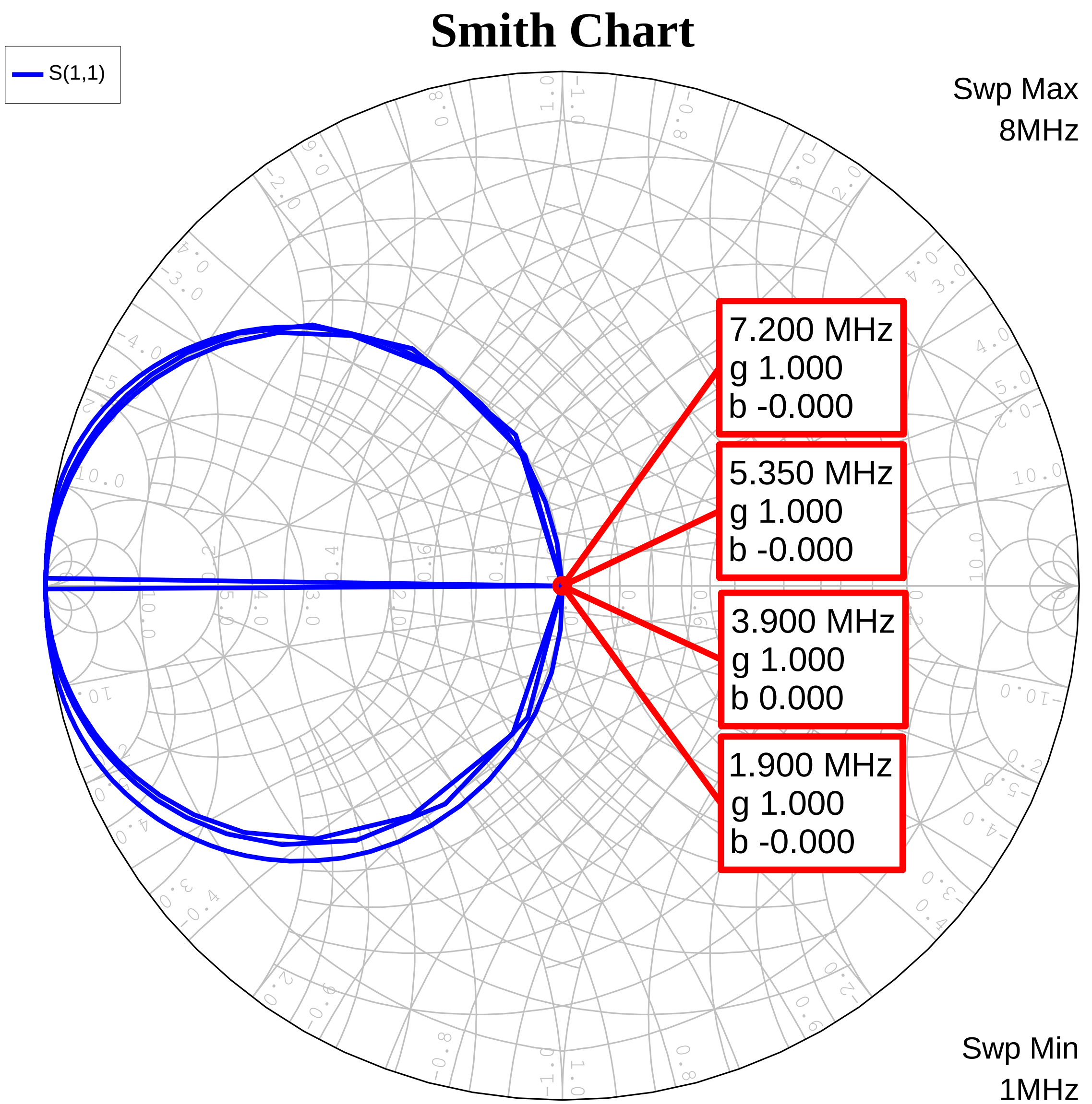
Optimized for 1.9, 3.9, 5.35, and 7.2 MHz

C1=83.7540679434925
L1=3927.33619221241
E1=24.2508350563102
Z1=170.813219148859
L2=11833.9813739146
C2=64.8031518197812
E2=87.5645615677362
Z2=50
F1=3.9
F2=5.35
F3=7.2
L3=5.39835878093928
L4=1.02642834805556
L5=0.476779486423607
C3=1e12/(L3*1e-9*(2*_PI*F1*1e6)^2)
C4=1e12/(L4*1e-9*(2*_PI*F2*1e6)^2)
C5=1e12/(L5*1e-9*(2*_PI*F3*1e6)^2)
C3: 308496
C4: 862192
C5: 1.02484e6

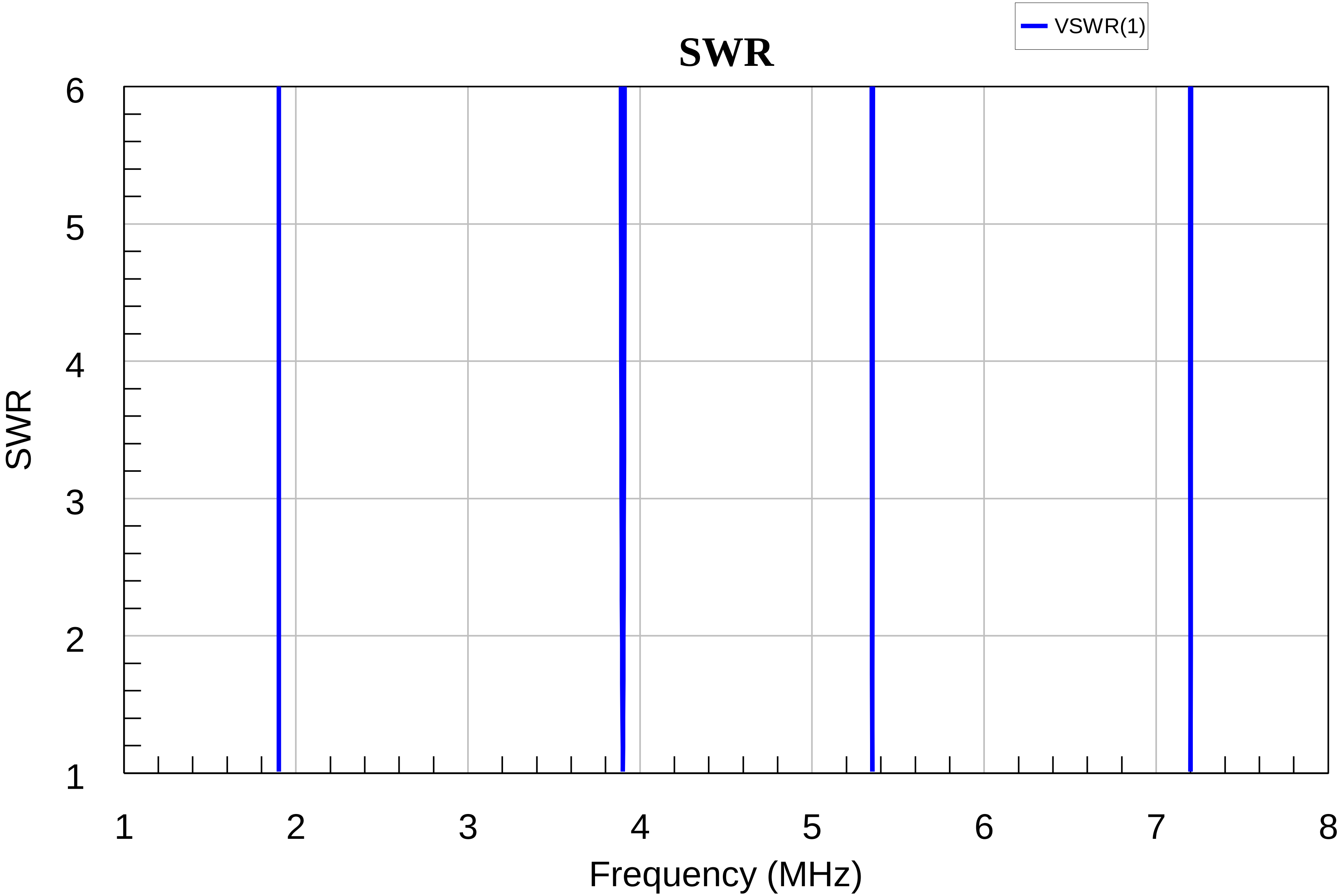
Theory

- The 50-ohm transmission line segment rotates the 1.9 MHz point clockwise to the bottom of the g = 1 circle without disturbing the match at 3.9, 5.35, and 7.2 MHz
- The three parallel LC circuits connected in series are resonant near 3.9, 5.35, and 7.2 MHz but provide shunt inductance at 1.9 MHz sufficient for matching that point

Input Impedance of 4-Band Network on Smith Chart



Perfect 4-Frequency Matching but No Bandwidth!



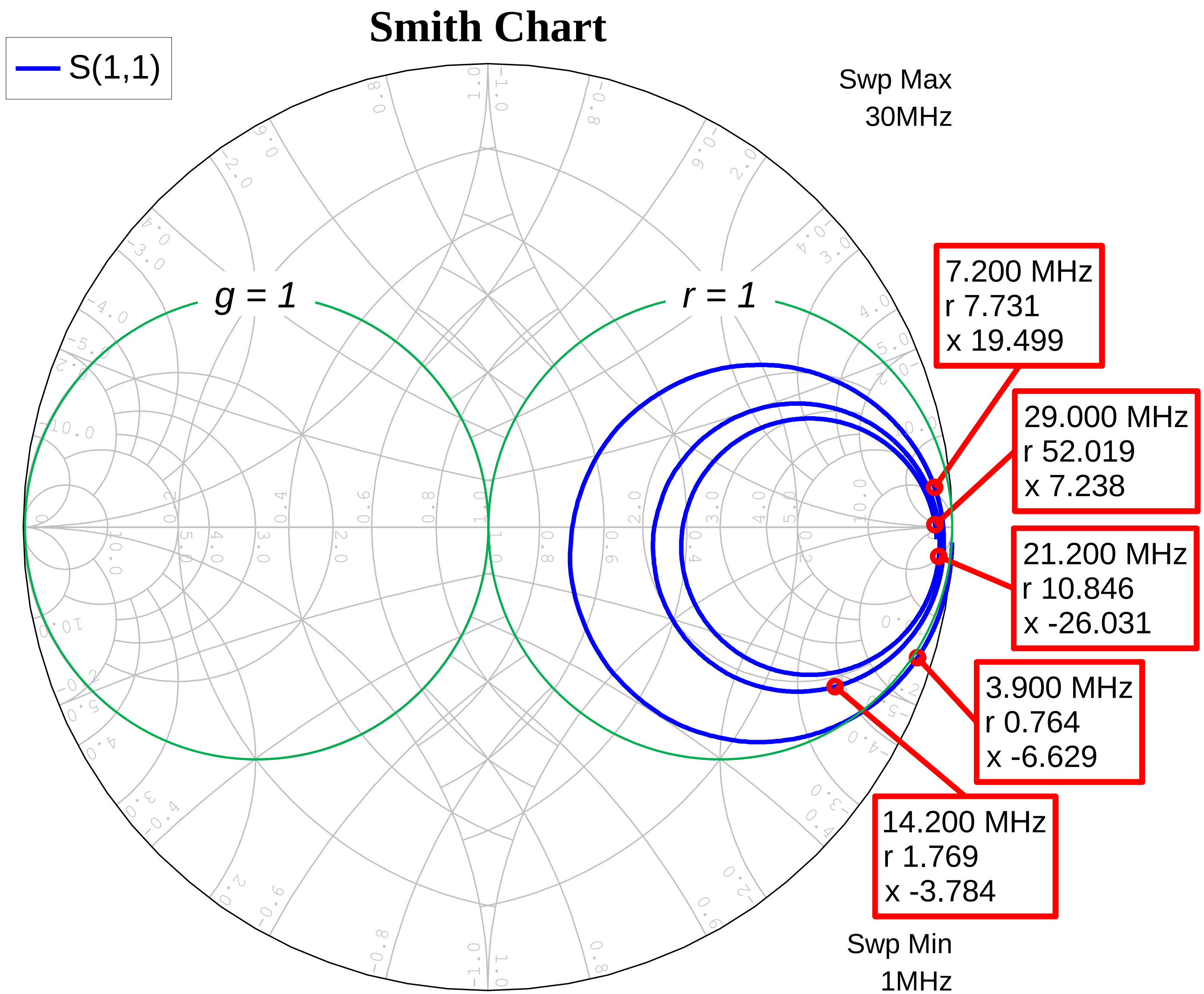
Iterative Synthesis

Multi-Frequency Matching by Distributed Network Loads Along Main Line

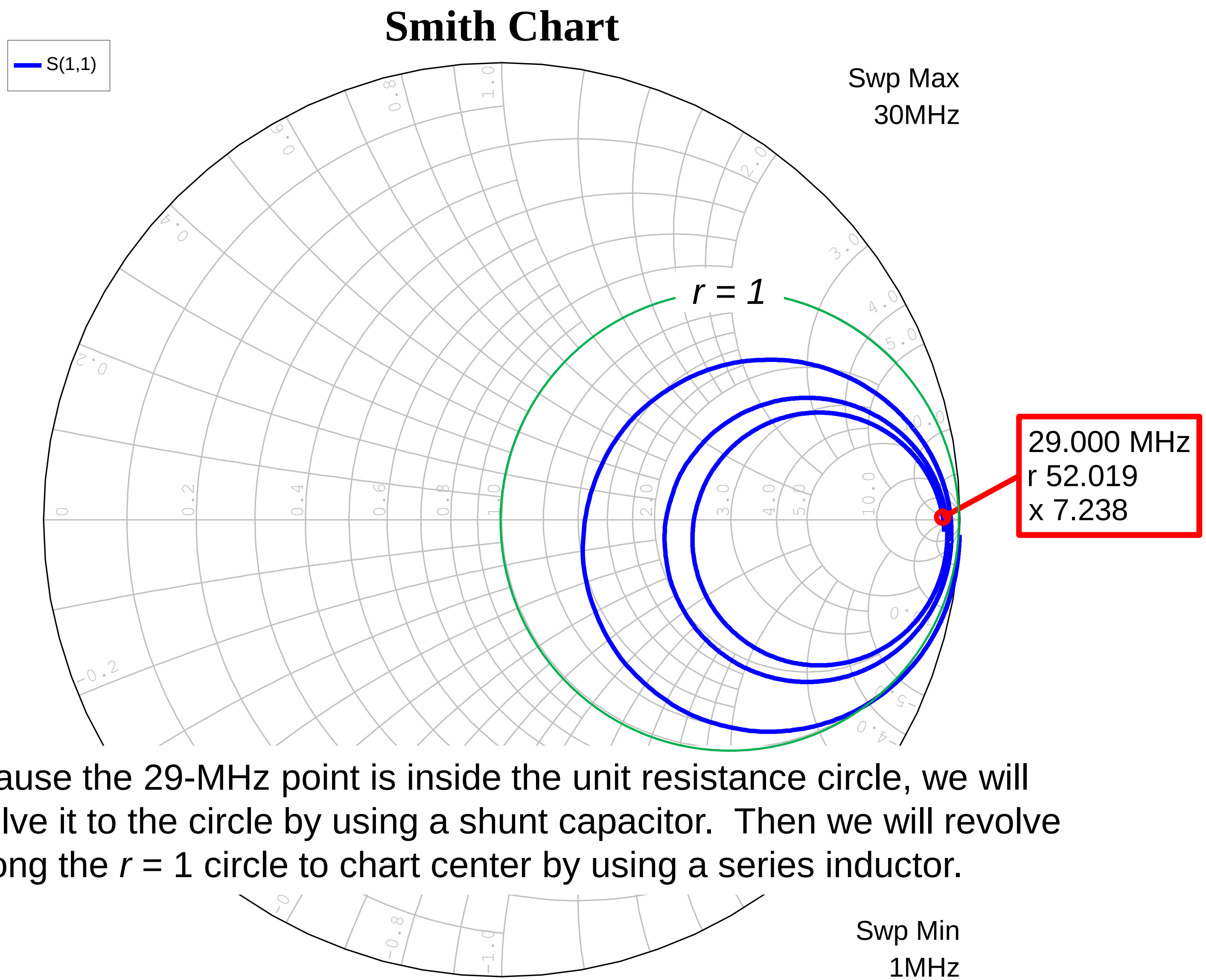
Examples 8 and 9: Four-Band and Five-Band Matching on 75, 40, 20, 15, and 10

- **Antenna: 98.4 ft. dipole**
- **Objectives:**
  - Example 8: 4-band match at 3.9, 7.2, 14.2, and 29 MHz
  - Example 9: 5-band match at 3.9, 7.2, 14.2, 21.2, and 29 MHz
- **Topology:**
  - Multiply loaded feed line
  - All transmission lines are part of the main 50-ohm feed line (match network is “built into” the feed line)
  - No stubs allowed (impractical at HF)
  - All reactive elements are lumped element capacitors and inductors
- **Approach:**
  - Iterative, highest frequency first (but any order is permitted)
  - Use tuning sliders to add stages for each lower frequency in sequence
  - Use optimizer to fine-tune intermediate and final network structures

Antenna Impedance on Smith Chart

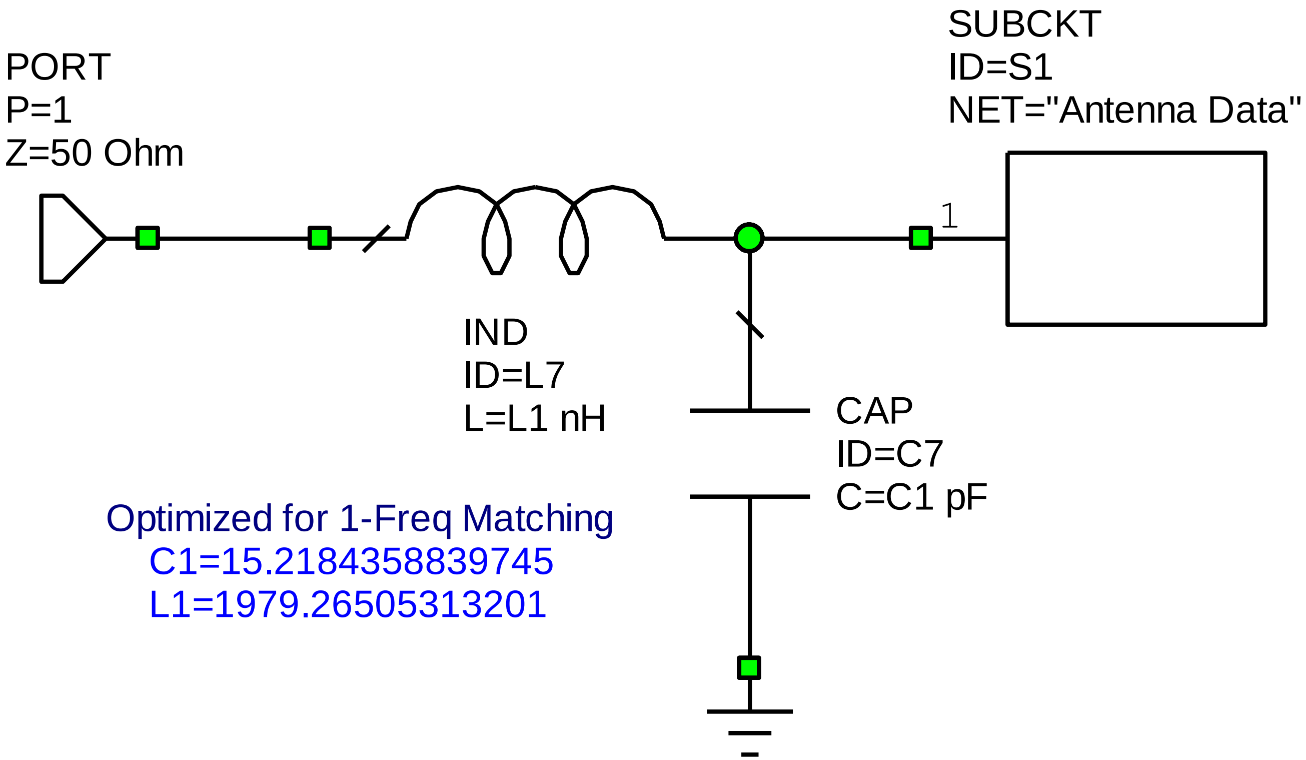


Initial Position of 29 MHz



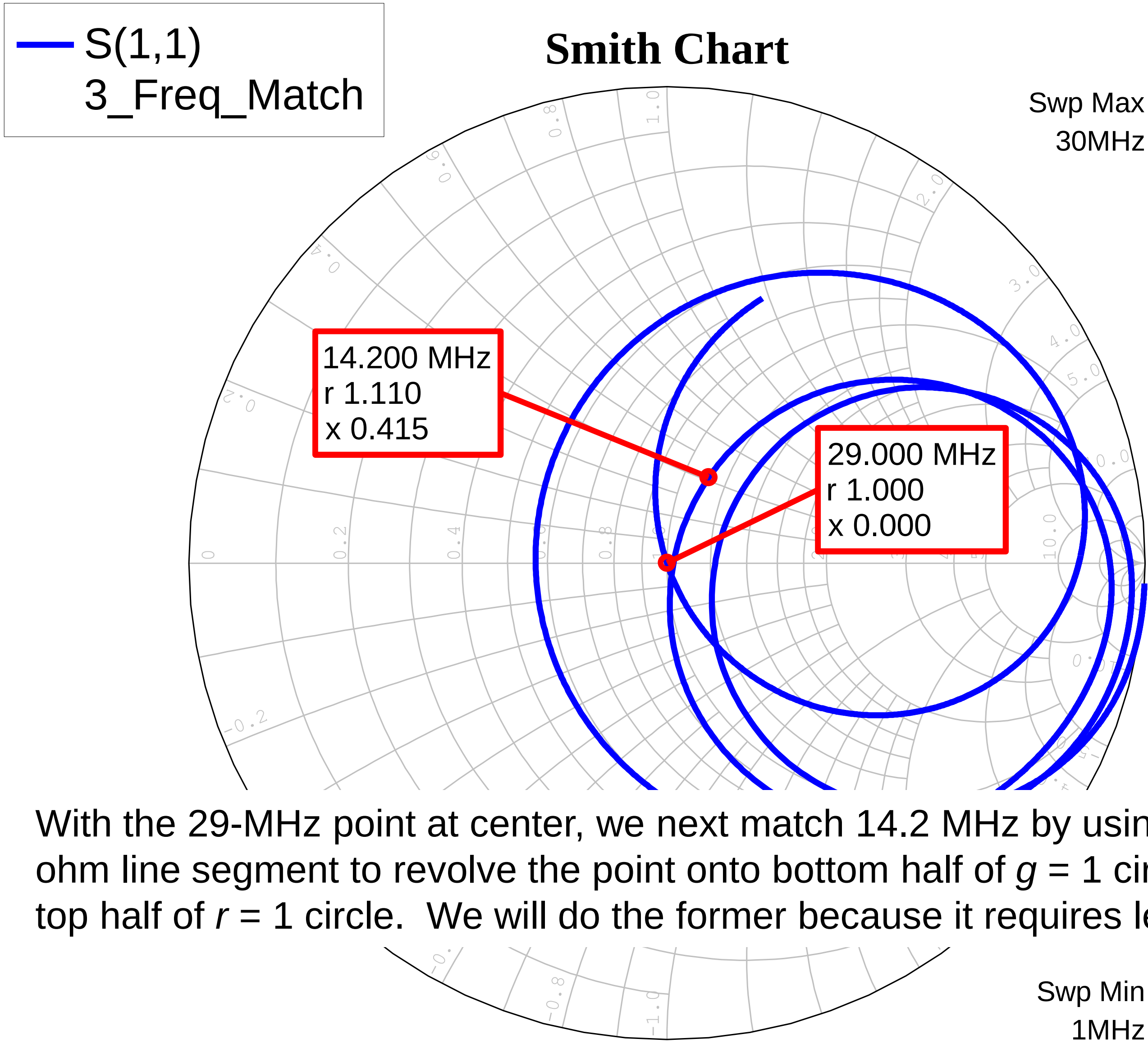
Because the 29-MHz point is inside the unit resistance circle, we will revolve it to the circle by using a shunt capacitor. Then we will revolve it along the *r* = 1 circle to chart center by using a series inductor.

Step 1: Match 29 MHz



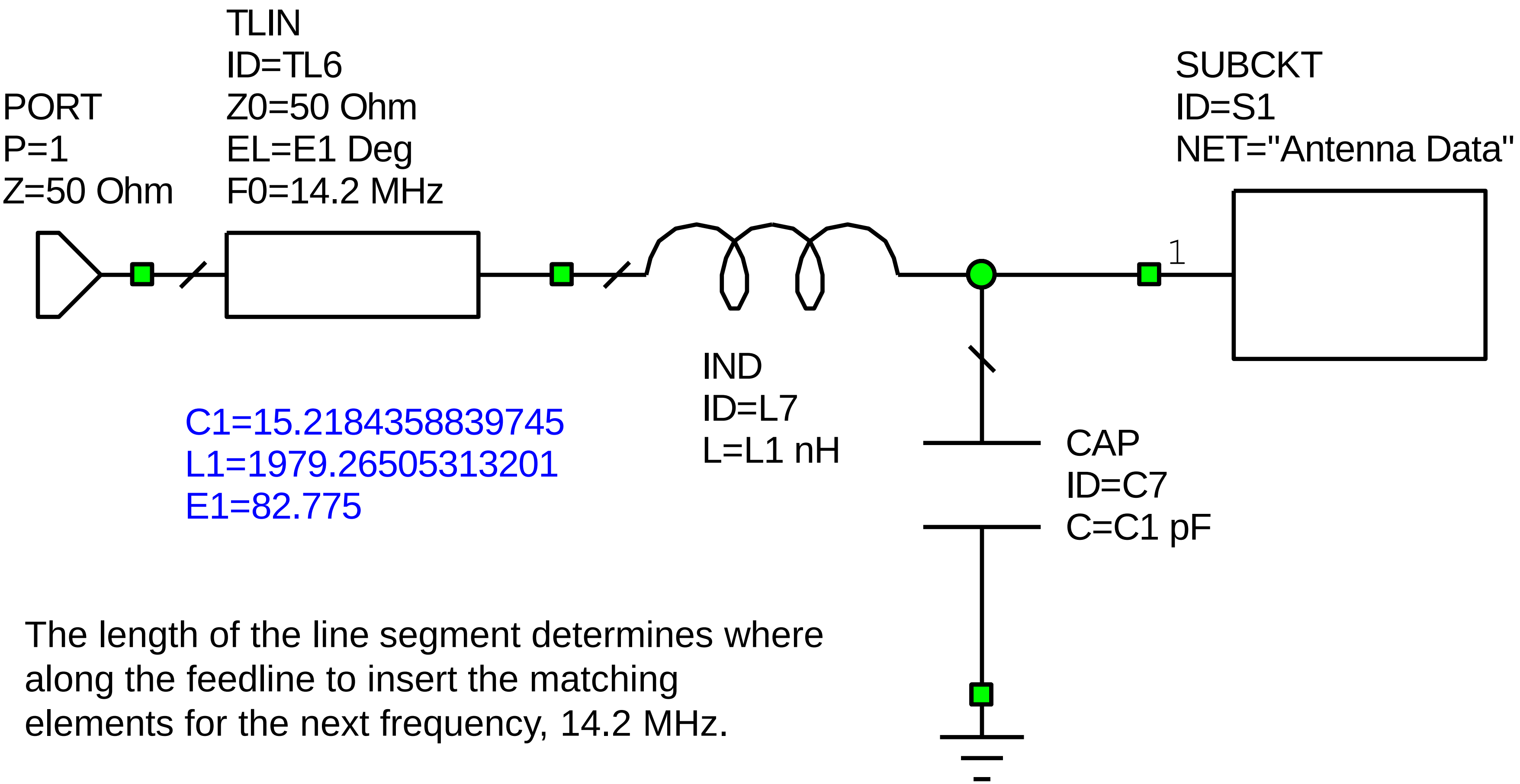
Optimized Match Network for 10 meters

Result of Step 1



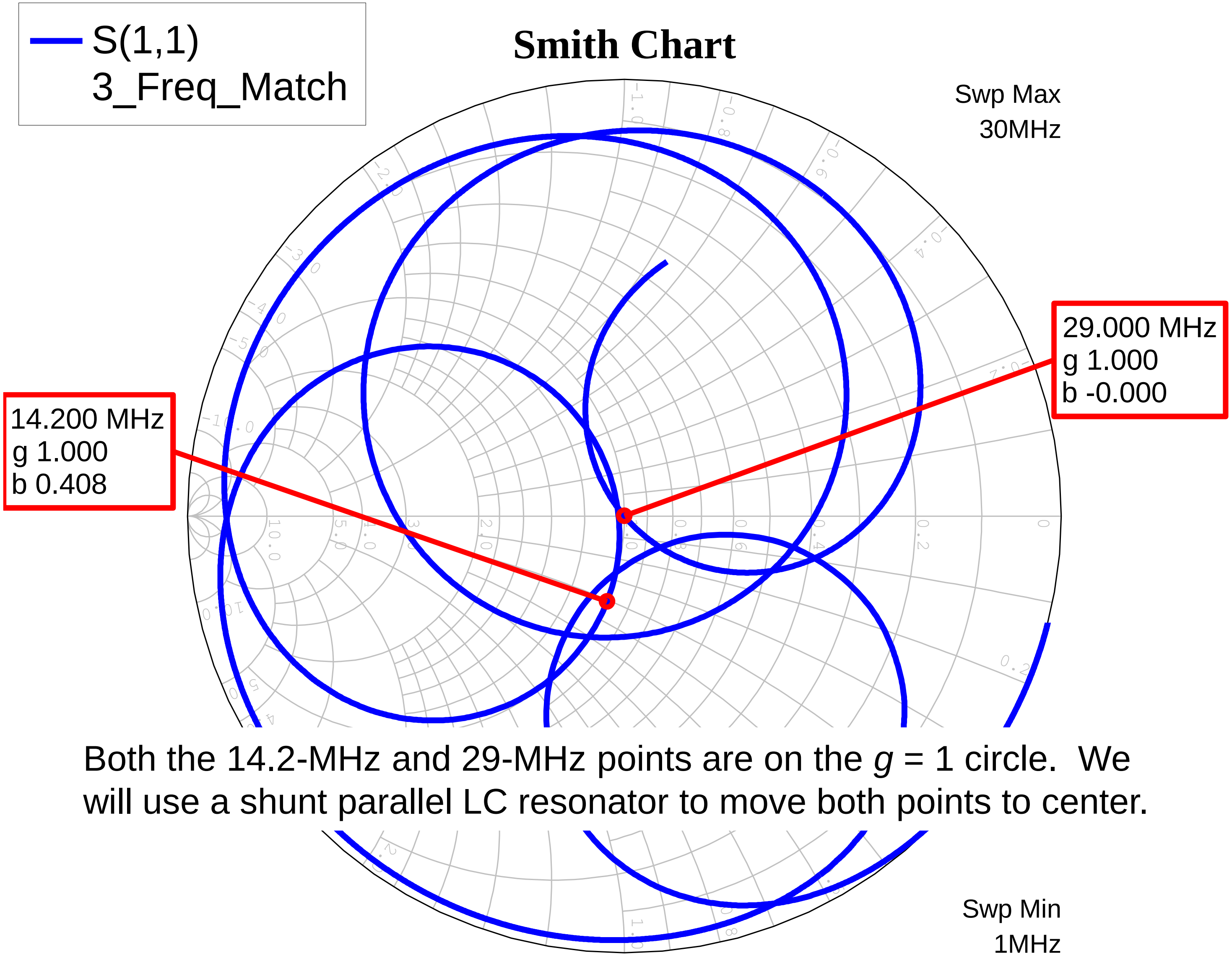
With the 29-MHz point at center, we next match 14.2 MHz by using a 50-ohm line segment to revolve the point onto bottom half of *g* = 1 circle or top half of *r* = 1 circle. We will do the former because it requires less line.

Match at 29 MHz with Added Line Segment



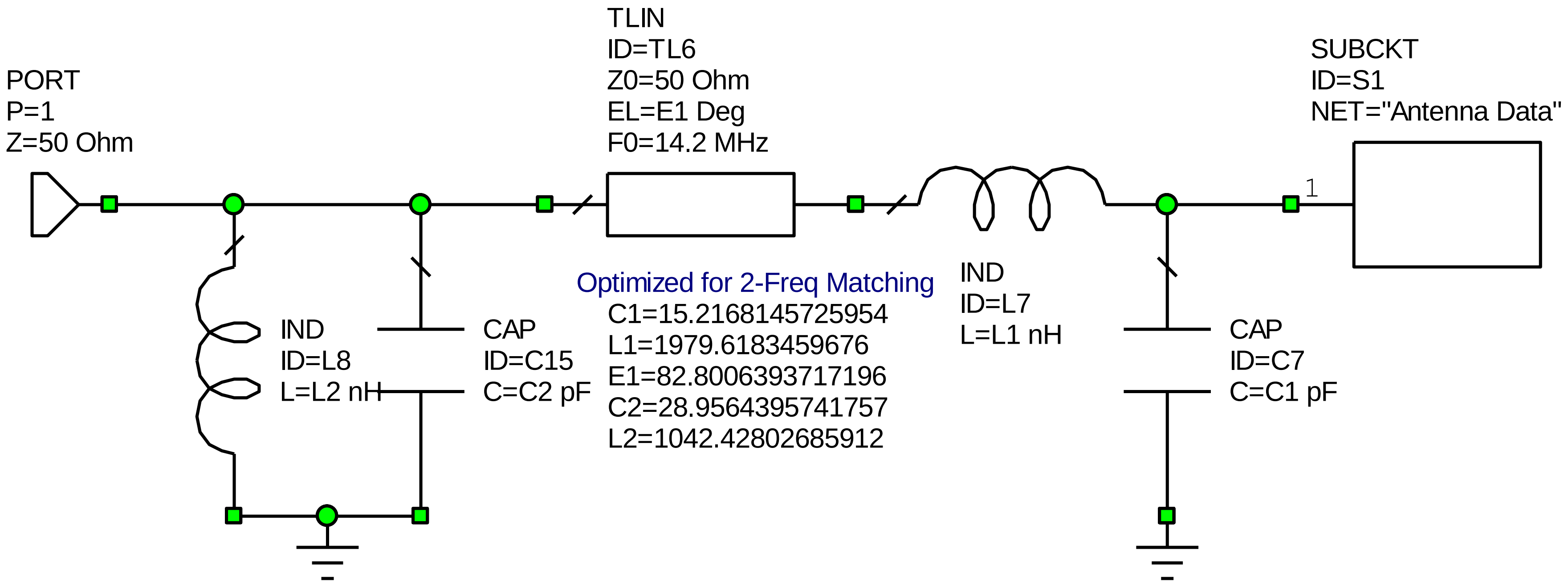
The length of the line segment determines where along the feedline to insert the matching elements for the next frequency, 14.2 MHz.

Result of Adding Line Segment



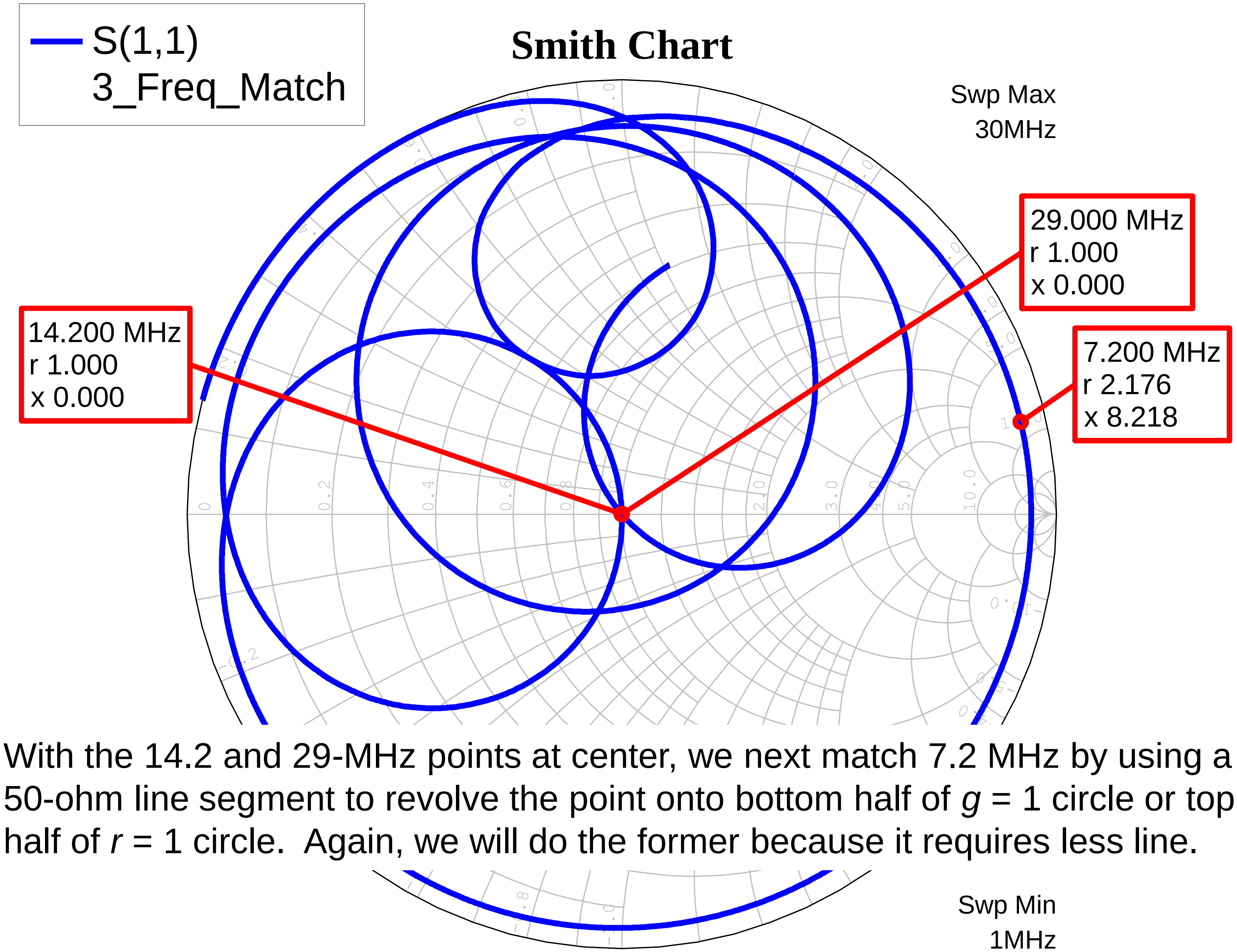
Both the 14.2-MHz and 29-MHz points are on the *g* = 1 circle. We will use a shunt parallel LC resonator to move both points to center.

Network for 2-Frequency Match at 14.2 and 29 MHz



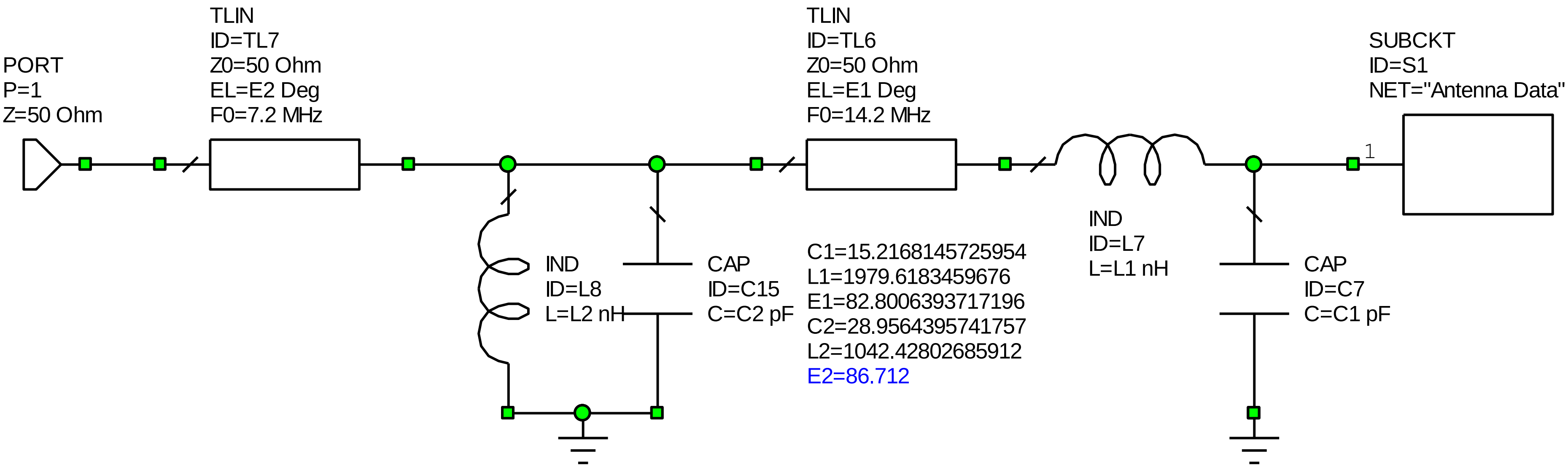
Optimized Match Network for 10 and 20 meters

Result of Step 2



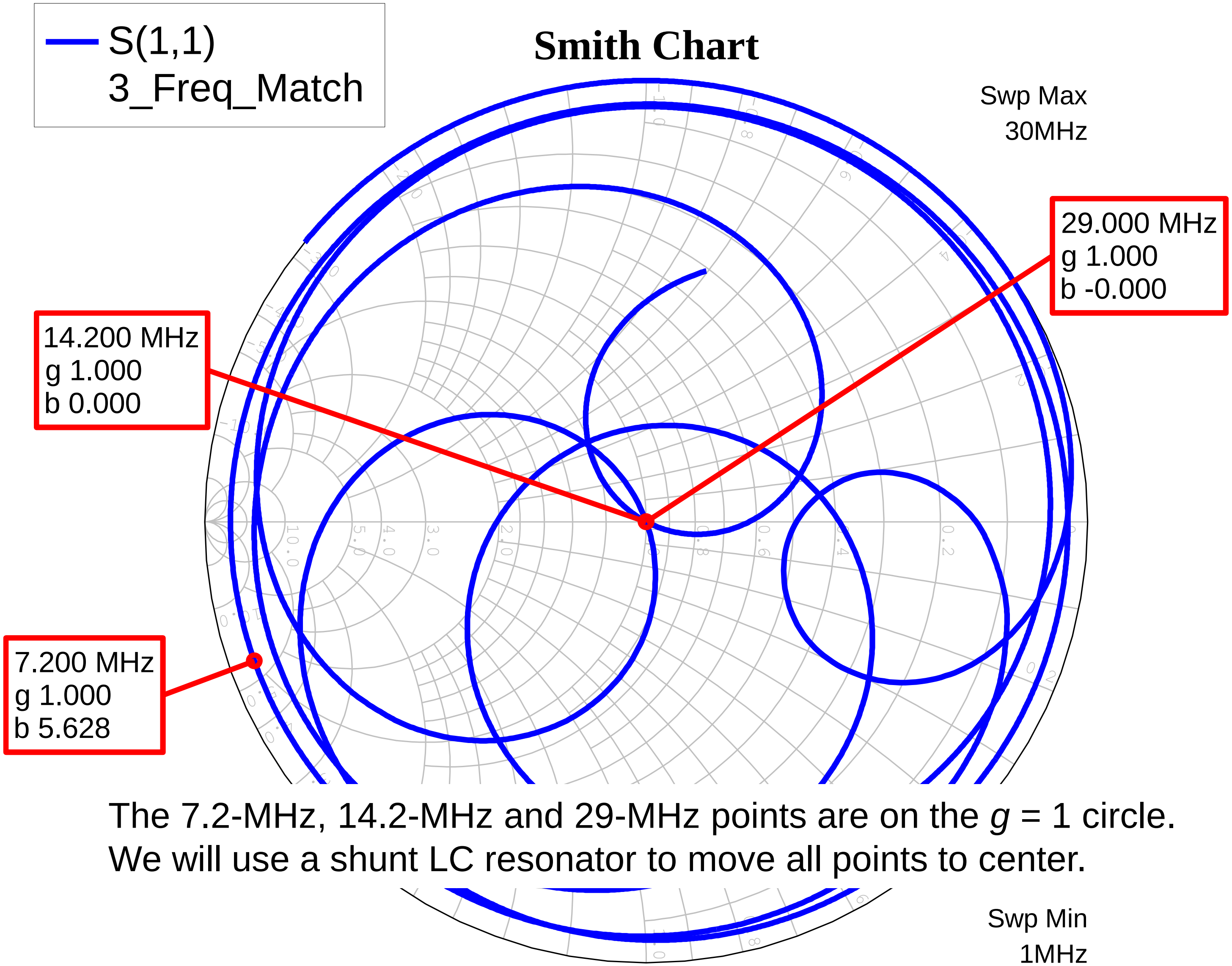
With the 14.2 and 29-MHz points at center, we next match 7.2 MHz by using a 50-ohm line segment to revolve the point onto bottom half of *g* = 1 circle or top half of *r* = 1 circle. Again, we will do the former because it requires less line.

Match at 14.2 and 29 MHz with 2nd Line Segment Added



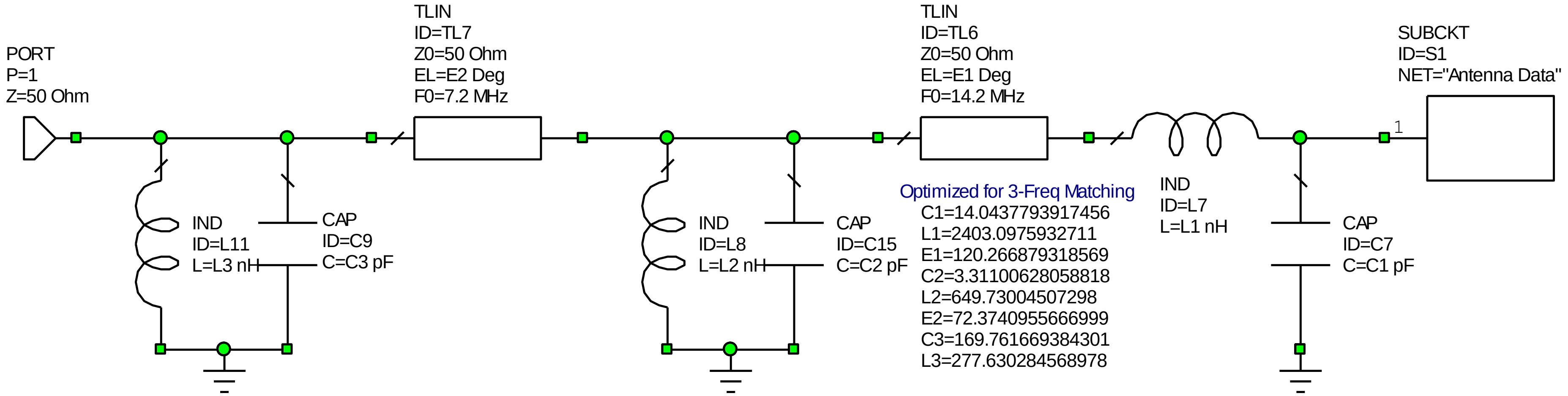
- The length of the line segment determines where along the feedline to insert the matching elements for the next frequency, 7.2 MHz

Result of Adding 2nd Line Segment



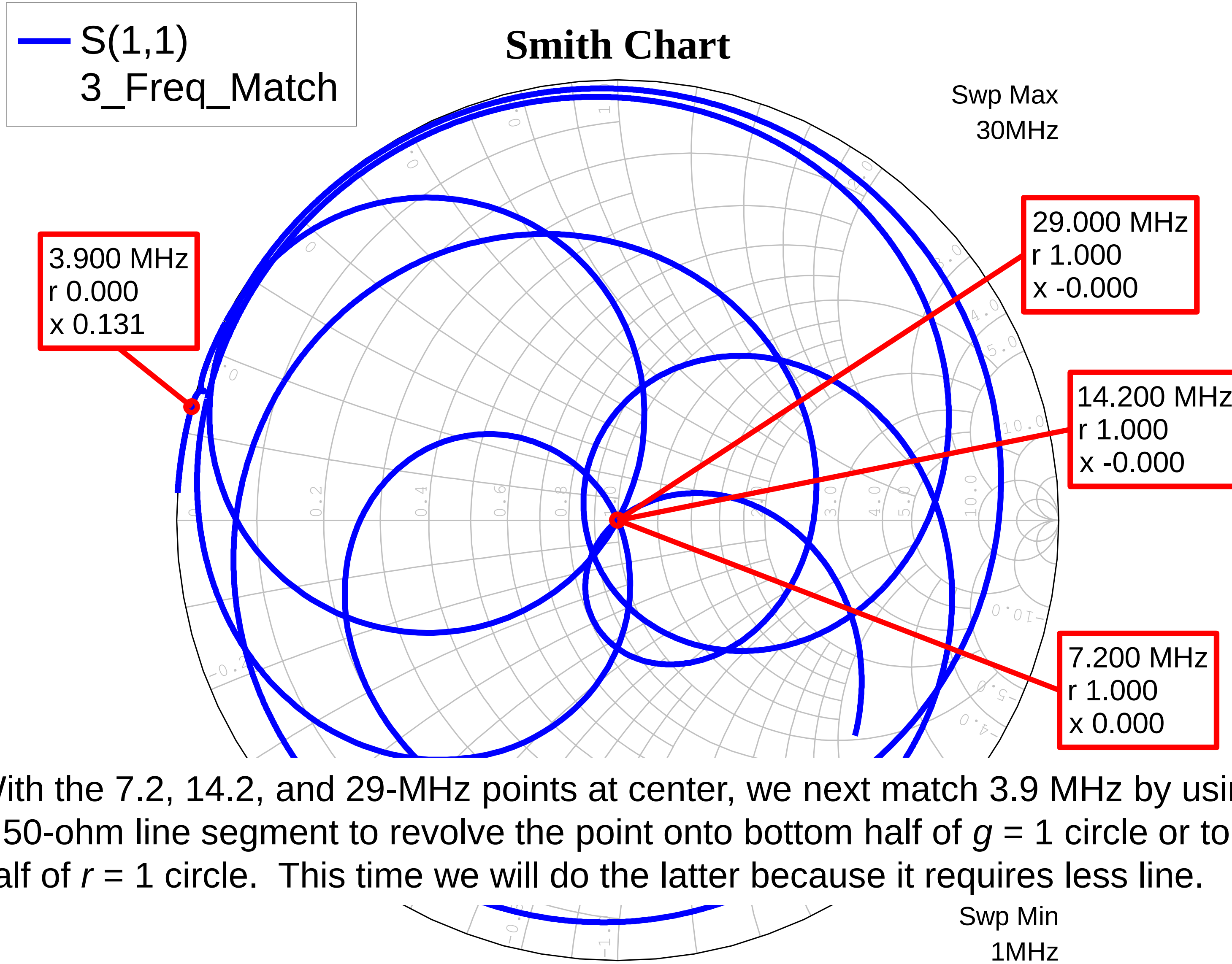
The 7.2-MHz, 14.2-MHz and 29-MHz points are on the *g* = 1 circle.
We will use a shunt LC resonator to move all points to center.

Network for 3-Frequency Match at 7.2, 14.2 and 29 MHz



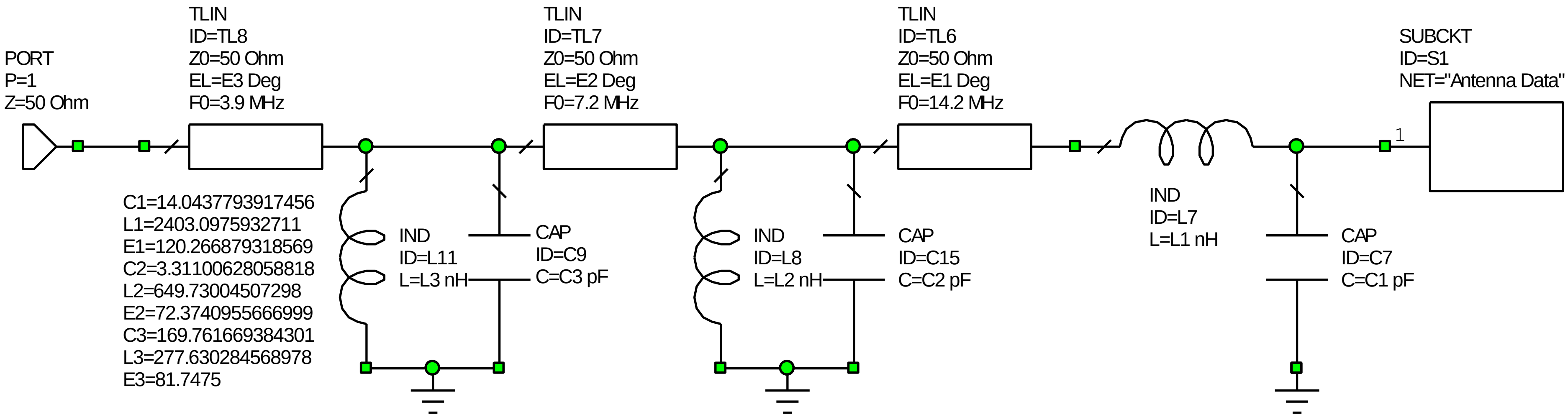
Optimized Match Network for 10, 20, and 40 meters

Result of Step 3



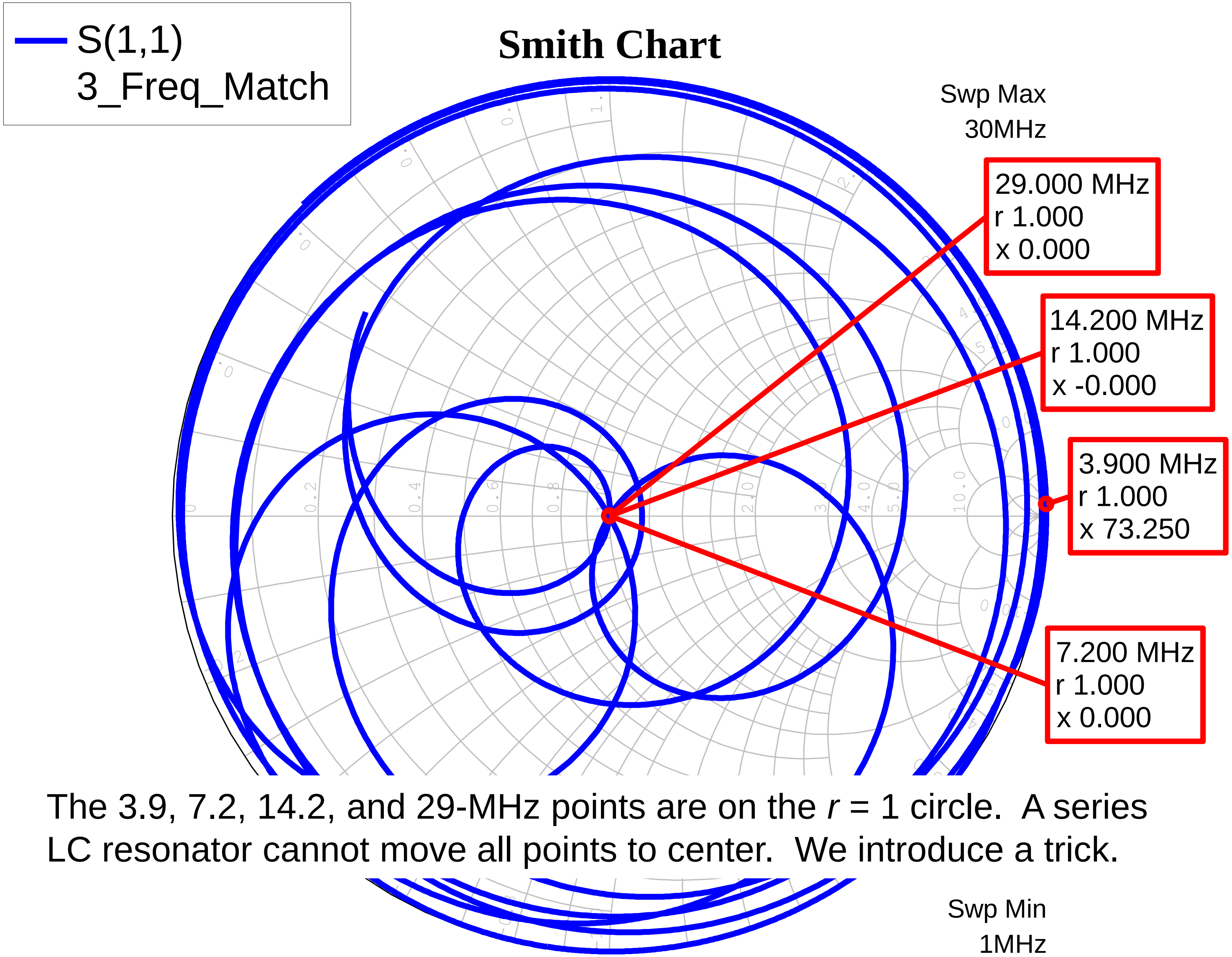
With the 7.2, 14.2, and 29-MHz points at center, we next match 3.9 MHz by using a 50-ohm line segment to revolve the point onto bottom half of *g* = 1 circle or top half of *r* = 1 circle. This time we will do the latter because it requires less line.

Match at 7.2, 14.2 and 29 MHz with 3rd Line Segment Added



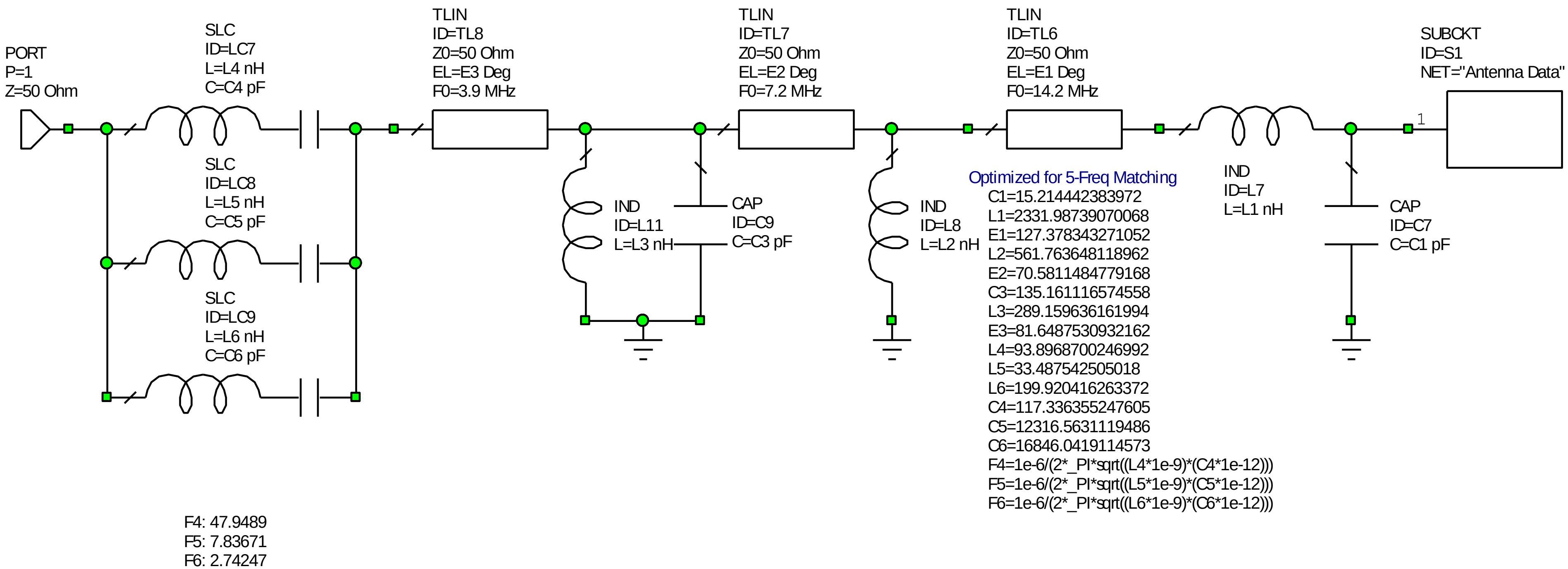
- The length of the line segment determines where along the feedline to insert the matching elements for the fourth frequency, 3.9 MHz

Result of Adding 3rd Line Segment



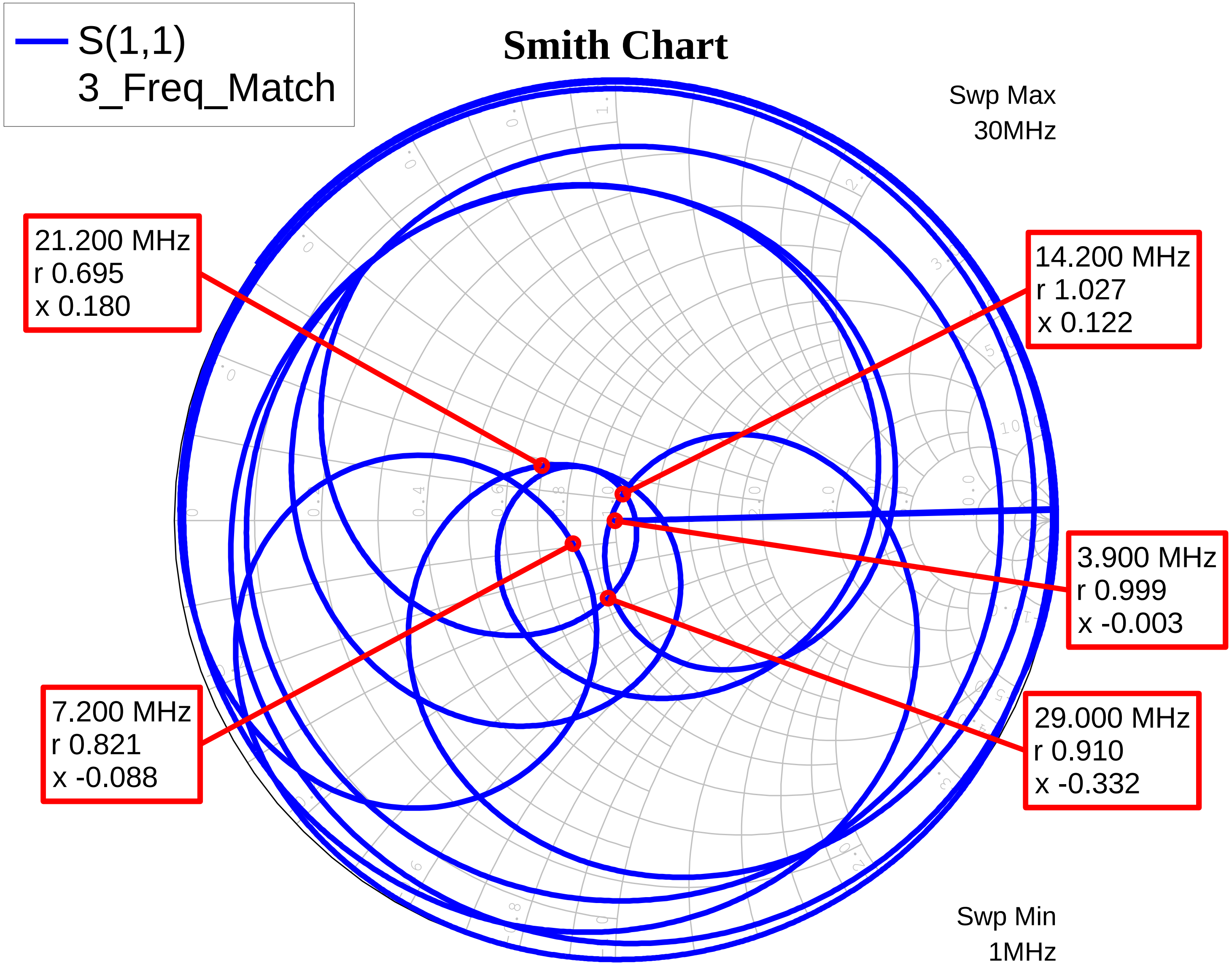
The 3.9, 7.2, 14.2, and 29-MHz points are on the *r* = 1 circle. A series LC resonator cannot move all points to center. We introduce a trick.

Resulting 5-Band, 14-Element Matching Network

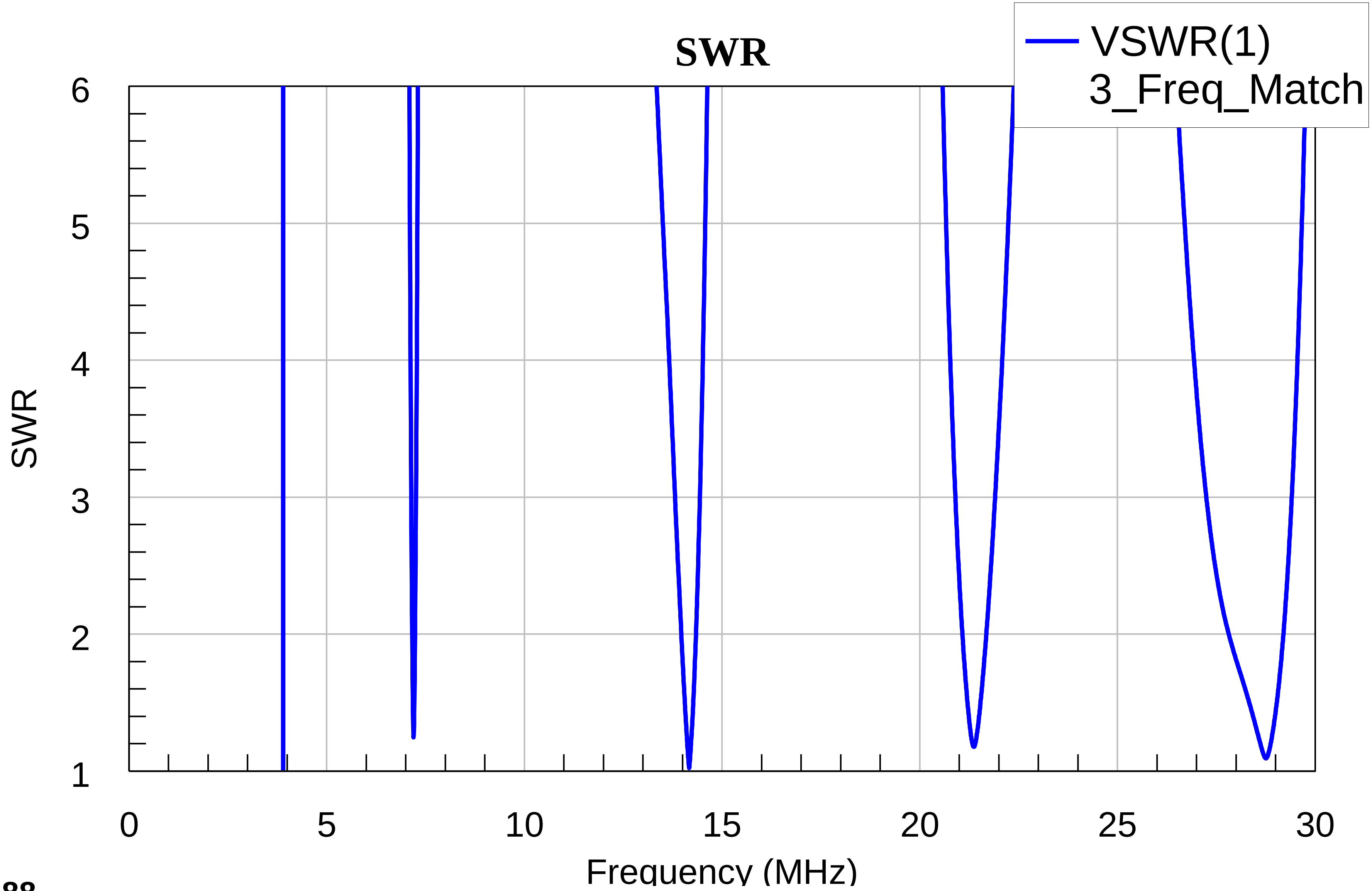


- Determine an L network that matches the frequency to be added
- In the series branch of the L put a bank of series LC resonators in parallel
  - 2nd Foster form, with impedance zeros at the frequencies matched already
- In the shunt branch of the L put a bank of parallel LC resonators in series
  - 1st Foster form, with impedance poles at the frequencies matched already
- The series and shunt elements can often be absorbed into the resonator bank, although this requires calculation or an optimizer to get right

Result for 5-Band Matching Network



SWR of 5-Band Match Network



Design Procedure for Iterative Multi-Frequency Matching

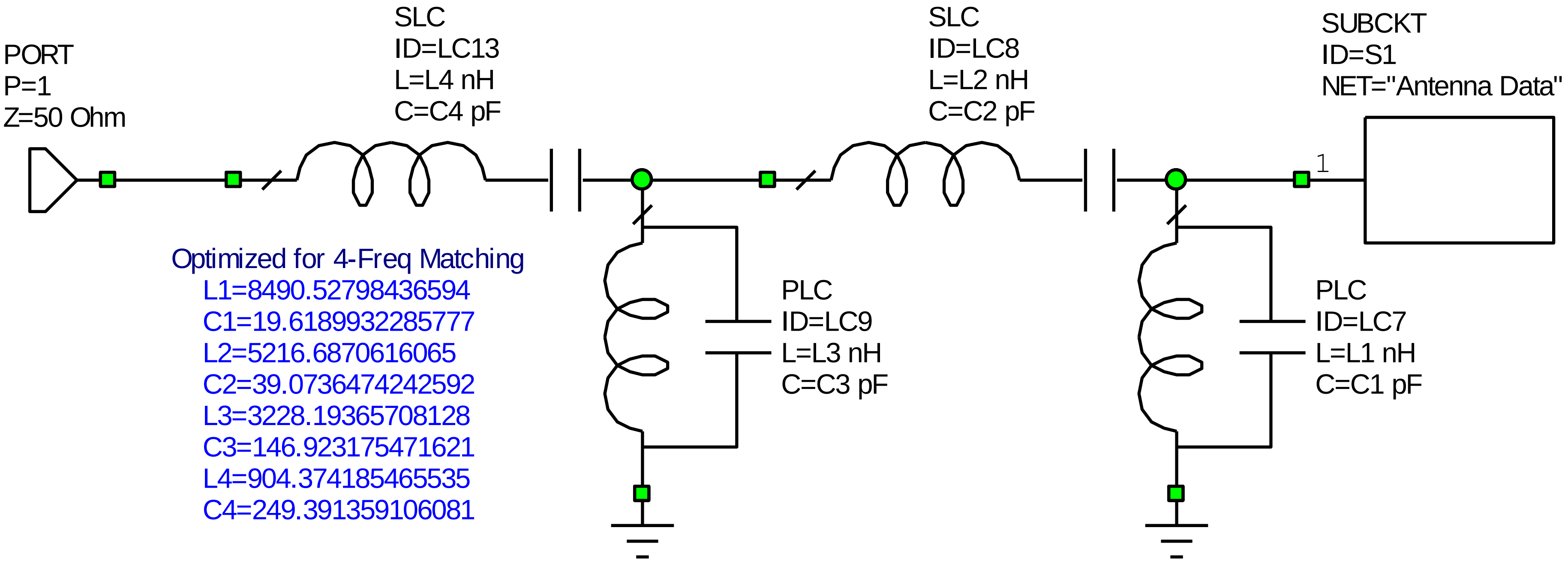
- **Step 0: Choose the order in which the points are to be matched, e.g. decreasing required bandwidth, decreasing frequency, increasing frequency, etc.**
- **Step 1: Use an L-network to move the first point to the center of the Smith chart**
- **Step 2: Add a Z_0-ohm transmission line segment to rotate the second point to the unit resistance (*r* = 1) circle or the unit conductance (*g* = 1) circle such that**
  - On the *r* = 1 circle, the low-frequency point is clockwise from the high-frequency one
  - On the *g* = 1 circle, the high-frequency point is counter-clockwise from the low-frequency one
- **Step 3: Add an LC resonator and adjust *L* and *C* to move both points to center**
  - For points on a resistance circle, use a series LC to move the points
  - For points on a conductance circle, use a shunt LC to move the points
  - This step requires that you label and keep track of each point's frequency
- **Step 4: Repeat Steps 2 and 3 for each remaining frequency point in order**
- **Three points in the proper order on the *r* = 1 or *g* = 1 circle can often be brought to center by using a single series or shunt LC resonator and optimizing all values**

Do the above steps manually viewing a Smith chart and using tuning sliders. When the points are close to center, use an optimizer to finish fine tuning the parts values.

Examples 10 and 11: Alternate 4-Band and 5-Band Matching on 75, 40, 20, 15, and 10 Meters

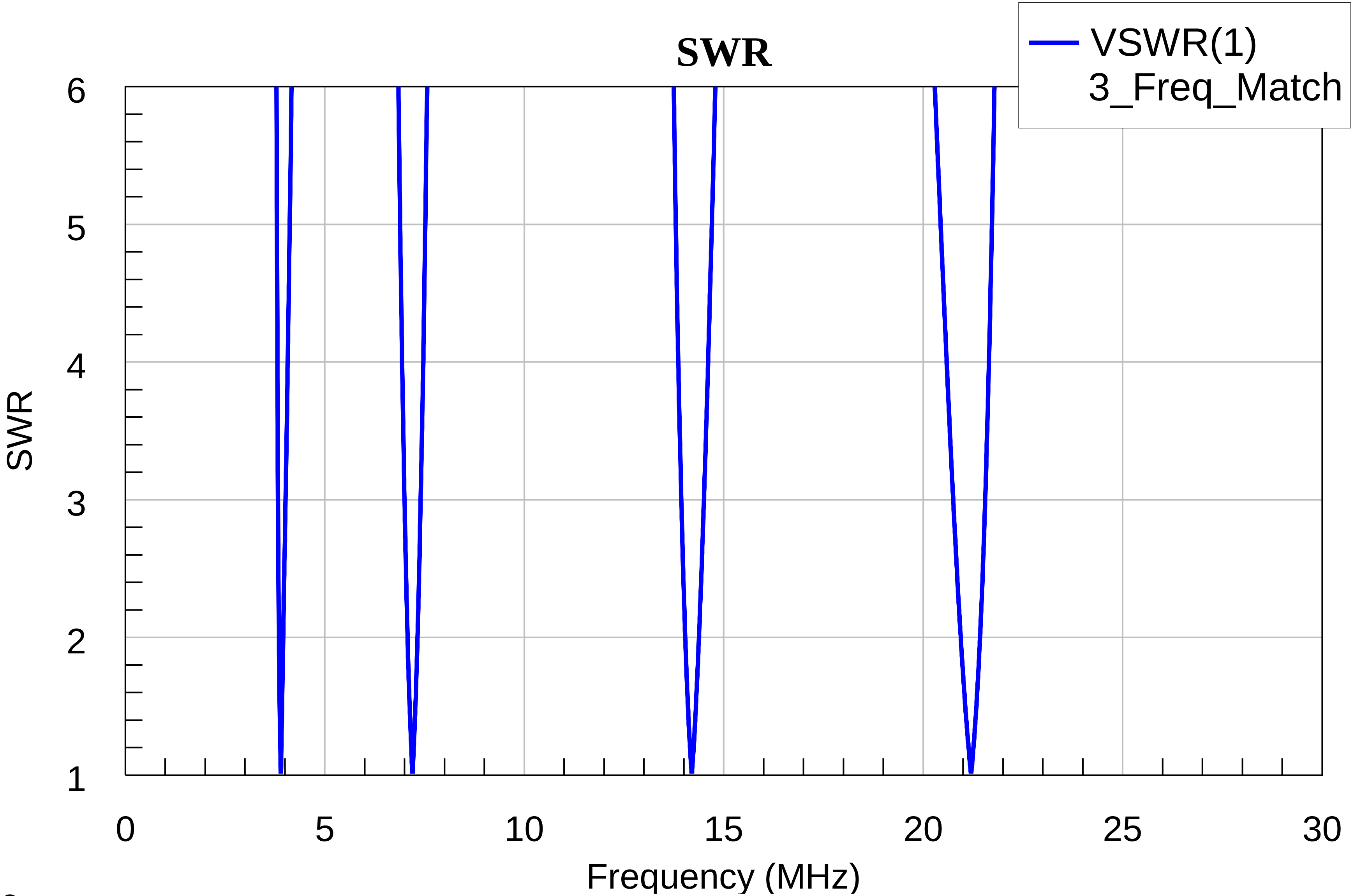
- **Antenna: 98.4 ft. dipole**
- **Objectives:**
  - Example 10: 4-band match at 3.9, 7.2, 14.2, and 21.2 MHz
  - Example 11: 5-band match at 3.9, 7.2, 14.2, 21.2, and 29 MHz
  - See if reversing the frequency order gives greater match bandwidth on 75 meters and less on 10 meters as compared to Examples 8 and 9
- **Topology:**
  - Example 4: Use lumped element capacitors and inductors
  - Example 5: Add a line section and resonator bank for fifth frequency
- **Approach:**
  - Match lowest four frequencies using an 8-element network
  - Add a resonator bank to match the 5^{th} (highest) frequency
  - Use tuning slider to hand tune the line section for 29 MHz
  - Use optimizer to fine-tune intermediate and final network structures

Example 10: Four-Band, 8-Element Lumped Match Network for 80, 40, 20, and 15 Meters

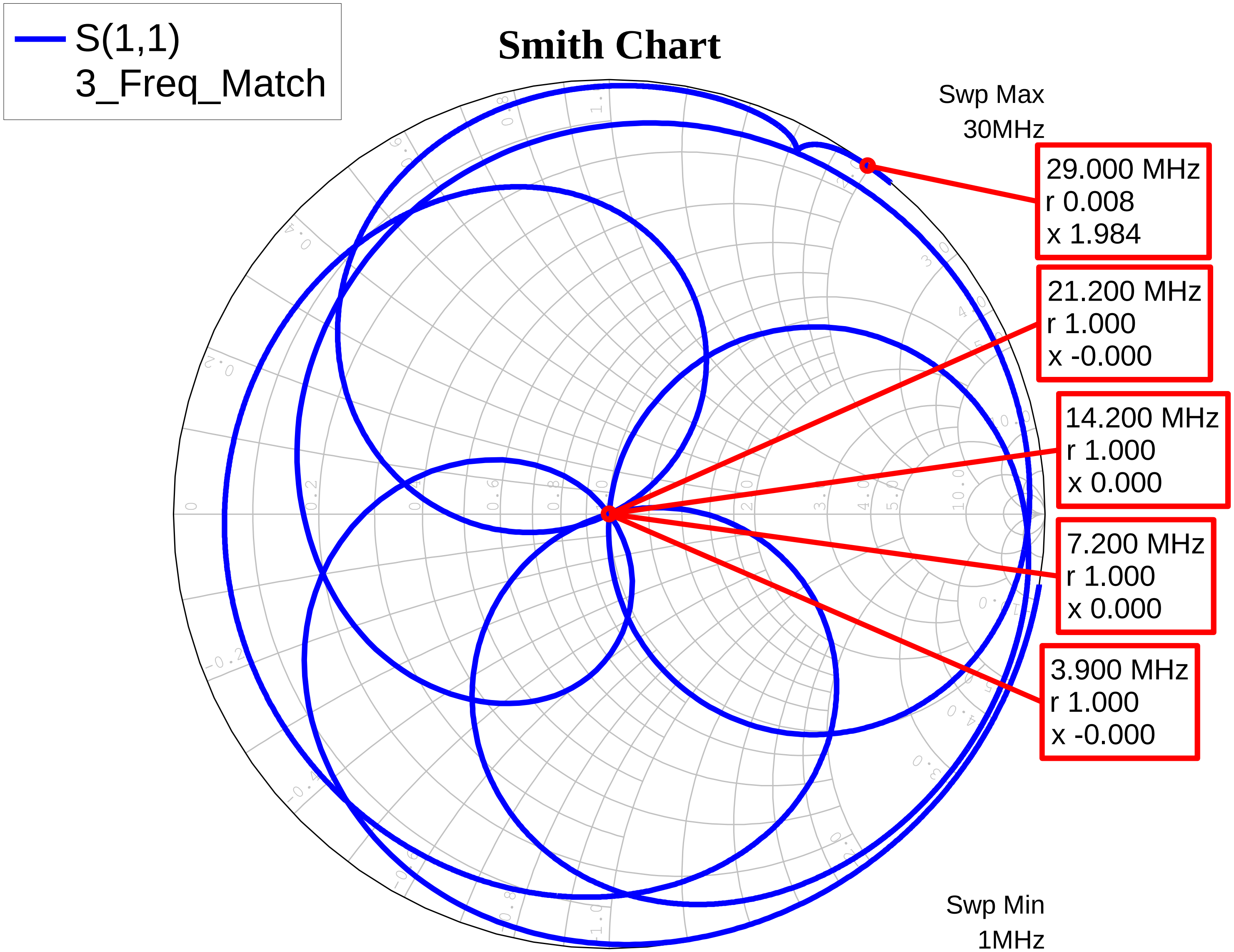


Optimized Match Network for 75, 40, 20, and 15 meters

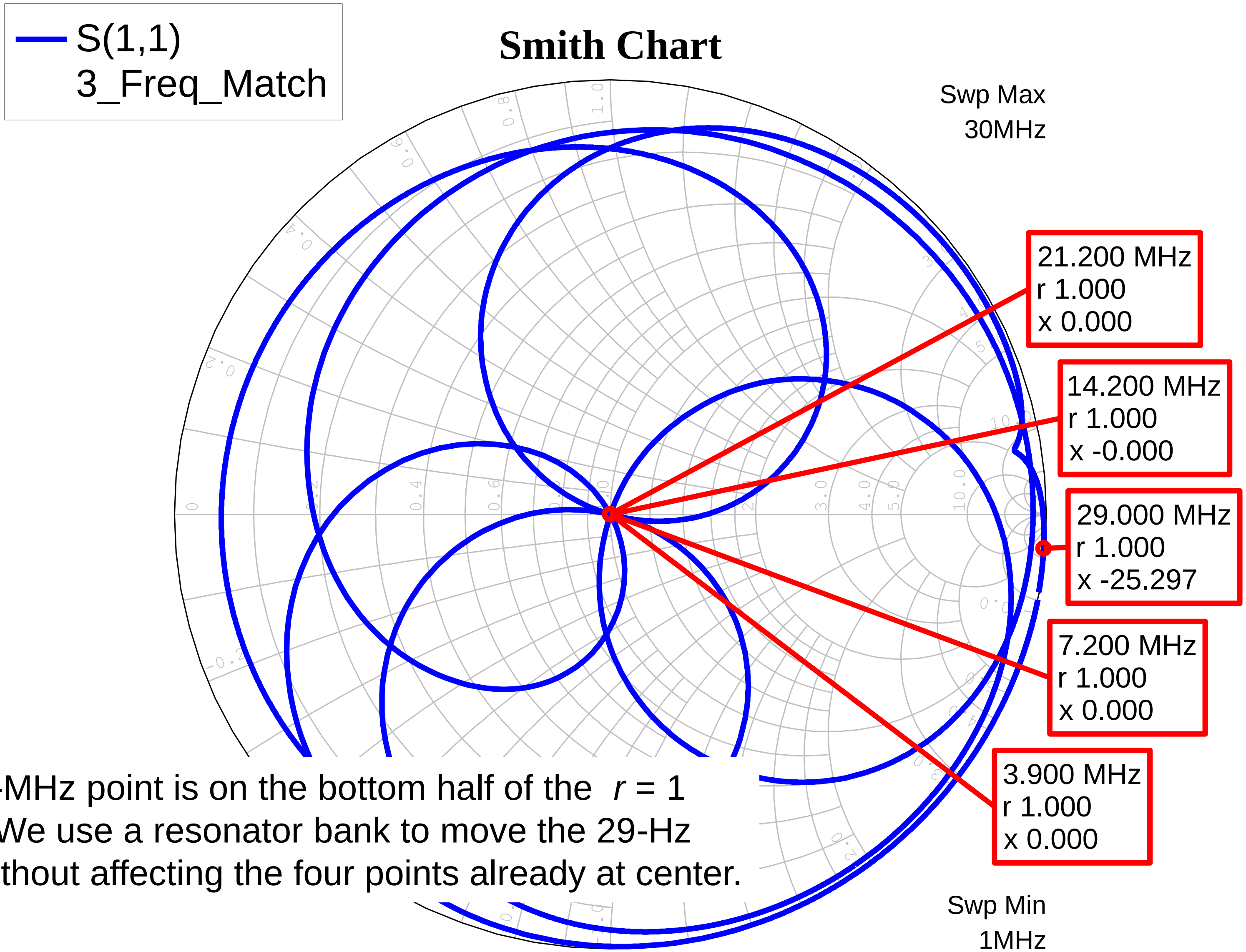
SWR of 4-Band Match



4 Bands Matched Using Lumped Network, 1 Band to Go

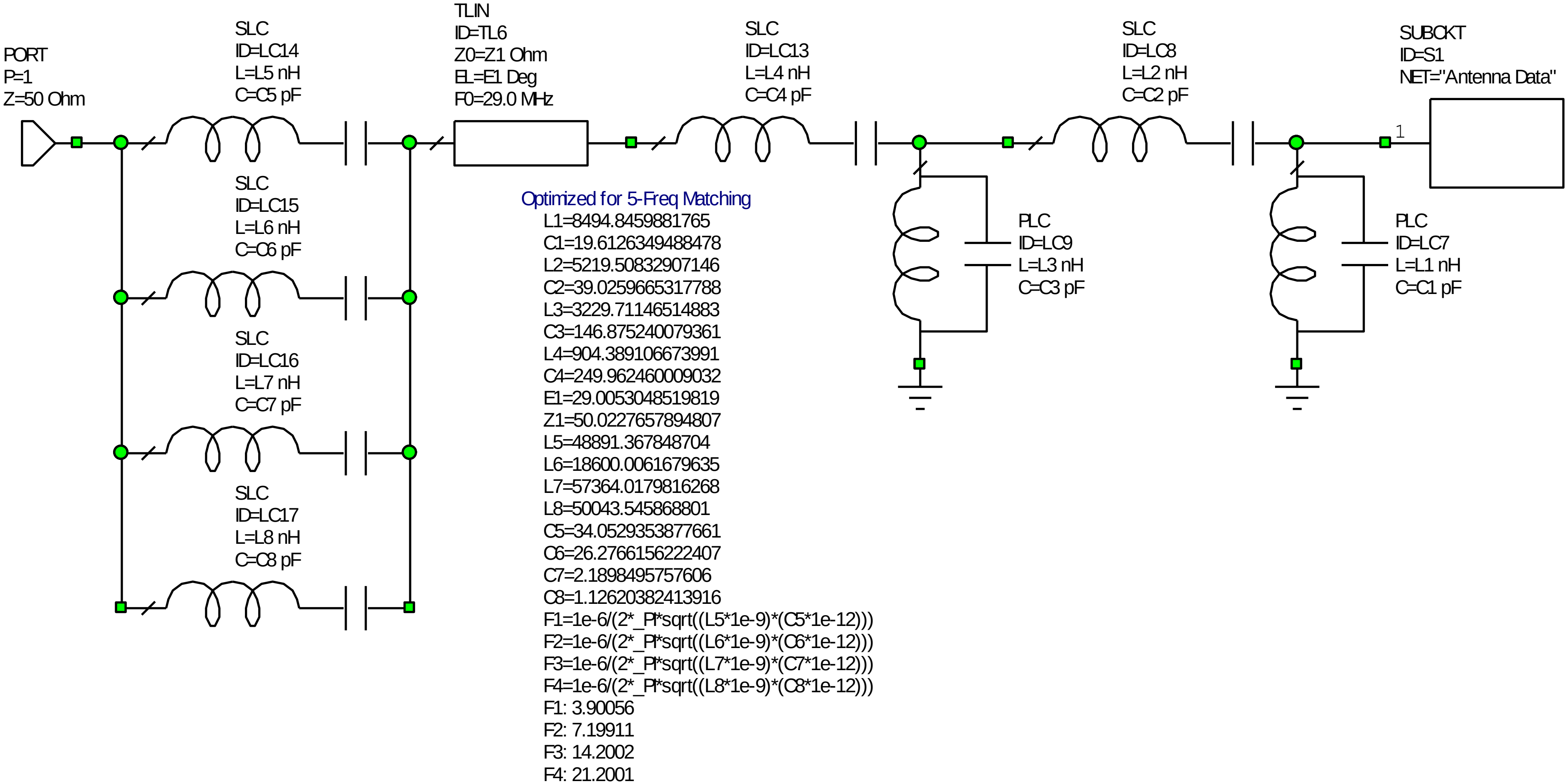


Result After Rotating the 29-MHz Point to the *r* = 1 Circle



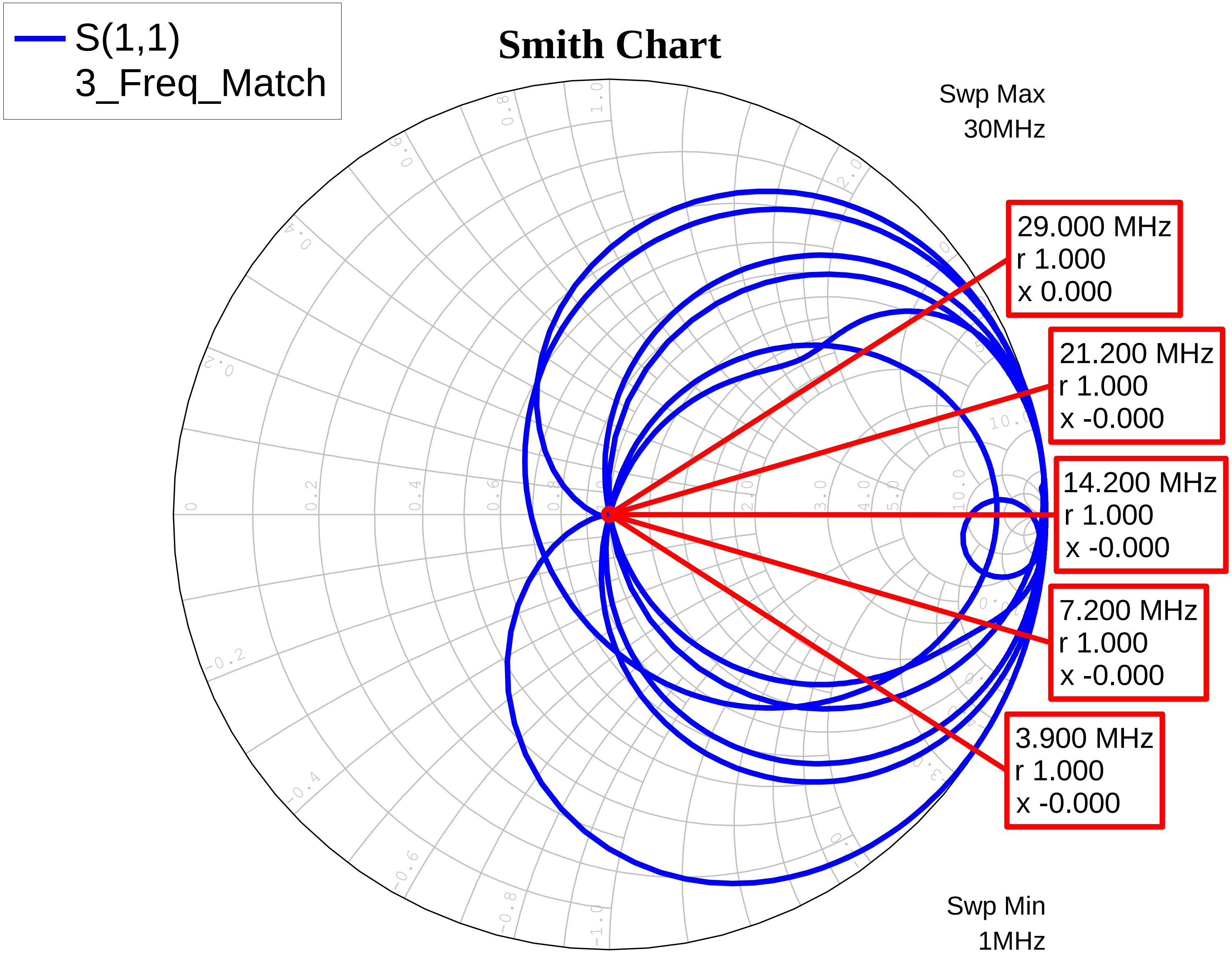
The 29-MHz point is on the bottom half of the *r* = 1 circle. We use a resonator bank to move the 29-Hz point without affecting the four points already at center.

Example 11: Final 5-Band, 17-Element Matching Network

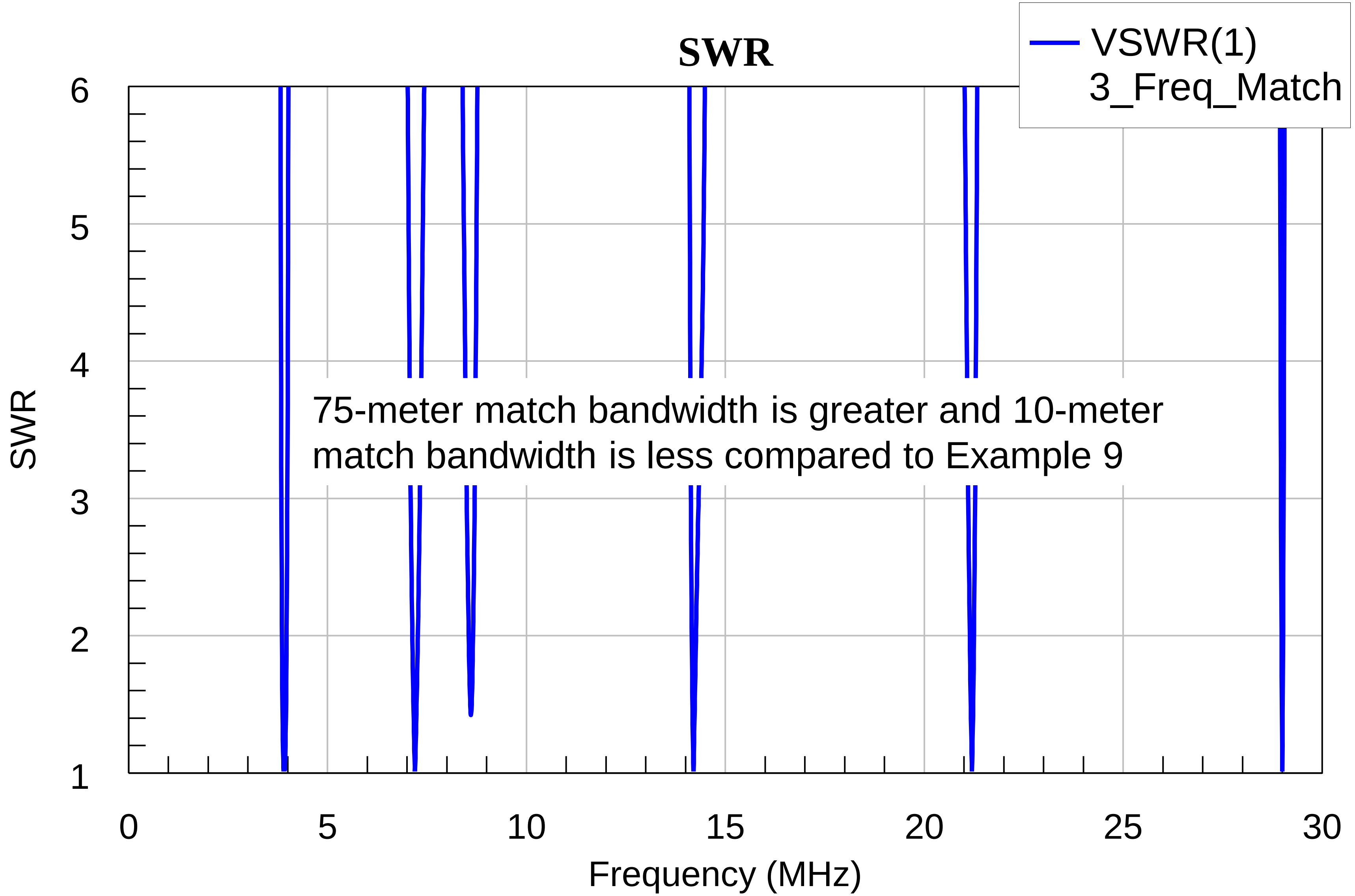


Optimized Match Network for 75, 40, 20, 15 and 10 meters

Impedance of 5-Band Match Network



SWR



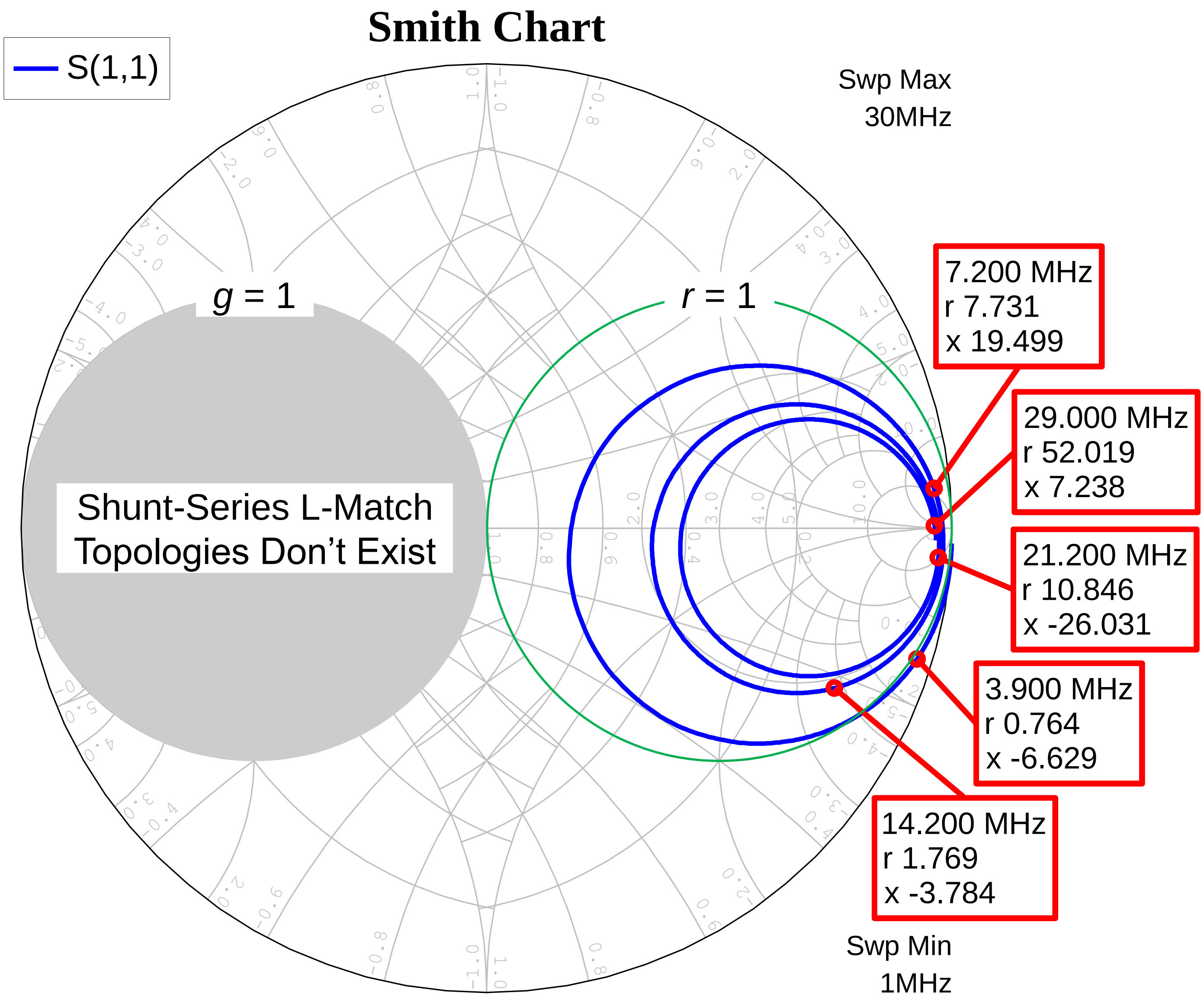
SWR
VSWR(1)
3_Freq_Match
6
5
4
3
2
1
SWR
0
5
10
15
20
25
30
Frequency (MHz)
75-meter match bandwidth is greater and 10-meter match bandwidth is less compared to Example 9

Matching an Arbitrary Number of Frequencies Using L-Network Topologies

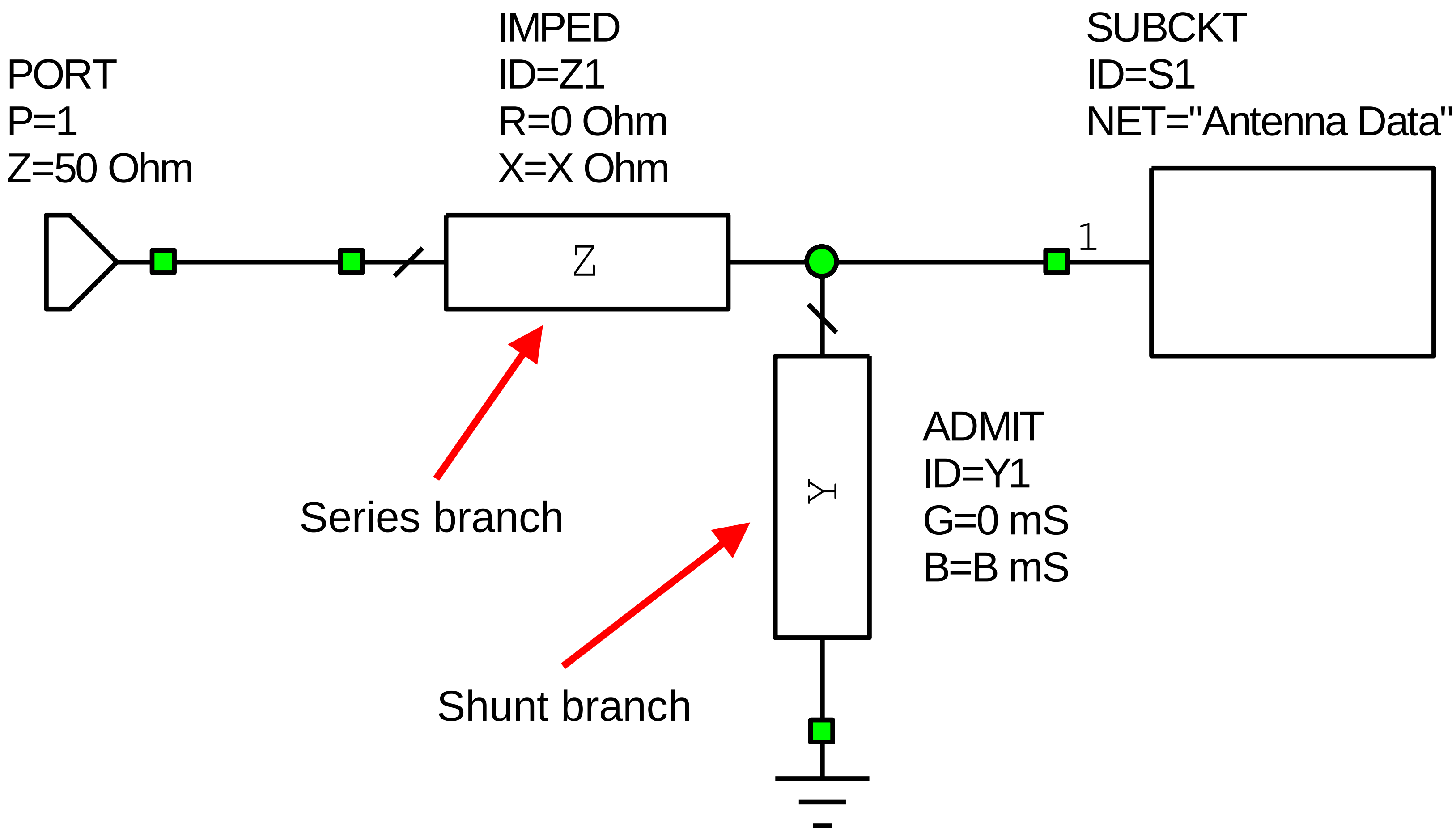
Antenna Impedance on Smith Chart

Step 0:

- Make sure either r = 1 or g = 1 circle is empty of points to be matched
- If necessary, shift points left or right by using an ideal transformer
- After match, you can shift back with a second transformer



Shunt-Series L-Network Topology

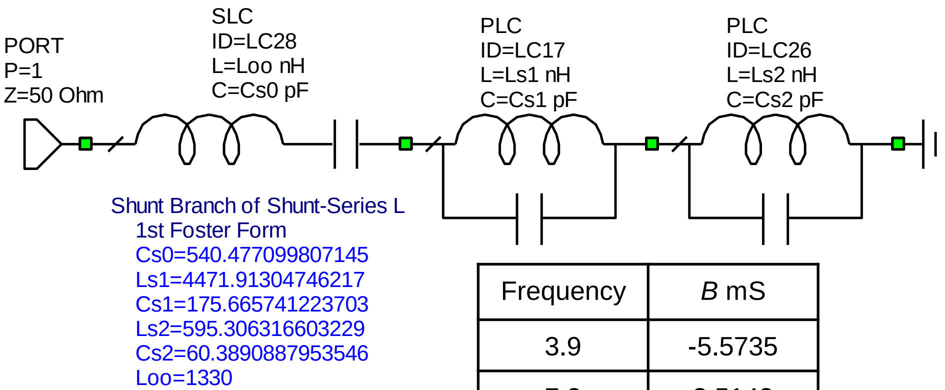


Step 1: Decide On the Sequence of Match Points to Realize

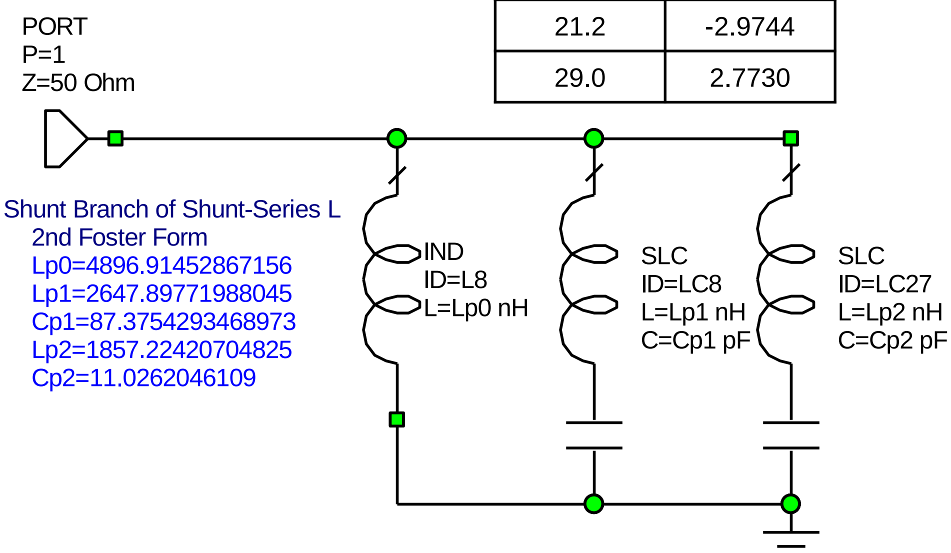
	Top Half of *r* = 1 Circle		Lower Half of *r* = 1 Circle	
Frequency	*B* mS	*X* ohms	*B* mS	*X* ohms
3.9	-5.5735	-378.57	-0.3811	378.57
7.2	-1.7414	-373.86	3.5142	373.86
14.2	-10.375	-148.84	1.6997	148.84
21.2	-2.9744	-425.20	1.6651	425.20
29.0	-2.6680	-360.65	2.7730	360.65

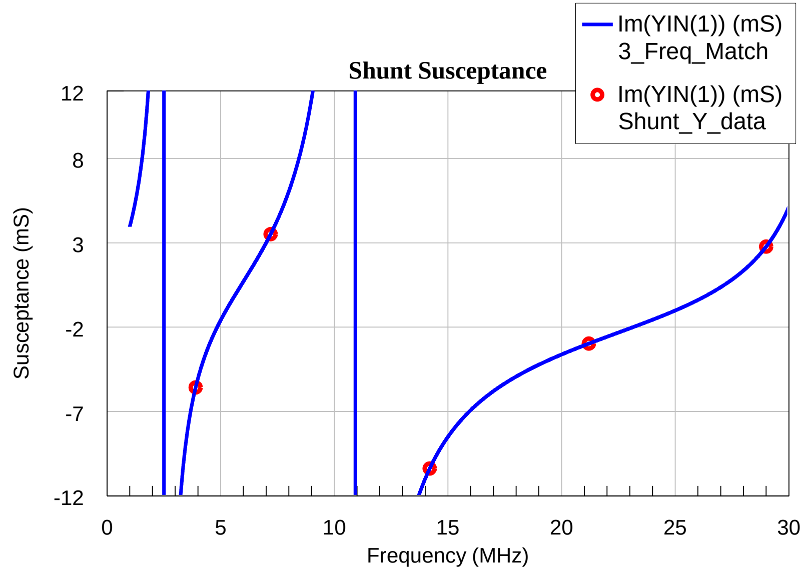
- Susceptance and reactance functions are tailored to fit chosen match point sequences by using Foster synthesis
- Susceptance and reactance sequences should increase monotonically with as few downward jumps as possible
- Downward jumps should be well placed, avoiding bands or regions where large match bandwidth is desired

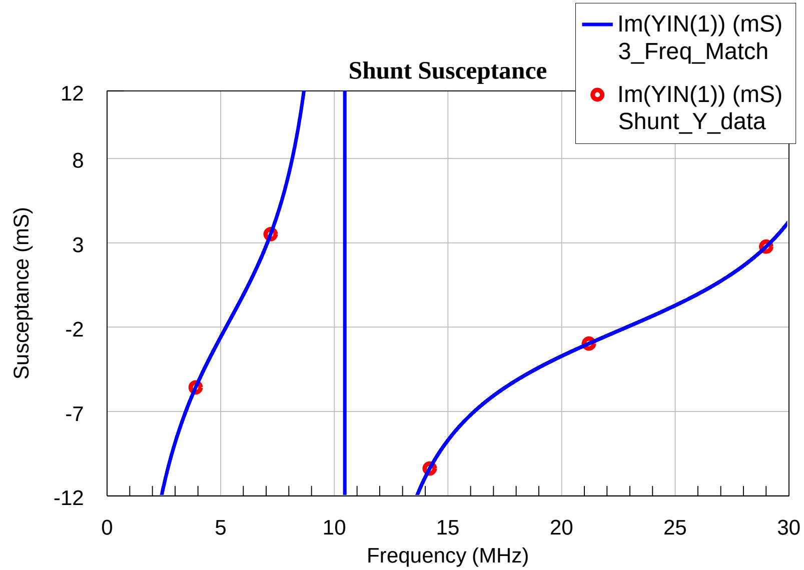
Shunt Branch of L Realized in 1st and 2nd Foster Forms



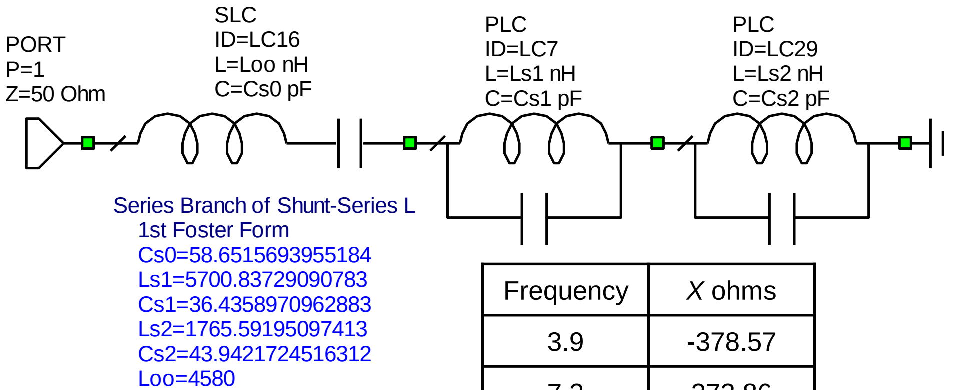
Frequency	*B* mS
3.9	-5.5735
7.2	3.5142
14.2	-10.375
21.2	-2.9744
29.0	2.7730



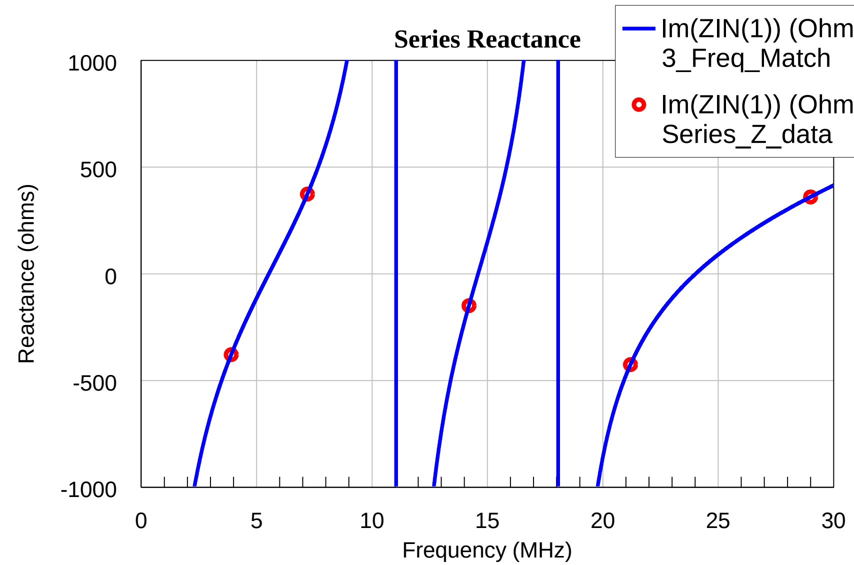


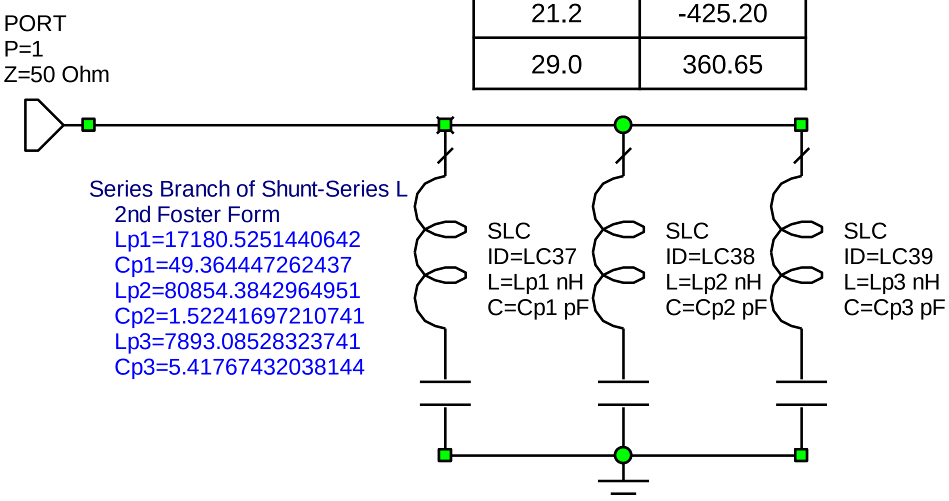


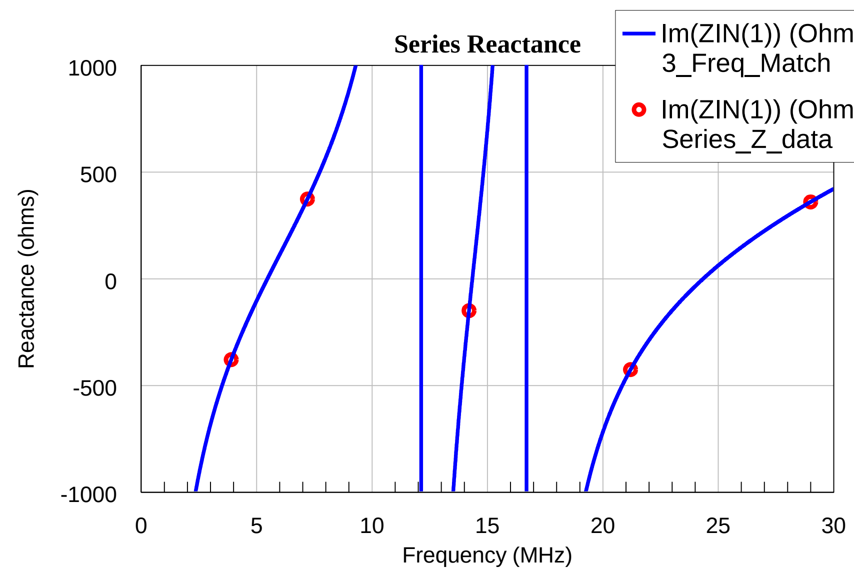
Series Branch of L Realized in 1st and 2nd Foster Forms



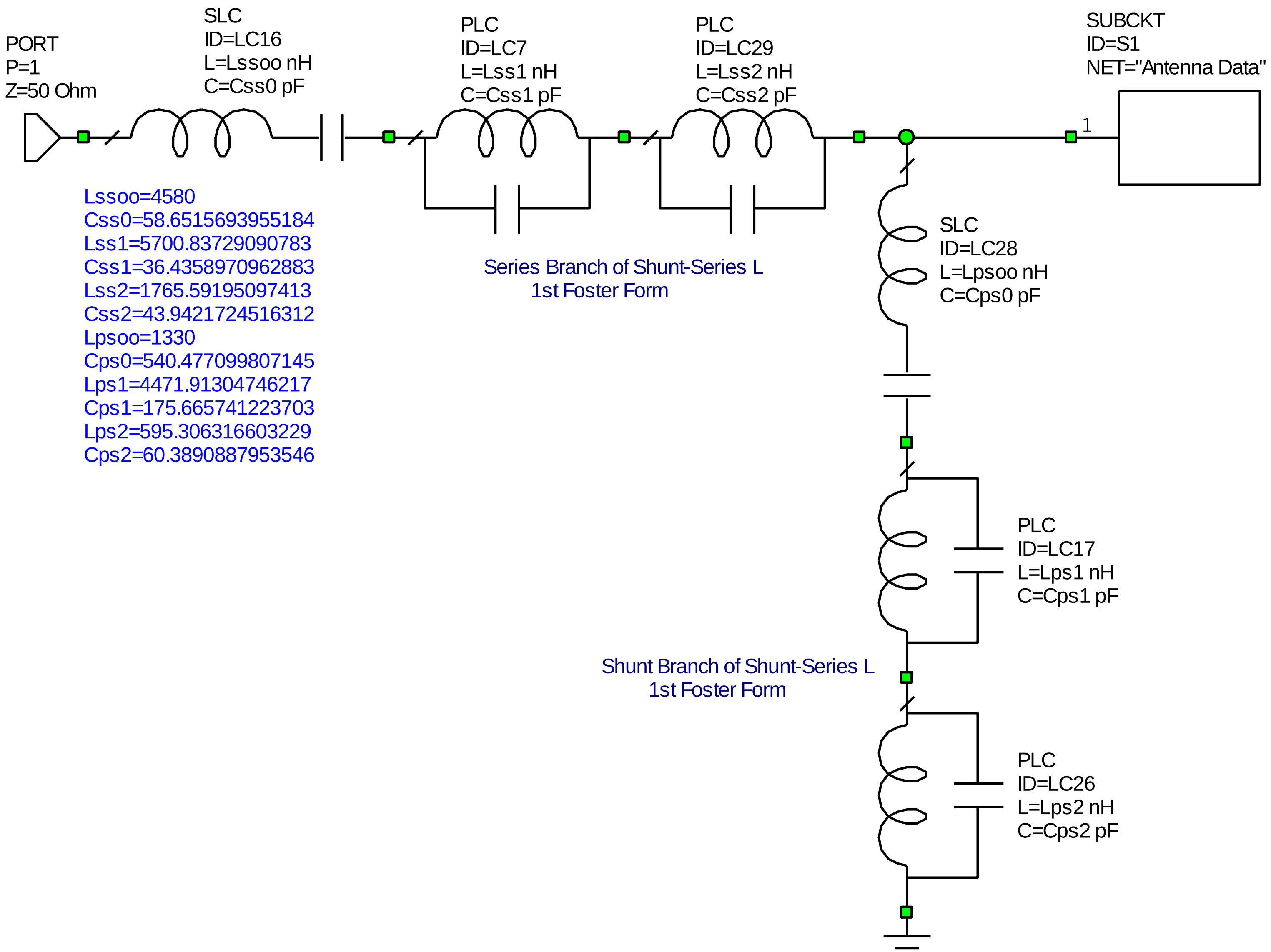
Frequency	*X* ohms
3.9	-378.57
7.2	373.86
14.2	-148.84
21.2	-425.20
29.0	360.65



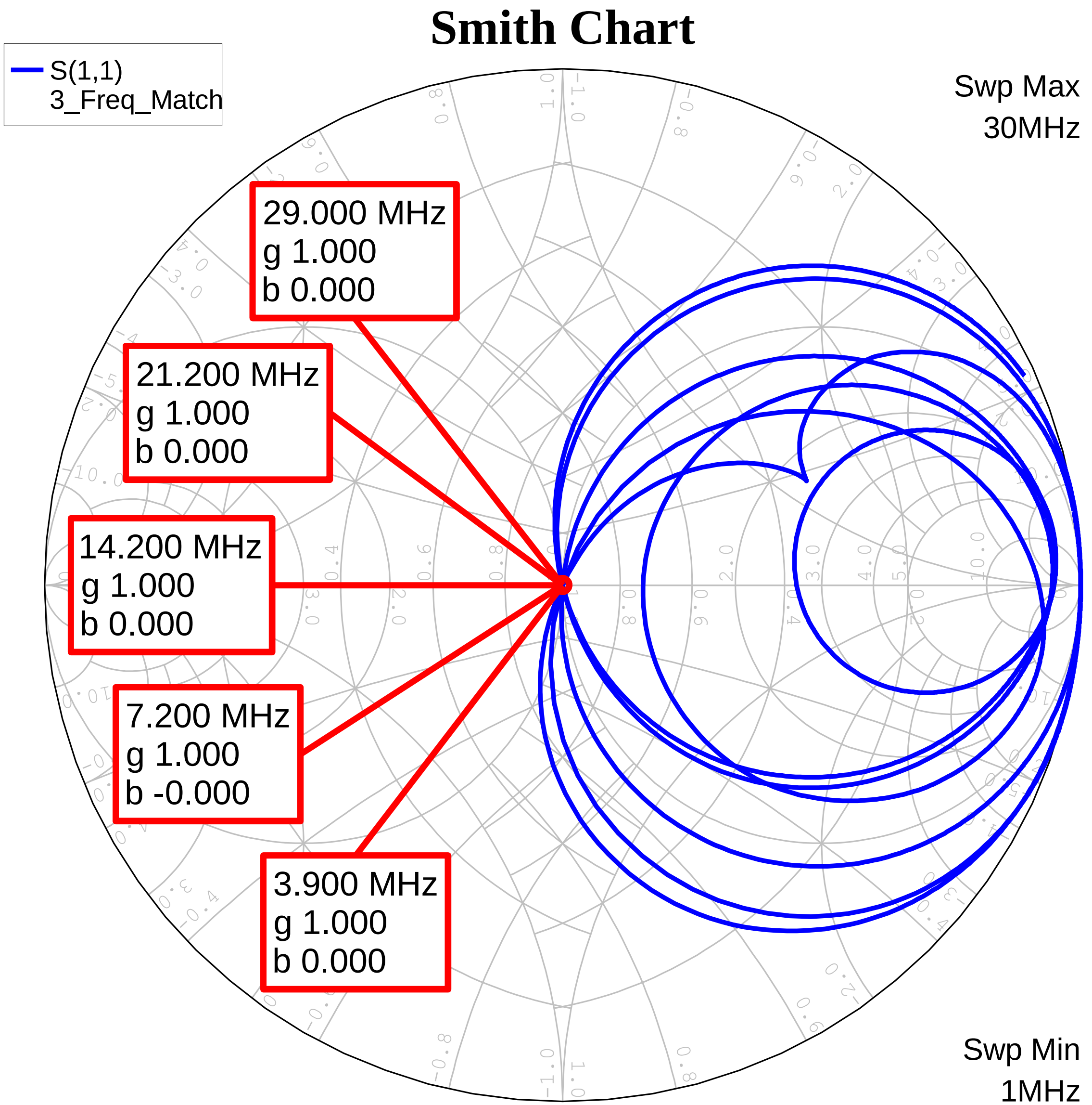




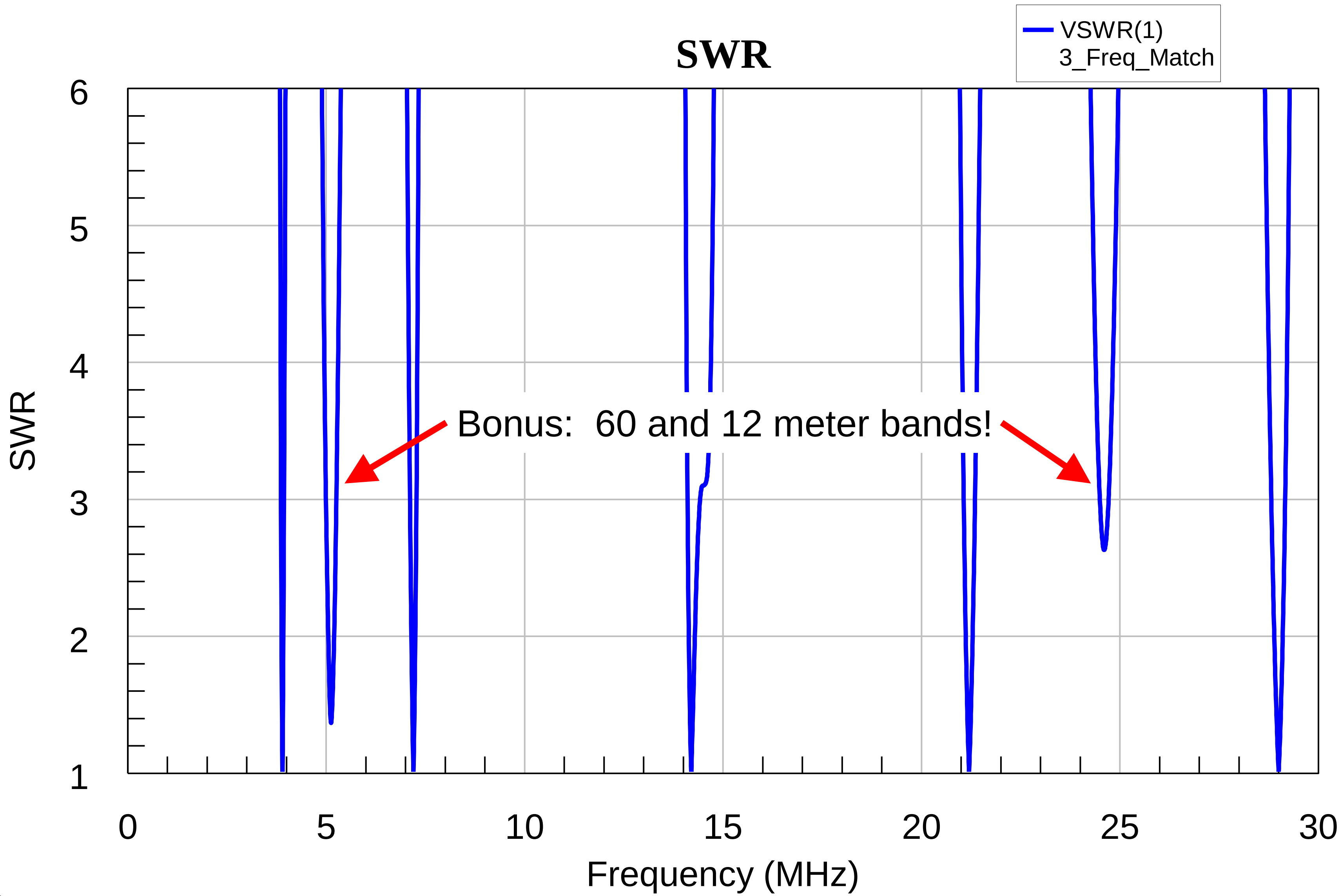
Match Network No. 1: Shunt 1FF and Series 1FF



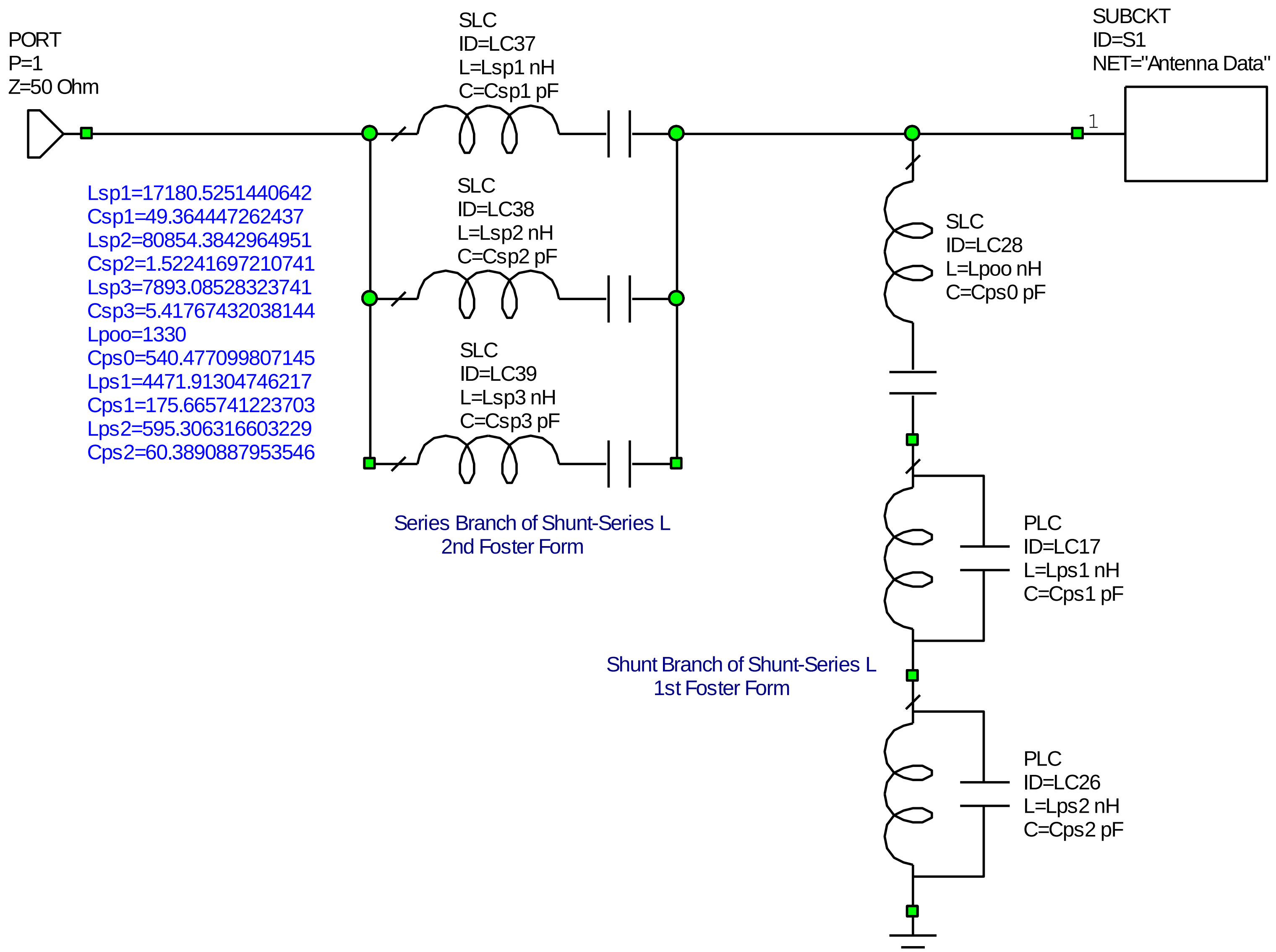
Match Result on Smith Chart – 5 Frequencies Matched!



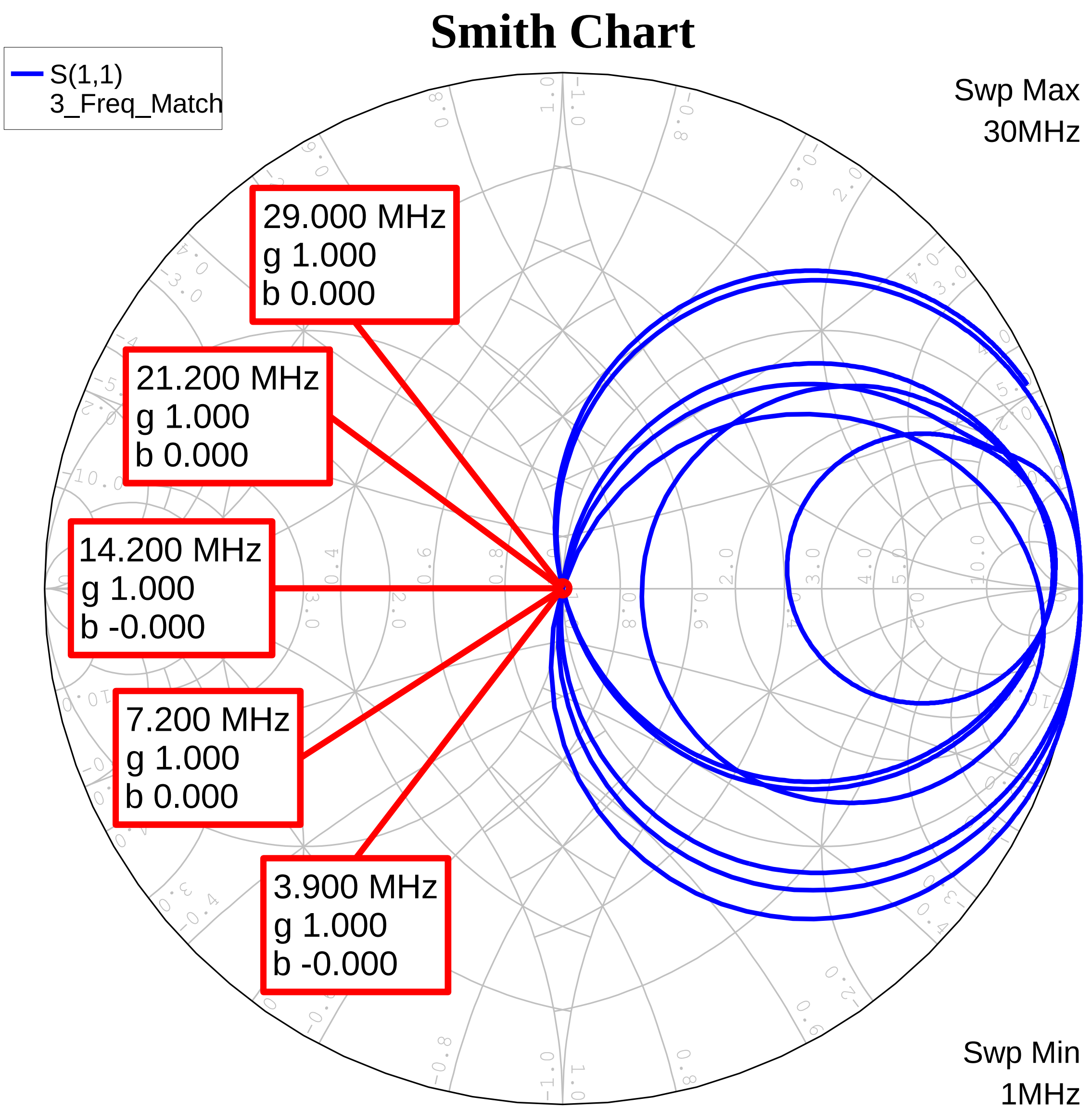
SWR – Five Bands and, Surprize! Two More!



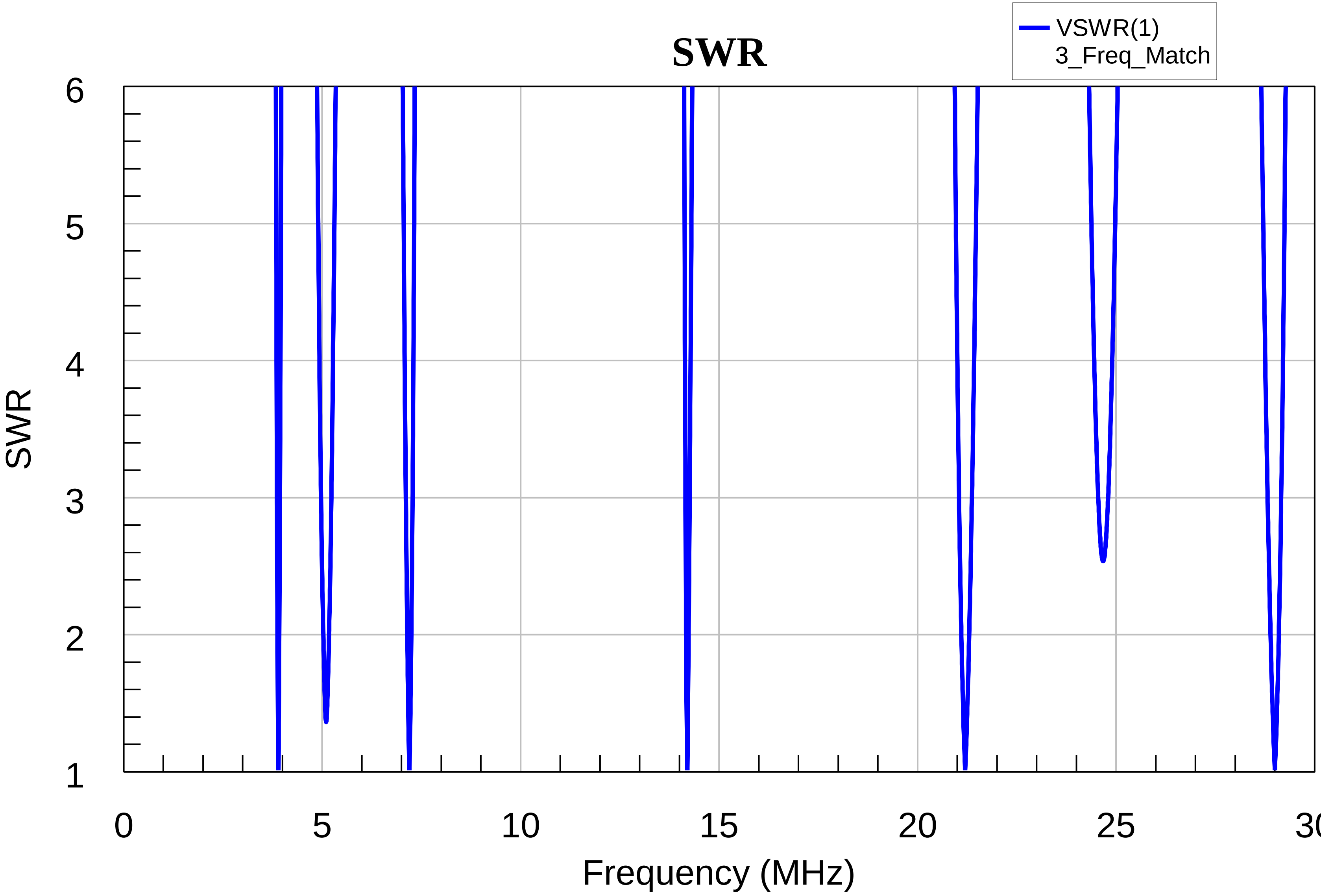
Match Network No. 2: Shunt 1FF and Series 2FF



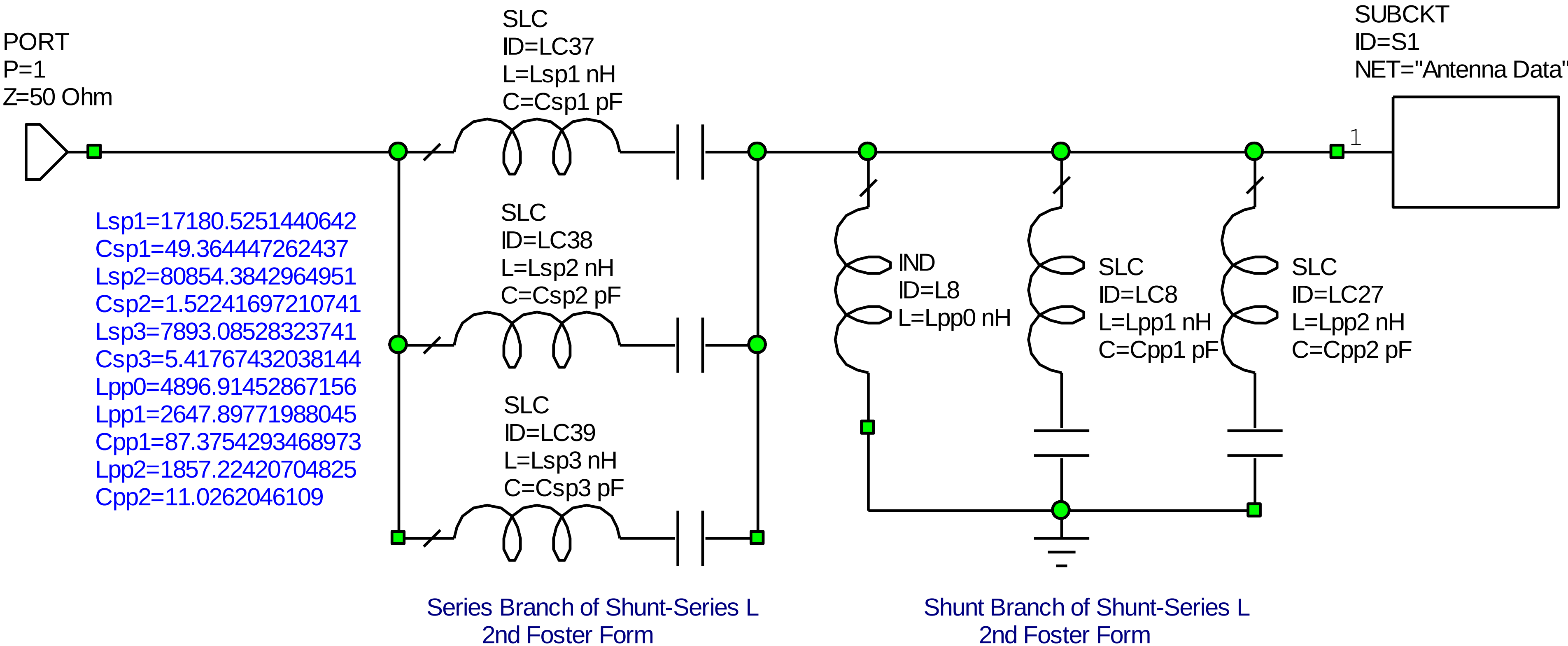
Five-Frequency Match



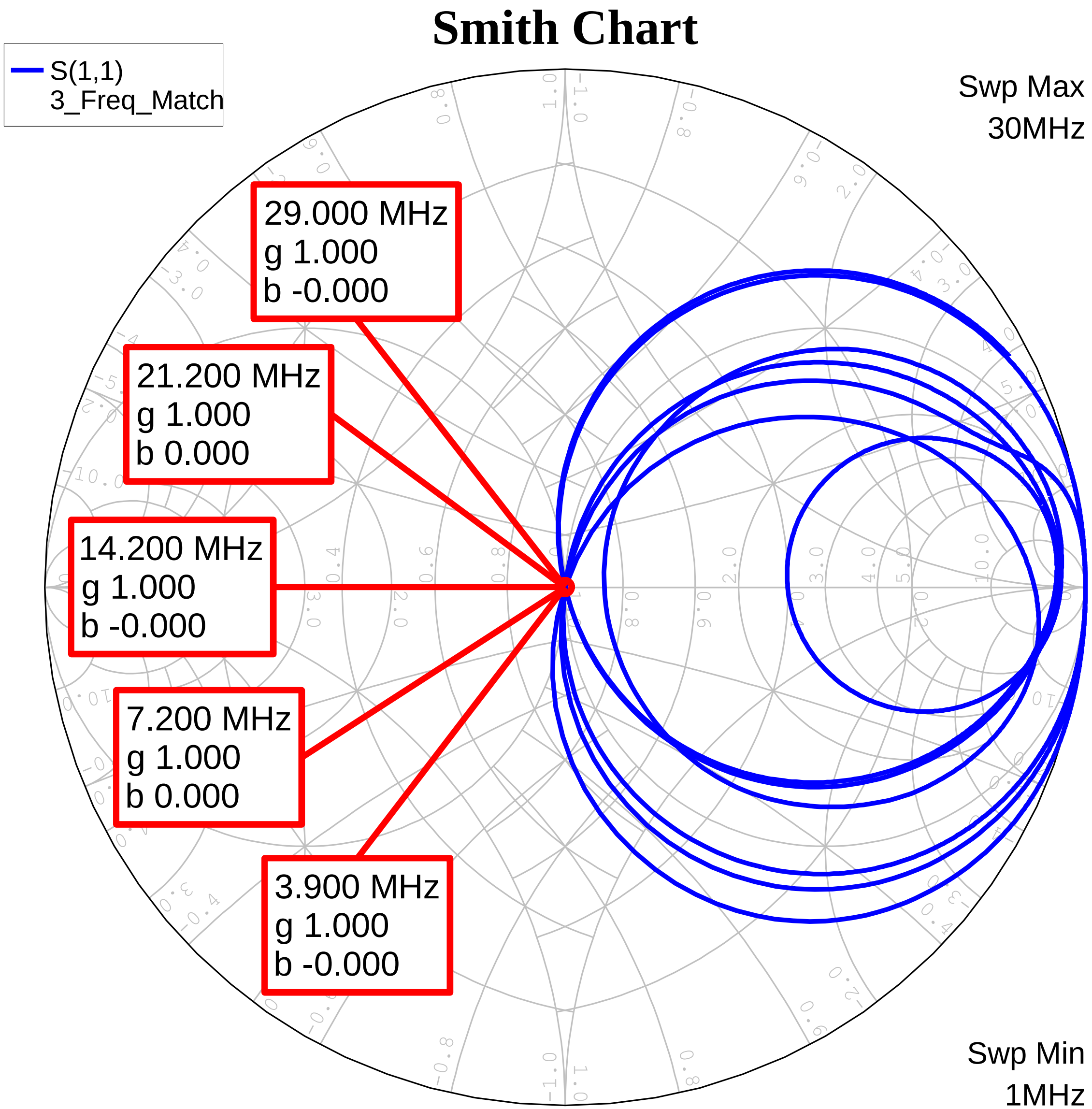
SWR


SWR
VSWR(1)
3_Freq_Match
6
5
4
3
2
1
SWR
0
5
10
15
20
25
30
Frequency (MHz)

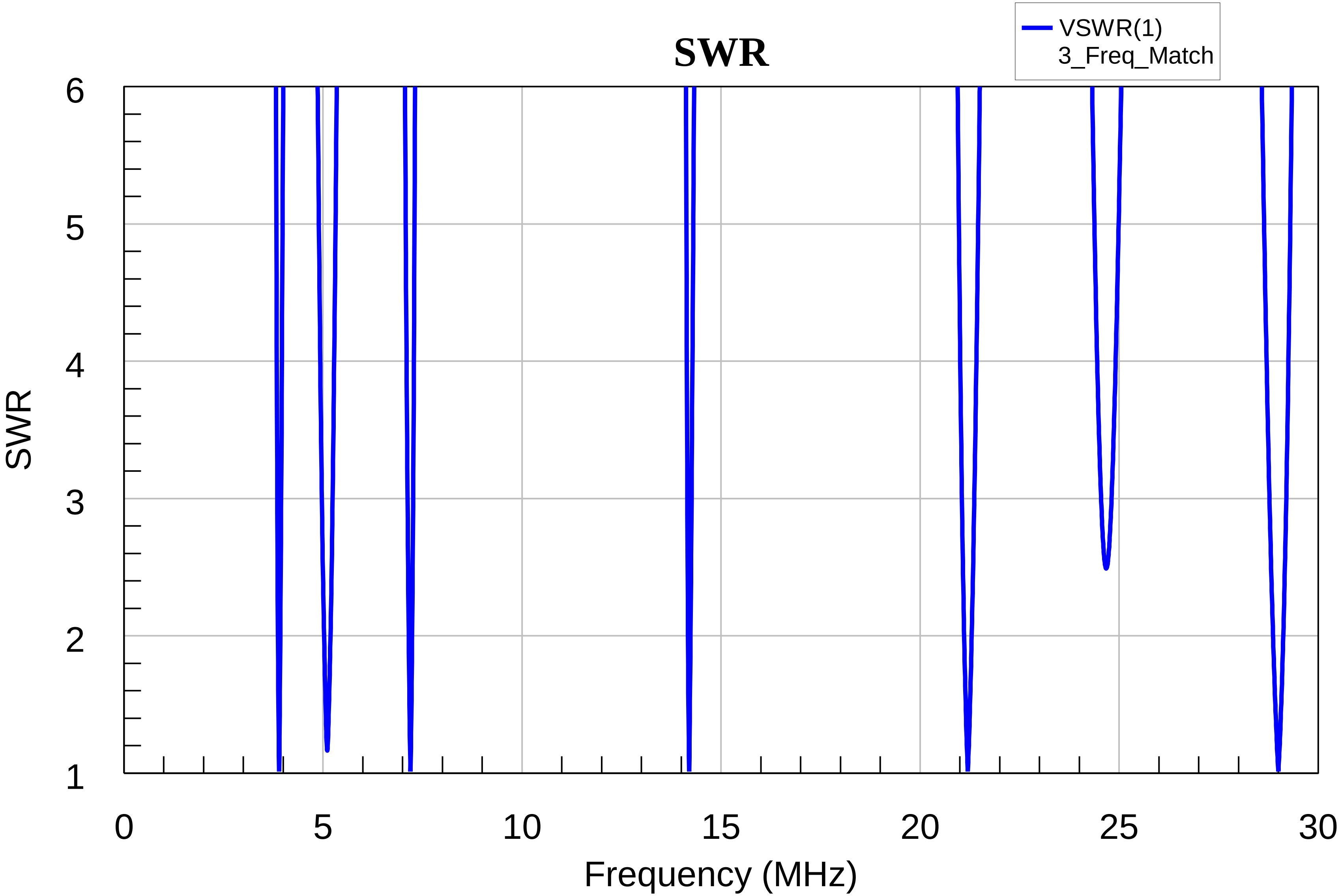
Match Network No. 3: Shunt 2FF and Series 2FF



Five-Frequency Match

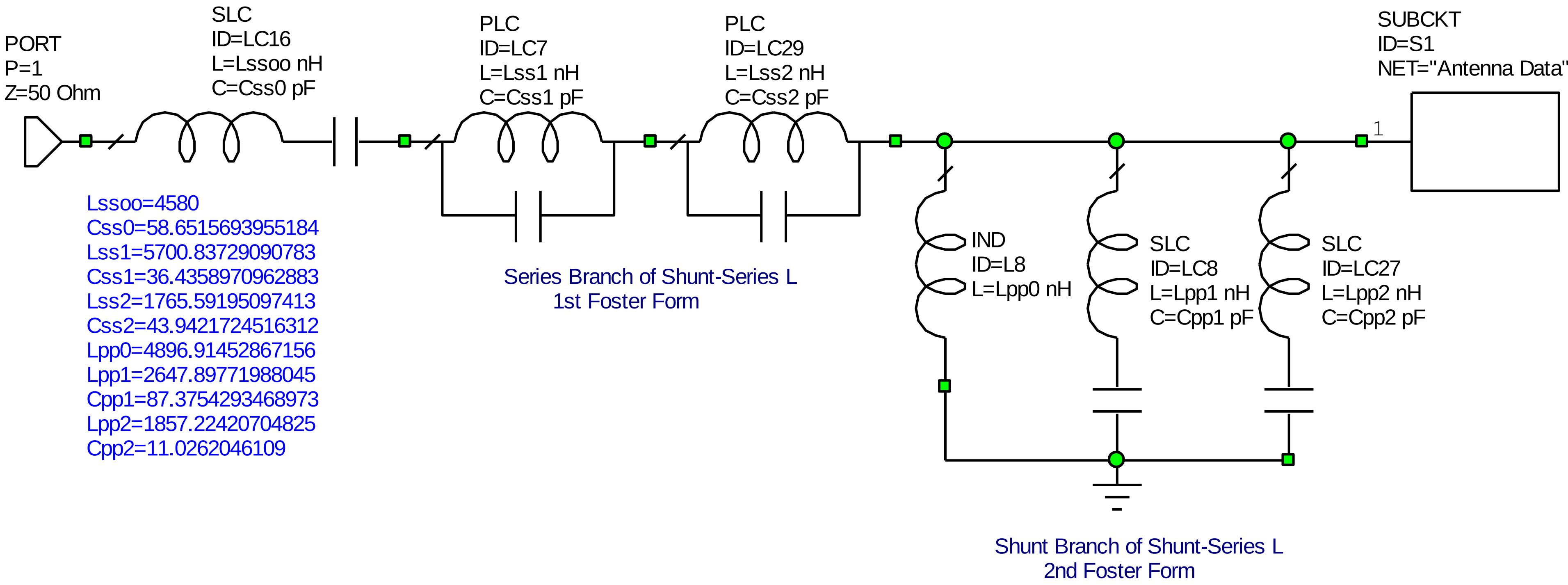


SWR

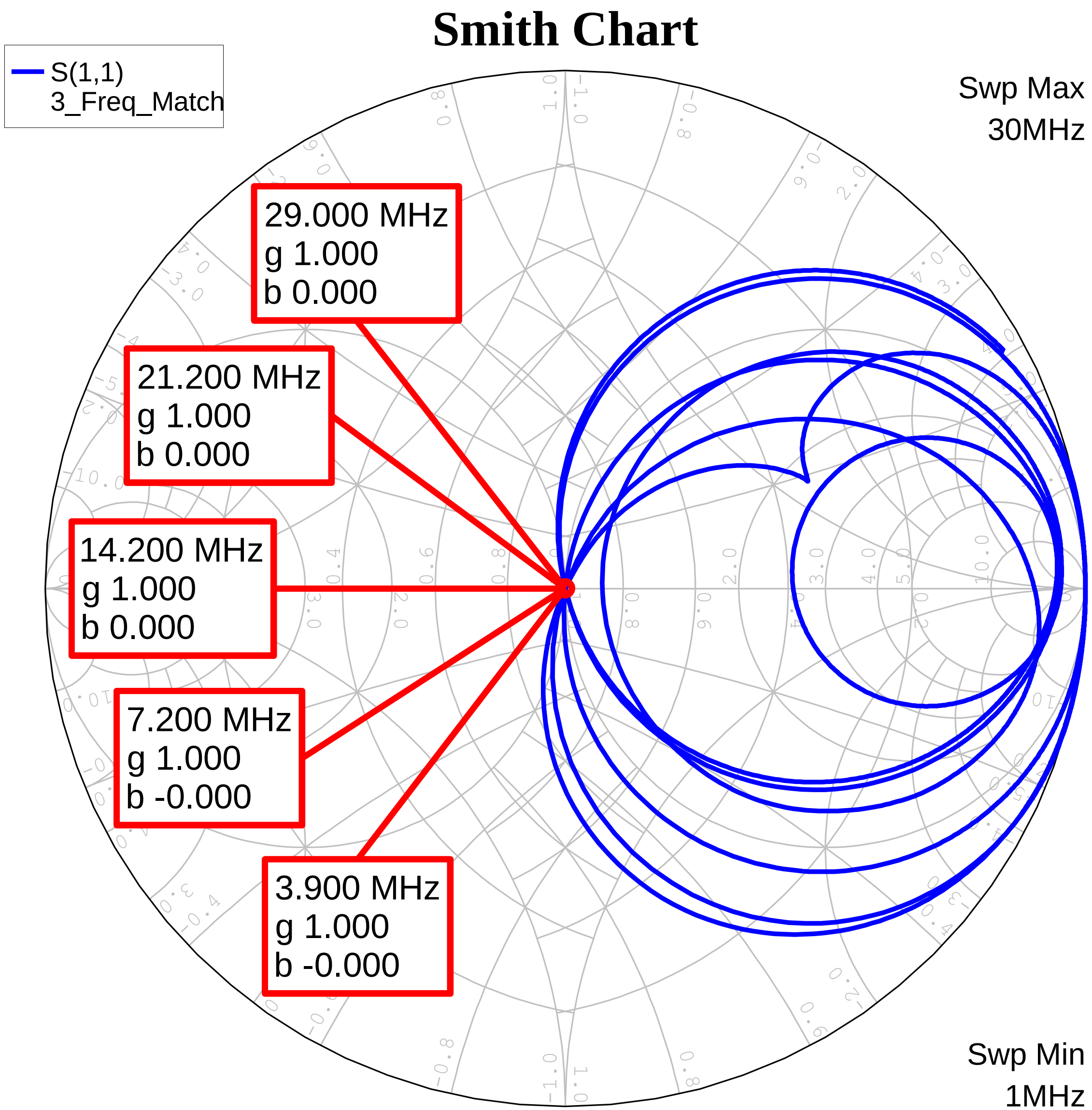


SWR
VSWR(1)
3_Freq_Match
6
5
4
3
2
1
SWR
0
5
10
15
20
25
30
Frequency (MHz)

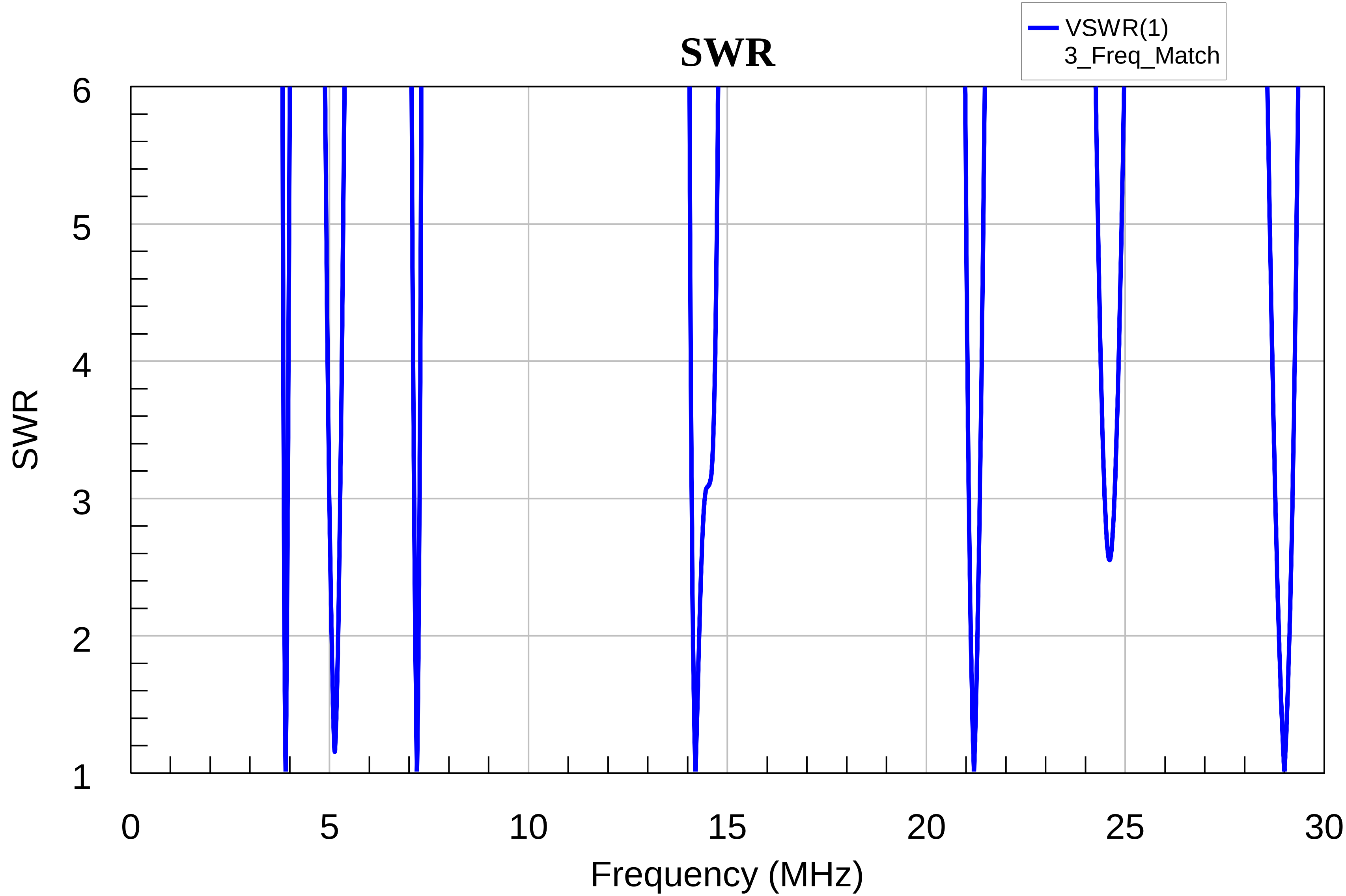
Match Network No. 4: Shunt 2FF and Series 1FF



Five-Frequency Match



SWR

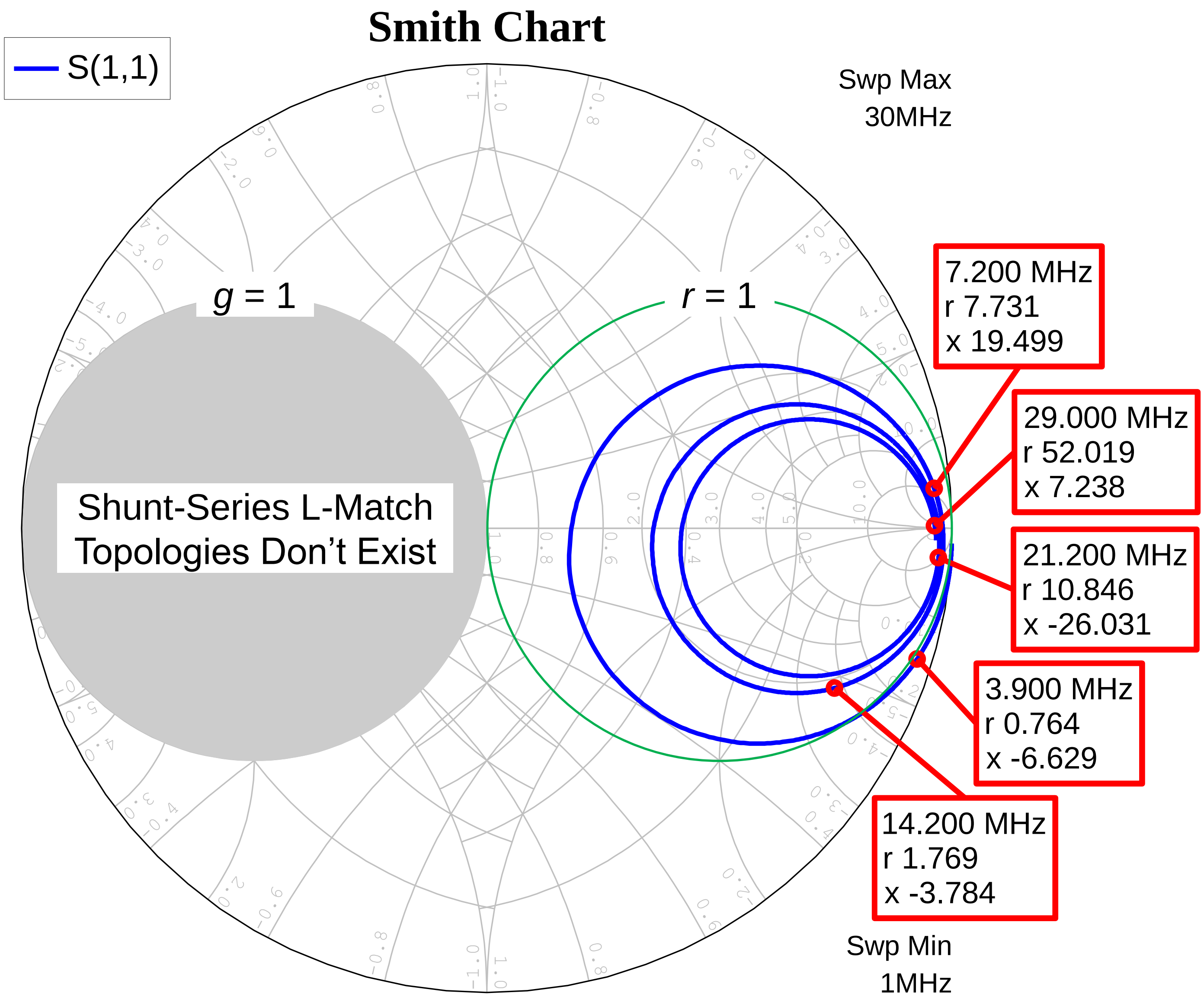


SWR
VSWR(1)
3_Freq_Match
6
5
4
3
2
1
SWR
0
5
10
15
20
25
30
Frequency (MHz)

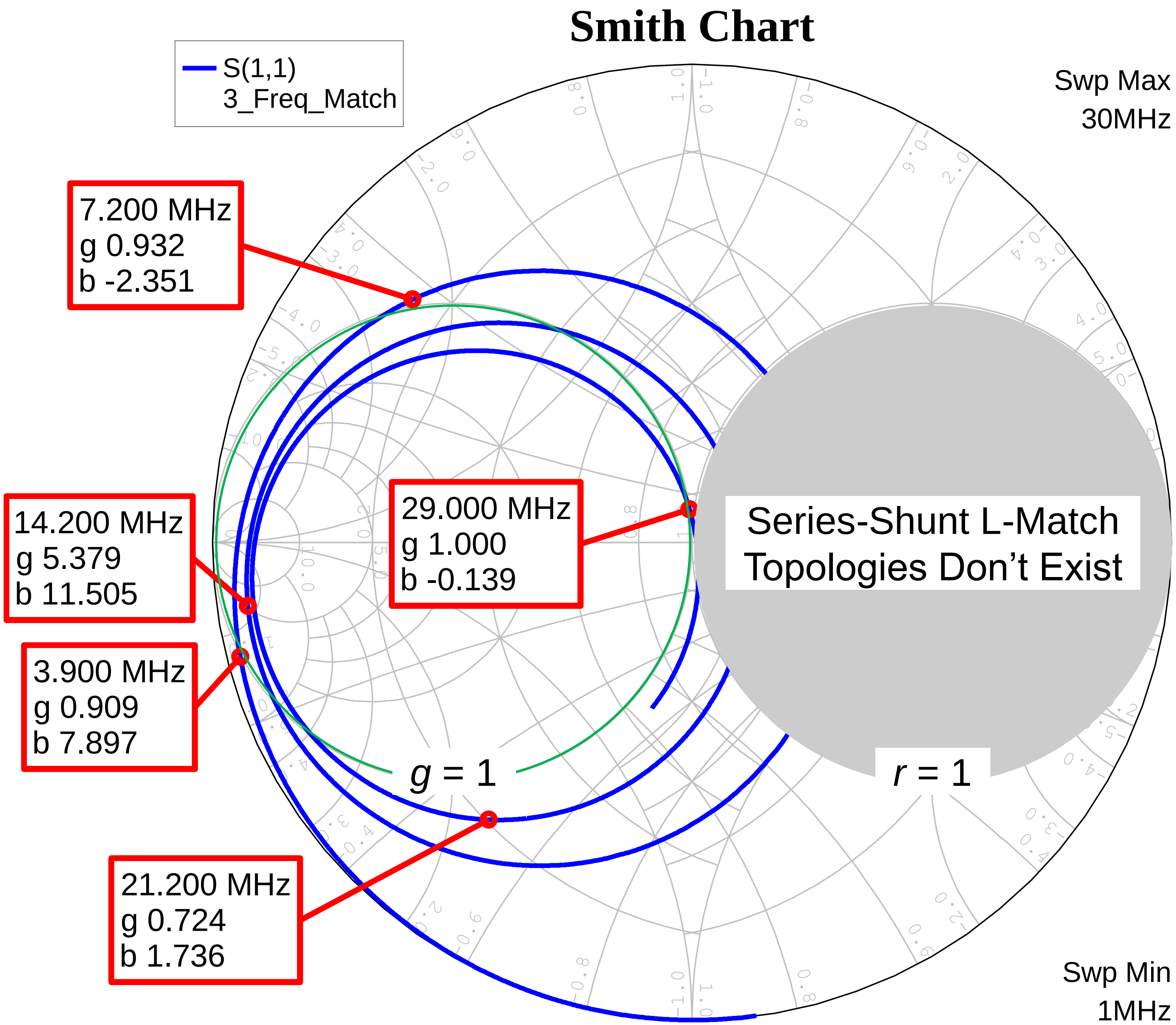
Shunt-Series L Works When Points Aren't in *g* = 1 Circle

<u>Step 0</u>:

- Make sure either *r* = 1 or *g* = 1 circle is empty of points to be matched
- If necessary, shift points left or right by using an ideal transformer
- Or change the normalization impedance
- After match, use a transformer to shift to 50 Ω



Series-Shunt L Works When Points Aren't in *r* = 1 Circle



- An ideal transformer, turns ratio 7.283, shifts all points left, lowering all impedances by a factor of 53
- Alternatively, match to Z_0 = 2650 Ω, and afterwards shift to 50 Ω by transformer
- Bottom line: when the *r* = 1 circle is empty, a series-shunt L topology can be used

Advanced Topics

Number of choices for L topologies

Matching with constraints

Iterative synthesis

Commutative match networks

Number of Choices for L Topologies

- Choice of shunt-series L or series-shunt L: 2
- Choice of sequence of *n* match points: 2^n
- Choice of 1st vs 2nd Foster form for series and shunt branches: 2^2
- Choice of pole placements per branch: $< 2^{n+2}$
- Maximum number of L match networks possible for *n* frequencies

$$N \leq 2^{2n+5}$$

- For the examples we've shown, $n = 5$ and $N > 33$ million!
- A designer's skill, experience, and intuition reduces the number of candidate networks to consider

Matching with Constraints

- **For an L network, some frequencies can be designated for which the network does not affect the load impedance**
- **These frequencies are "fixed points" with respect to impedance transformation by the network**

$$Z_{match}(f) = Z_{load}(f)$$

- **Such fixed or "do not move" points are created by adding constraints to the reactance and susceptance tables that specify an L network's branches**

Frequency	*B* mS	*X* ohms
3.9	-5.5735	-378.57
7.2	0	0
14.2	-10.375	-148.84
21.2	0	0
29.0	2.7730	360.65

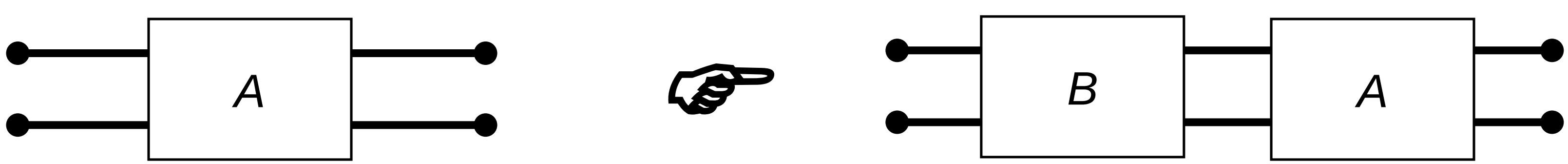
⇨ Match network will match 3.9, 14.2, and 29 MHz and have no effect on the load impedance at 7.2 and 21.2 MHz

Application 1: "Add-A-Band" Match Networks

- **Given a multi-frequency match network A, new frequencies can be added to the set of match frequencies by a second match network B in cascade**

Frequency	*B* mS	*X* ohms
3.9	-5.5735	-378.57
7.2	3.5142	373.86
14.2	-10.375	-148.84

Frequency	*B* mS	*X* ohms
3.9	0	0
7.2	0	0
14.2	0	0
21.2	TBD	TBD
29.0	TBD	TBD



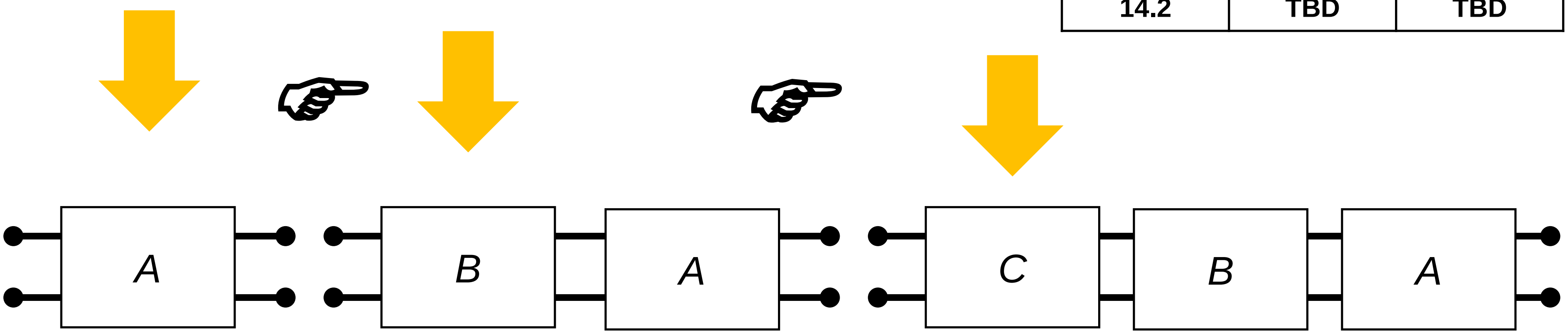
Application 2: "Iterative" Match Network Synthesis

- **Frequencies are added to the set of match frequencies one at a time, each by adding a network to a cascade chain**

Frequency	*B* mS	*X* ohms
3.9	-5.5735	-378.57

Frequency	*B* mS	*X* ohms
3.9	0	0
7.2	TBD	TBD

Frequency	*B* mS	*X* ohms
3.9	0	0
7.2	0	0
14.2	TBD	TBD



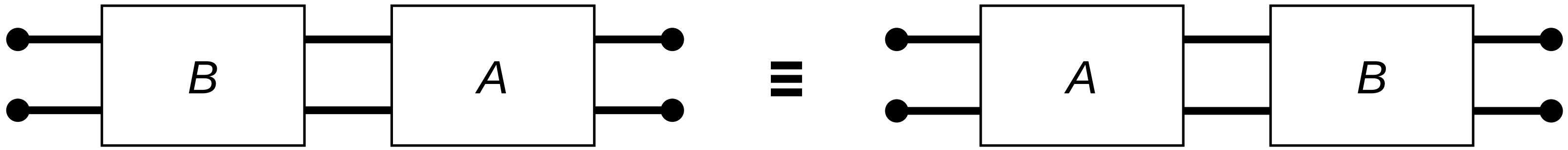
Application 3: Commutative Match Networks

- **Two networks are bi-complementary if the match frequencies of each are the fixed points of the other**

Frequency	*B* mS	*X* ohms
3.9	-5.5735	-378.57
7.2	0	0
14.2	-10.375	-148.84
21.2	0	0
29.0	2.7730	360.65

Frequency	*B* mS	*X* ohms
3.9	0	0
7.2	3.5142	373.86
14.2	0	0
21.2	-2.9744	-425.20
29.0	0	0

- **Bi-complementary networks are commutative; frequency match does not depend on the order in which the networks are cascaded**



$$\forall k = 1, \cdots, n: B(A(Z_L(f_k))) = A(B(Z_L(f_k)))$$

Application 4: Generalized Commutative Match Networks

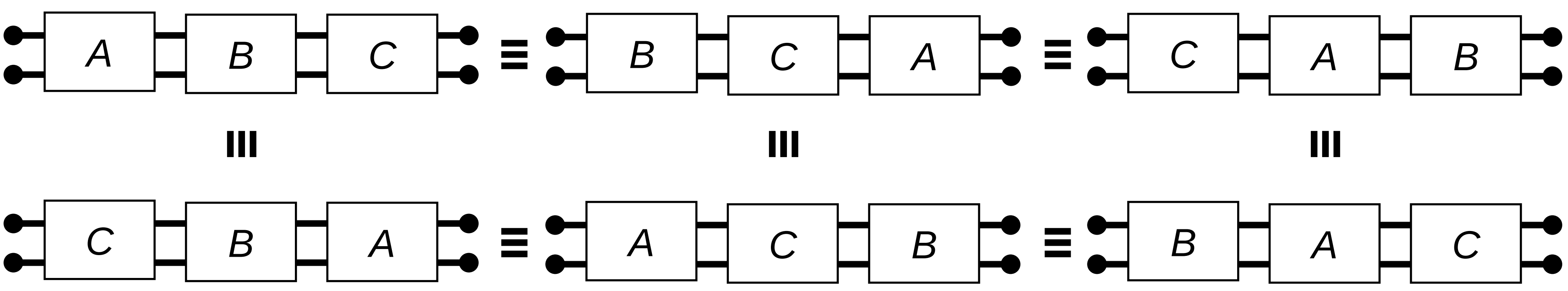
- **A collection of match networks is complementary if the match frequencies of every network are fixed points of the others**

Freq	*B* mS	*X* ohms
3.9	0	0
7.2	0	0
14.2	-10.375	-148.84
21.2	0	0
29.0	2.7730	360.65

Freq	*B* mS	*X* ohms
3.9	0	0
7.2	3.5142	373.86
14.2	0	0
21.2	0	0
29.0	0	0

Freq	*B* mS	*X* ohms
3.9	-5.5735	-378.57
7.2	0	0
14.2	0	0
21.2	-2.9744	-425.20
29.0	0	0

- **For complementary networks, frequency match does not depend on the order in which individual networks are cascaded**



Tips for Getting Maximum Match Bandwidth

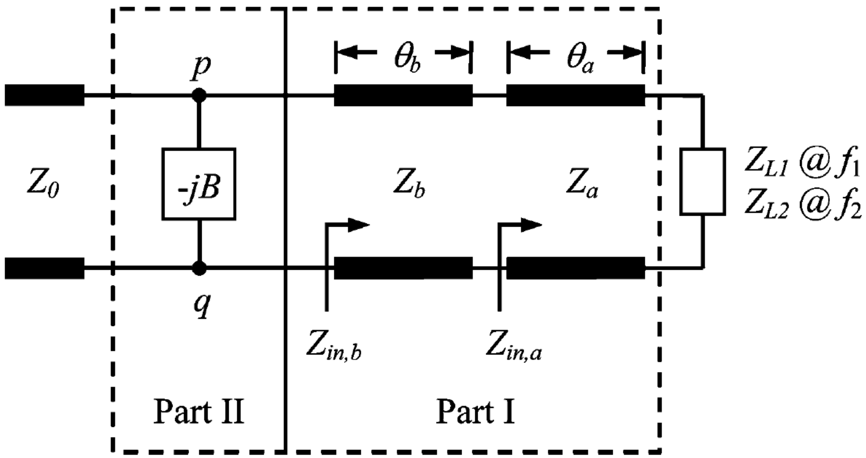
- **Put the network as close to antenna feedpoint as possible**
- **Keep match moves on the Smith chart as short and direct as possible**
- **Keep network order as low as possible; use the simplest network topology that gets the job done; keep transmission line sections short**
- **When using iterative synthesis, match the frequencies in order of importance, i.e. in order of decreasing match bandwidth**
- **When using L topologies, keep the slopes of the reactance and susceptance functions small at frequencies where bandwidth is desired**
  - Use as few resonators as possible and keep poles away from bands where match bandwidth is desired
- **An optimizer is essential when matching more than 3 frequencies but must be given a good starting point, viz. network topology and part values**
- **Be aware of the Fano bound: A single-frequency match network will have greater match bandwidth in a band than a multi-frequency match network can have**

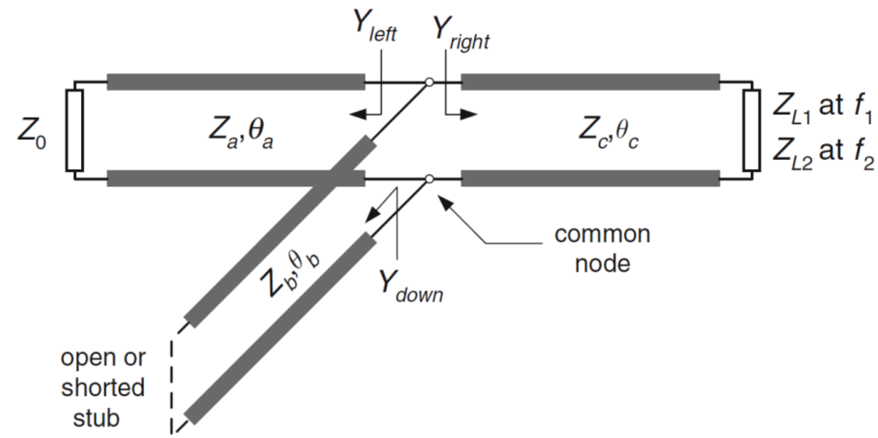
Multi-frequency and Multi-band Matching Using Line Sections and Stubs

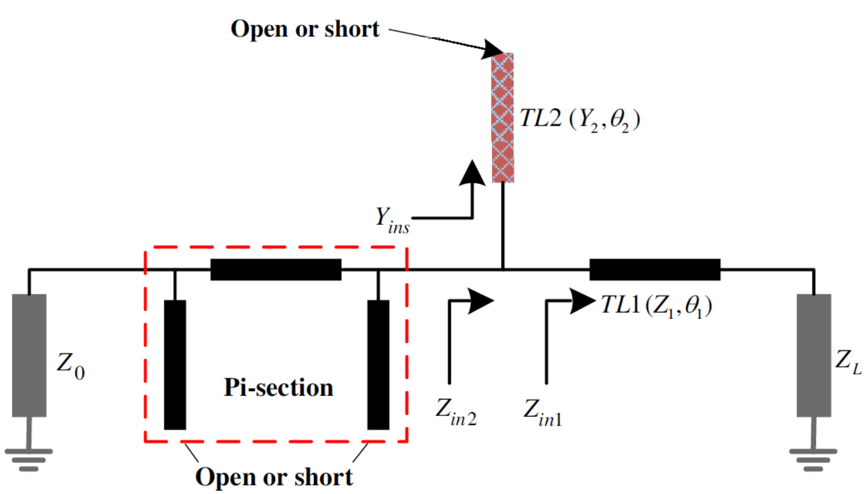
Brief history

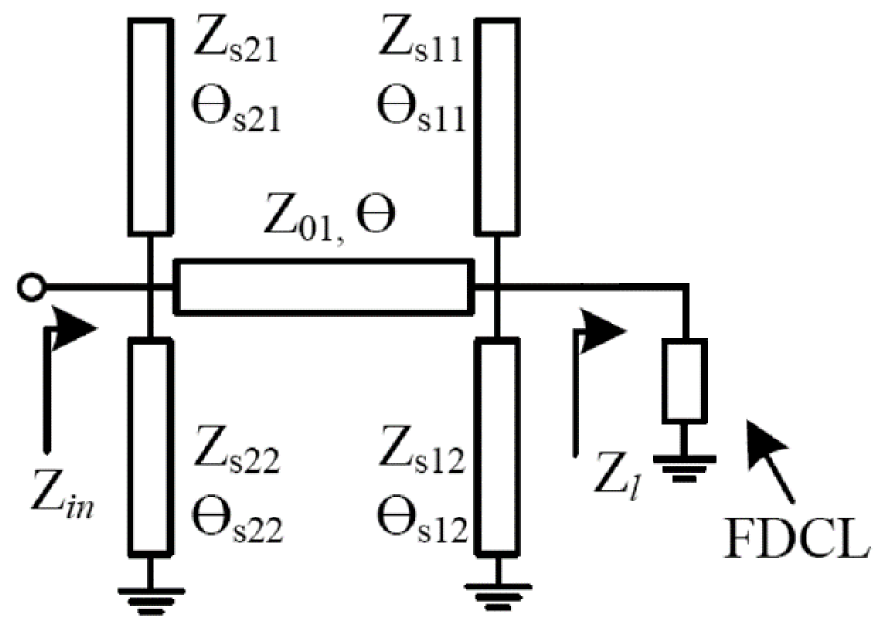
- **Early phase**
  - Cascaded quarter wave transformers matching real impedance
  - Chebyschev broadband quarter wave transformers
- **Middle phase**
  - Bramham's alternated-line, 12^{th} wave or $1/6^{th}$ wave transformers
  - Short-section transformers for matching a real impedance
  - Networks of line sections and stubs for matching at harmonically related frequencies a real impedance
  - Networks of line sections and stubs for matching at multiple comensurate frequencies multiple real impedances
- **Current phase**
  - Networks of line sections and stubs for matching at multiple arbitrary frequencies a complex frequency dependent load impedance

Topologies for General Dual and Multi-Band Matching

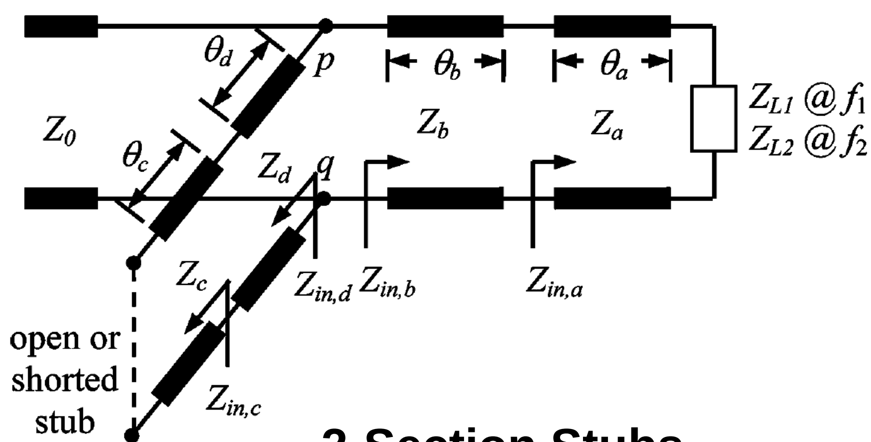




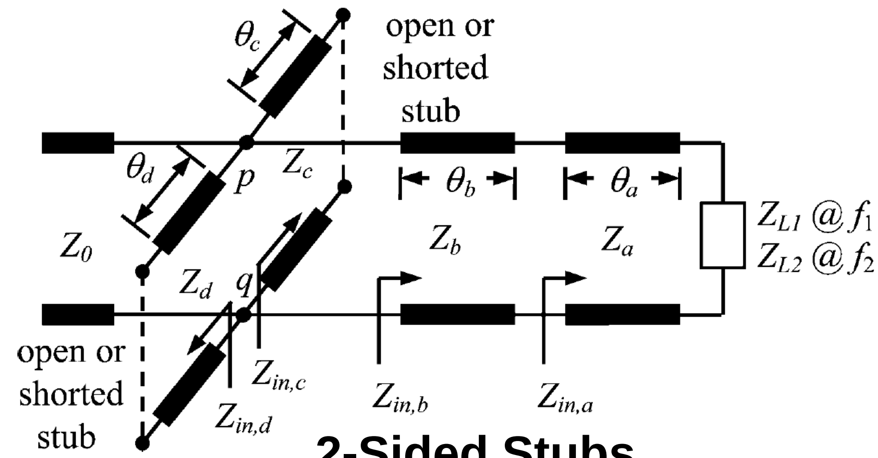




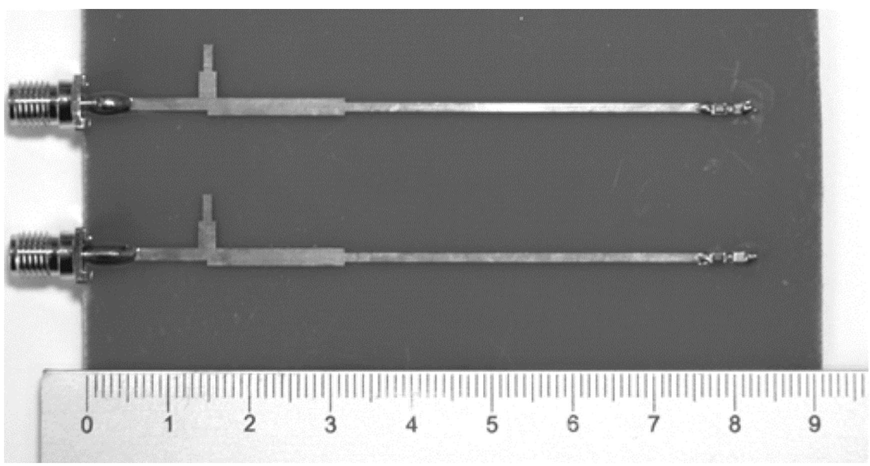
Two Realizations of Shunt *jB* and Layout Examples

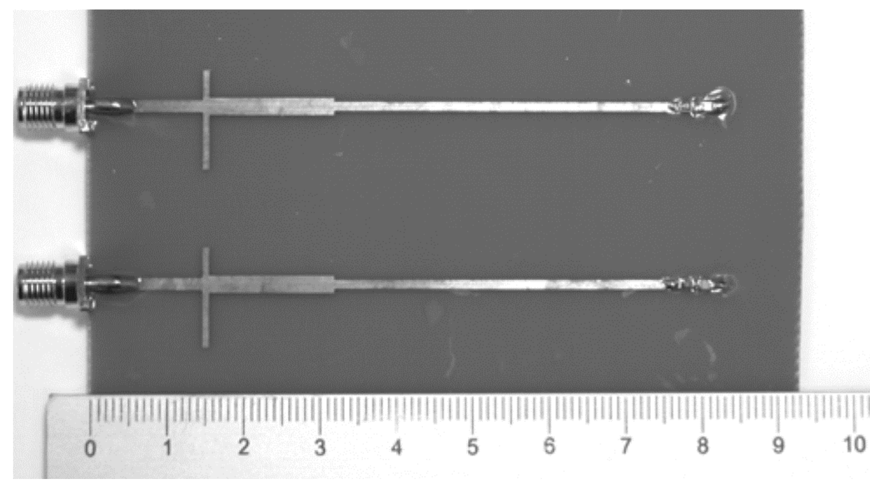


2-Section Stubs

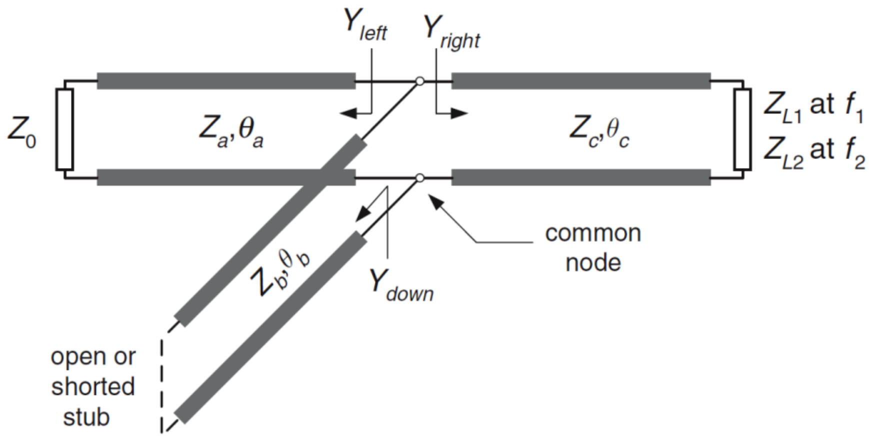


2-Sided Stubs

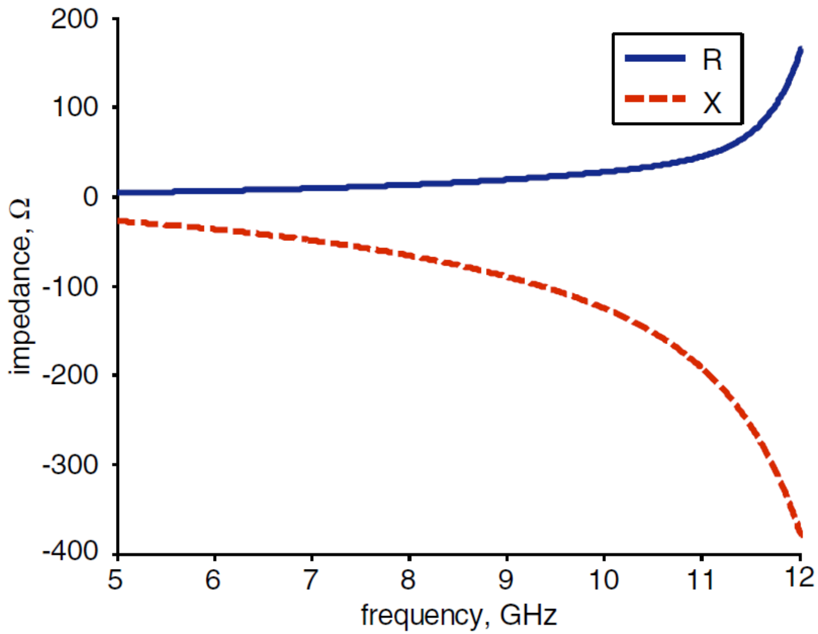


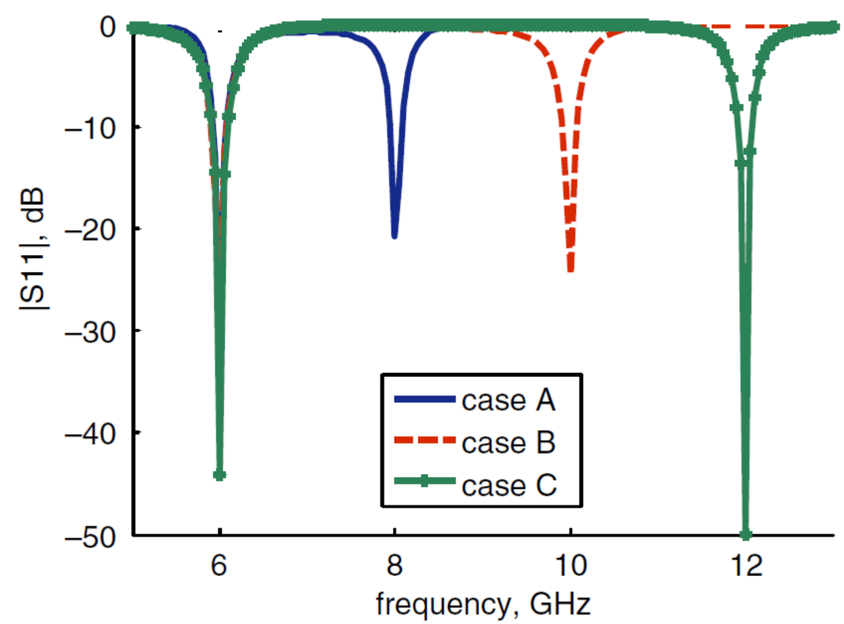


T-Topology for Dual Band Matching – 3 Examples

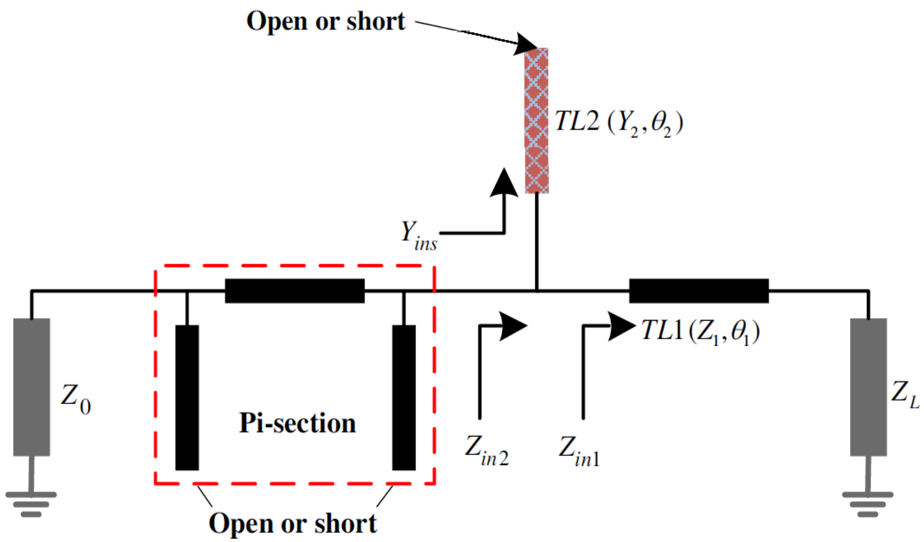


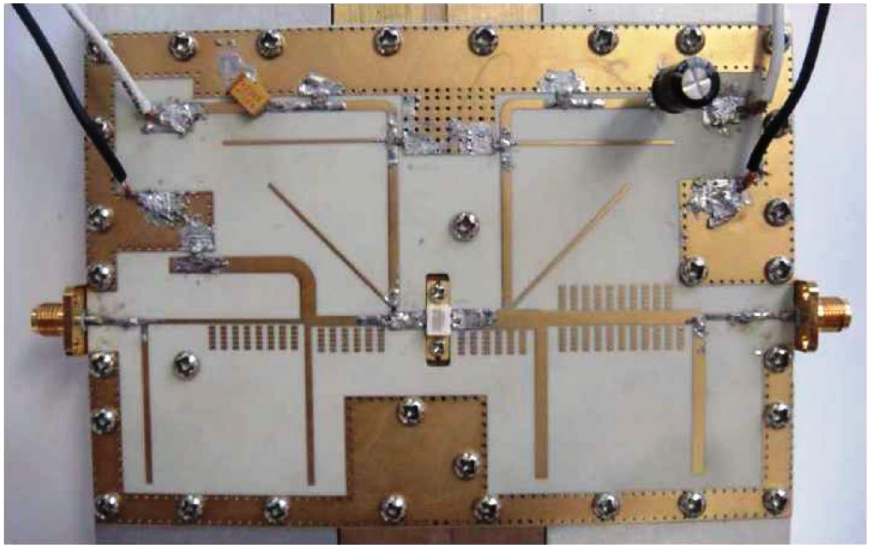
Case	f_1, GHz	f_2, GHz	Z_a, Ω	θ_a, deg	Z_b, Ω	θ_b, deg	Z_c, Ω	θ_c, deg	Stub type
A	6	8	68	77	10	154	43	196.6	Open
B	6	10	134	67.5	23	67.5	55	173	Short
C	6	12	34	60	27	60	71	216	Open

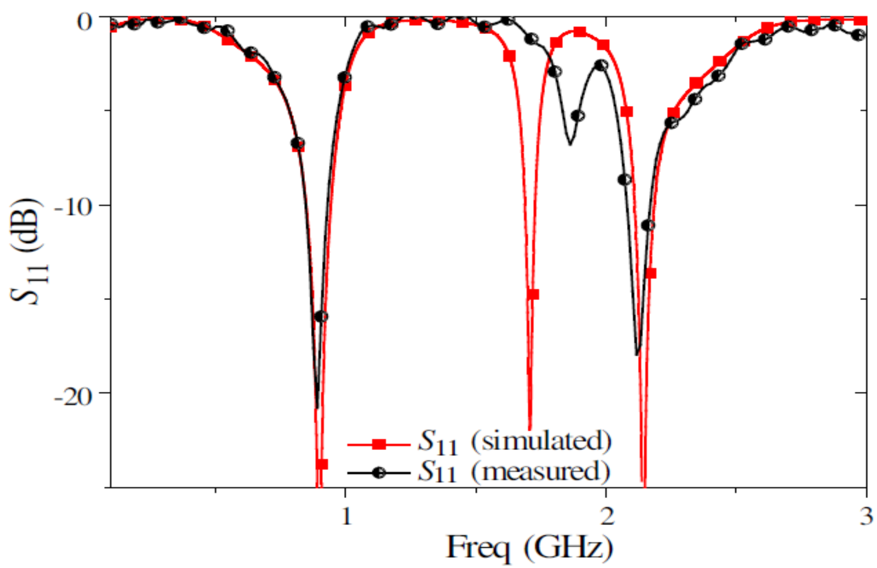


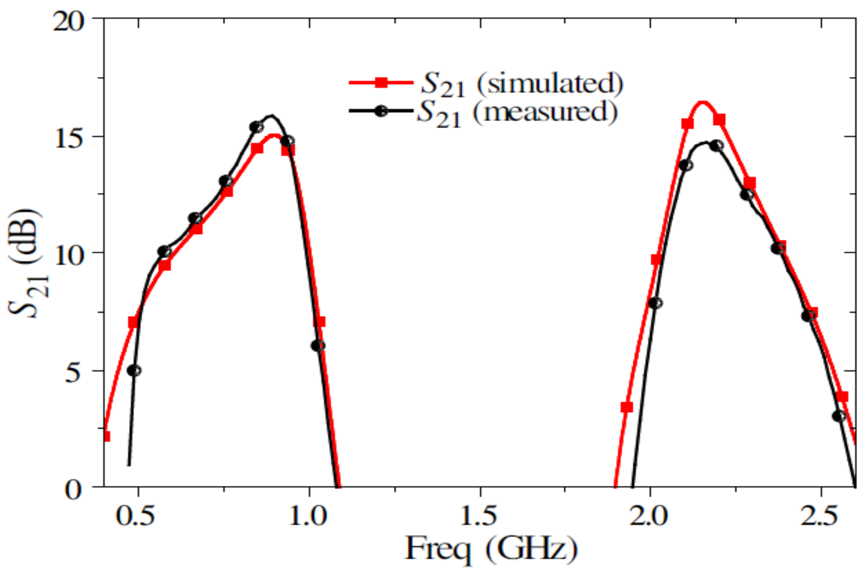


Pi-Topology for Dual-Band Matching – Amplifier

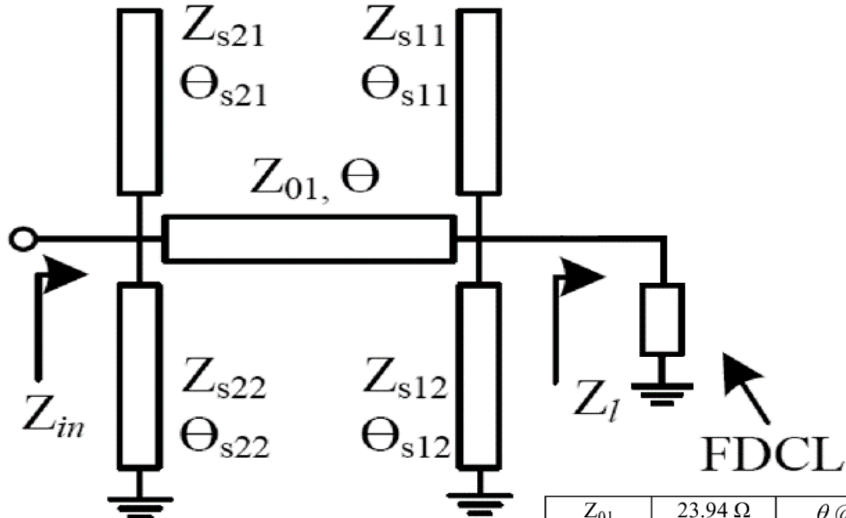


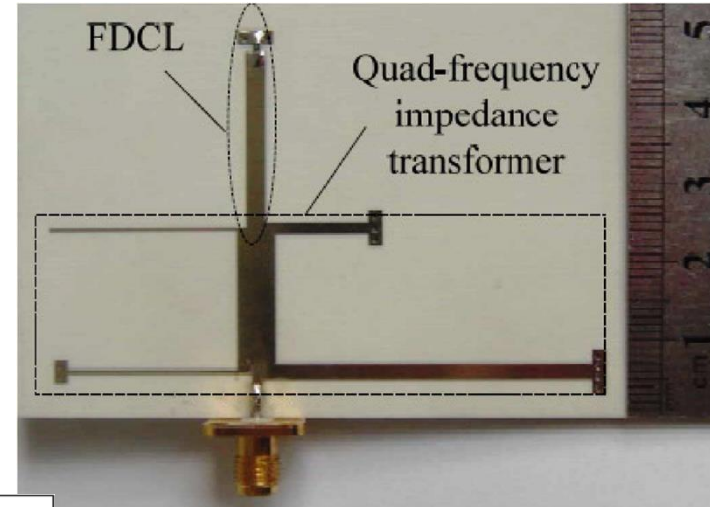




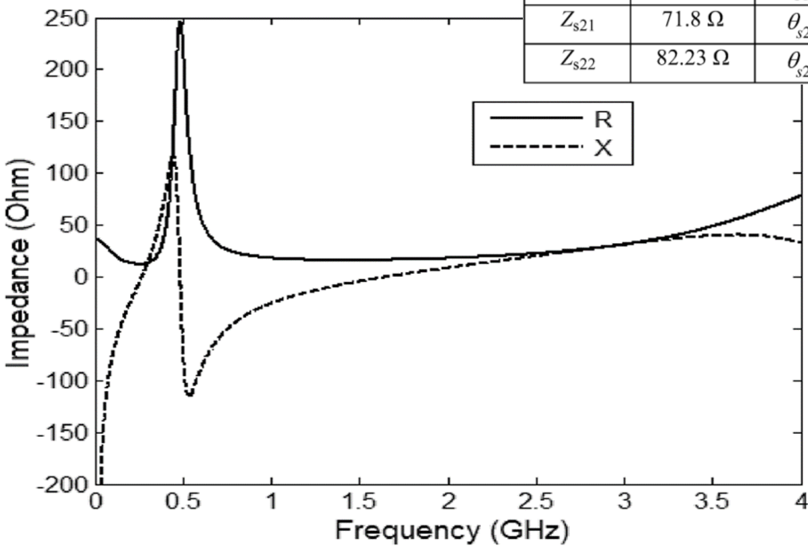


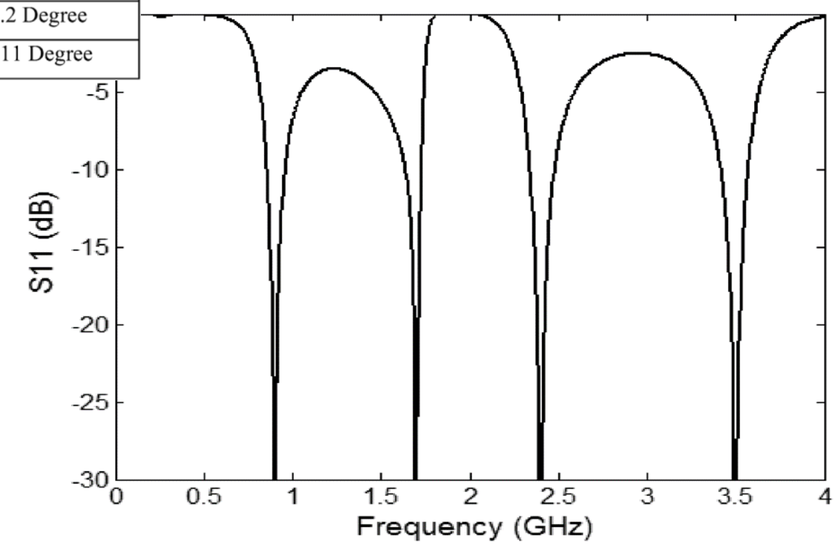
A Topology for General Quad and Multi-Band Matching





Z_{01}	23.94 Ω	θ @f_4	160 Degree
Z_{s11}	64.51 Ω	θ_{s11} @f_1	39.1 Degree
Z_{s12}	49.62 Ω	θ_{s12} @f_1	35.27 Degree
Z_{s21}	71.8 Ω	θ_{s21} @f_1	40.2 Degree
Z_{s22}	82.23 Ω	θ_{s22} @f_1	24.11 Degree





Broadband Continuous-Frequency Matching

Case Study No. 1

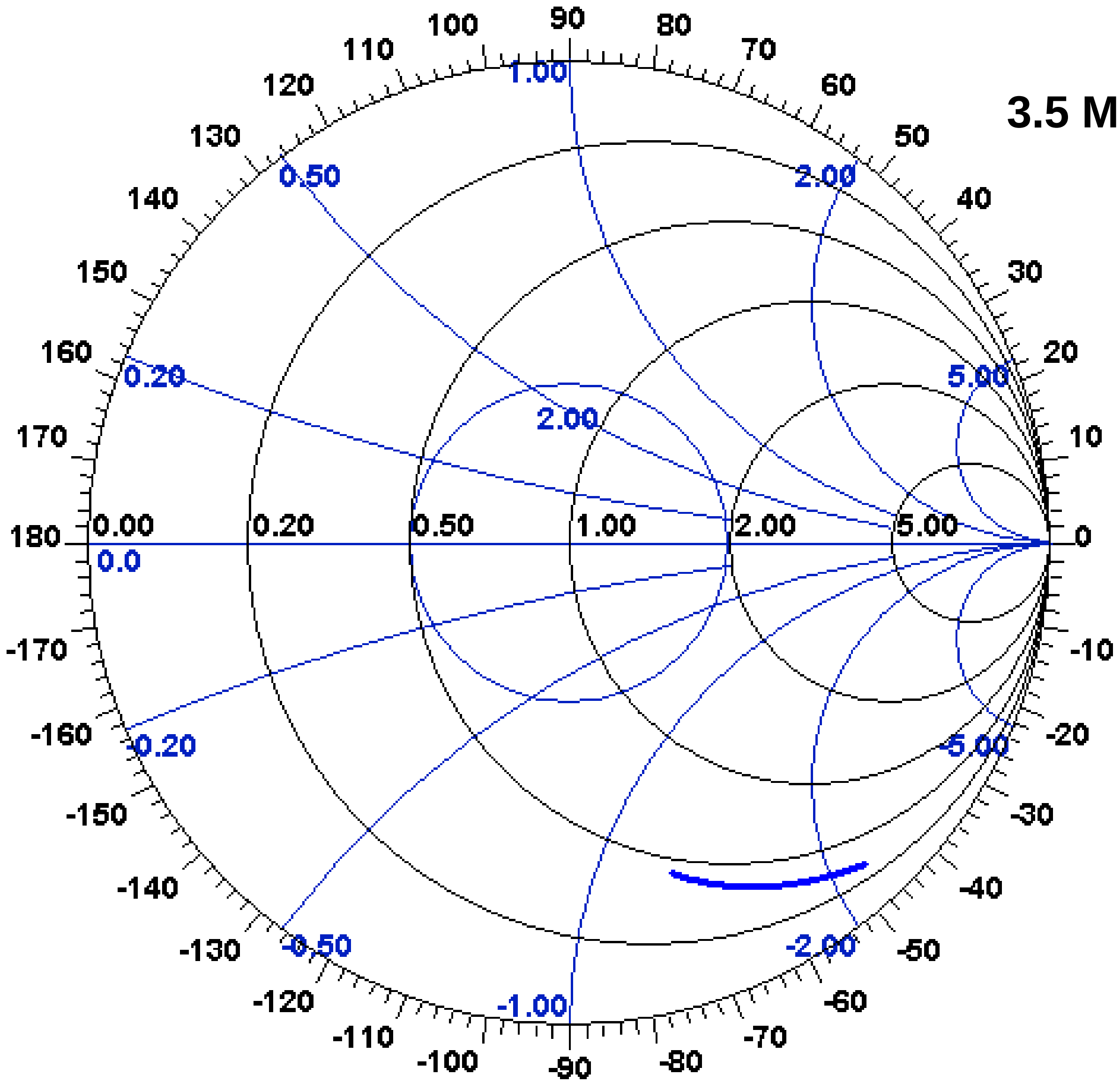
How Not to Do Broadband Matching

Object Lesson: How Not To Do It !
As published in *QEX* Sep/Oct 2001

- Illustrates pitfalls of attempting wideband matching at fewer discrete frequencies than the order of the network
- Antenna was an electrically-short vertical monopole
- Used only three data points between 3.5 MHz and 4 MHz

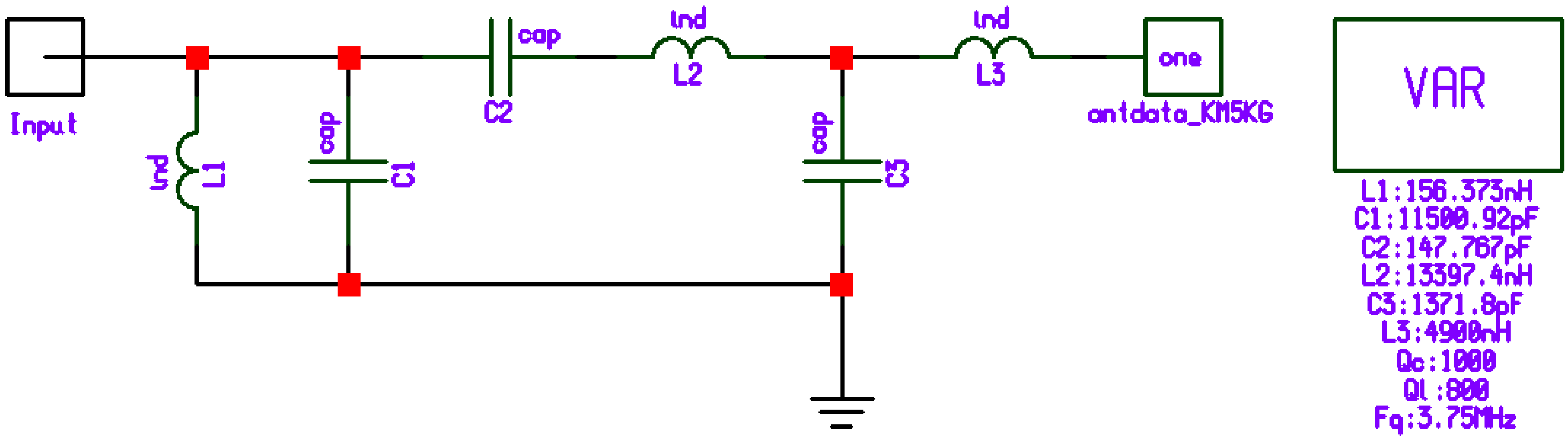
Frequency (MHz)	Real (ohms)	Imaginary (ohms)
3.500	15.9	-112.0
3.725	18.7	-89.1
3.750	19.0	-86.6
4.000	22.6	-62.6

Impedance of KM5KG Vertical Monopole

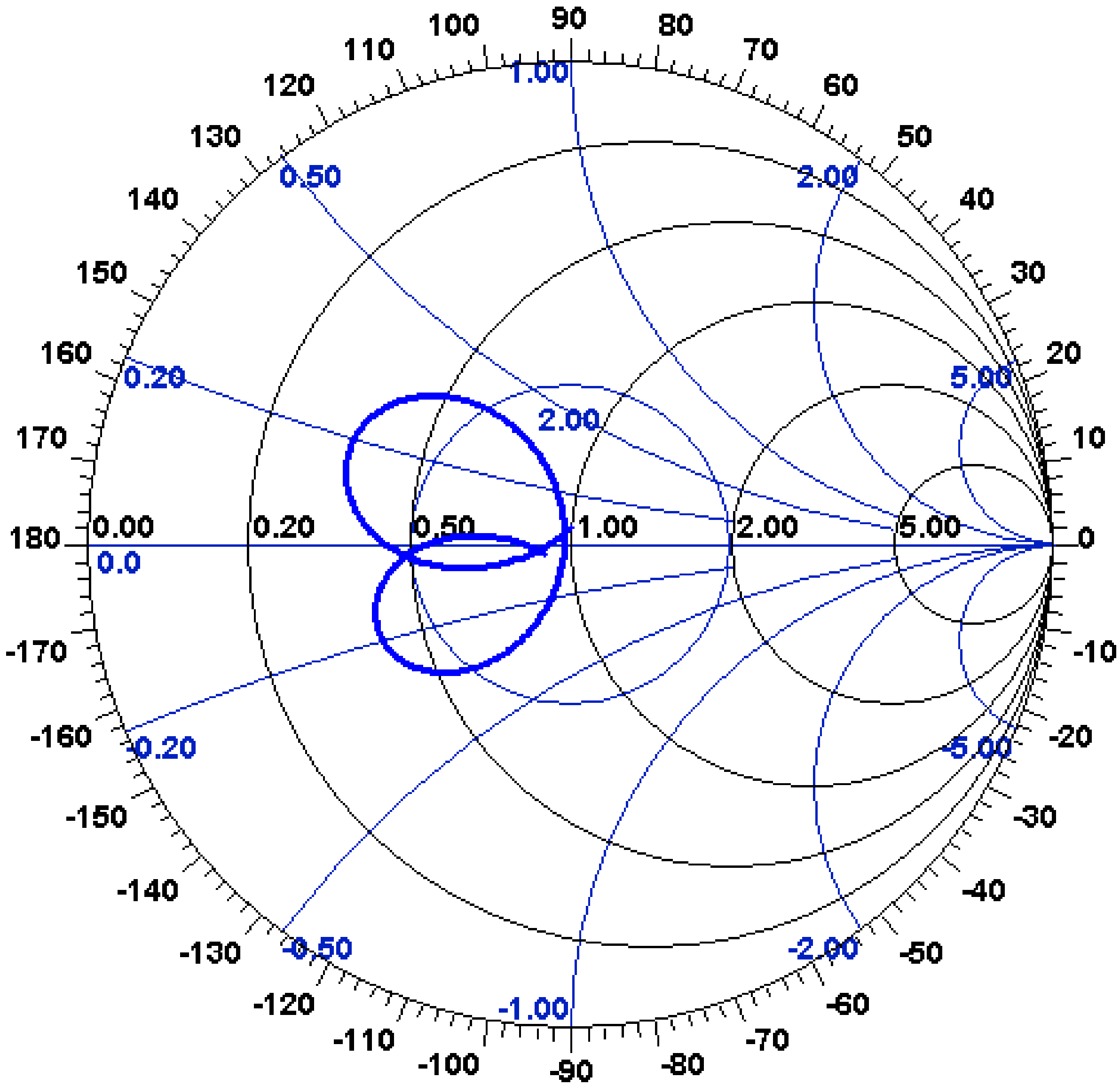


Published Six-Element Matching Network

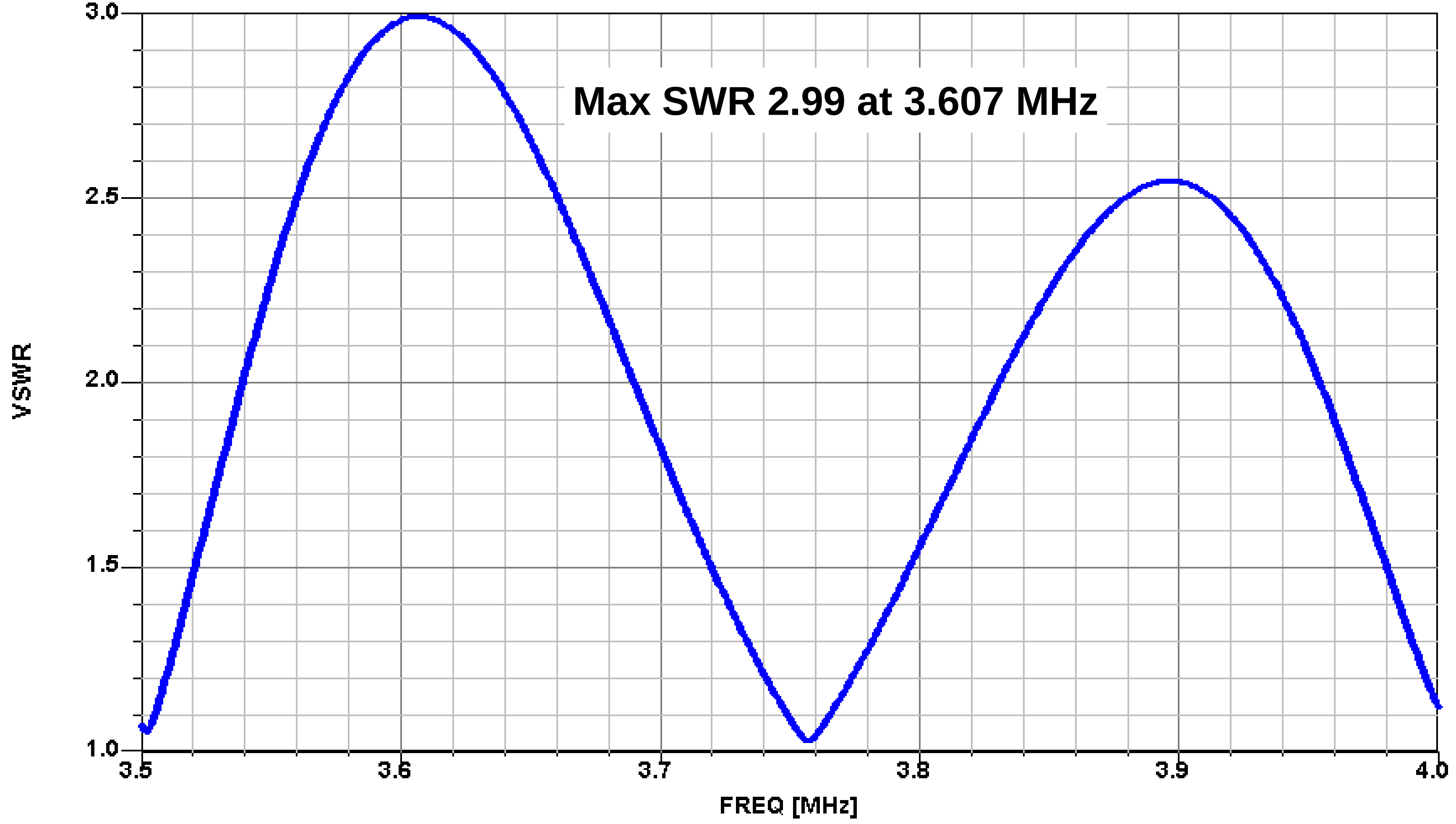
- **Component losses included**
  - Capacitors: Q_C = 1,000
  - Inductors: Q_L = 800
- **Components calculated for perfect three-frequency match at 3.5, 3.75, and 4 MHz**



Matching Result - Unpublished



Really a 3-Frequency Match, Not a Broadband Match

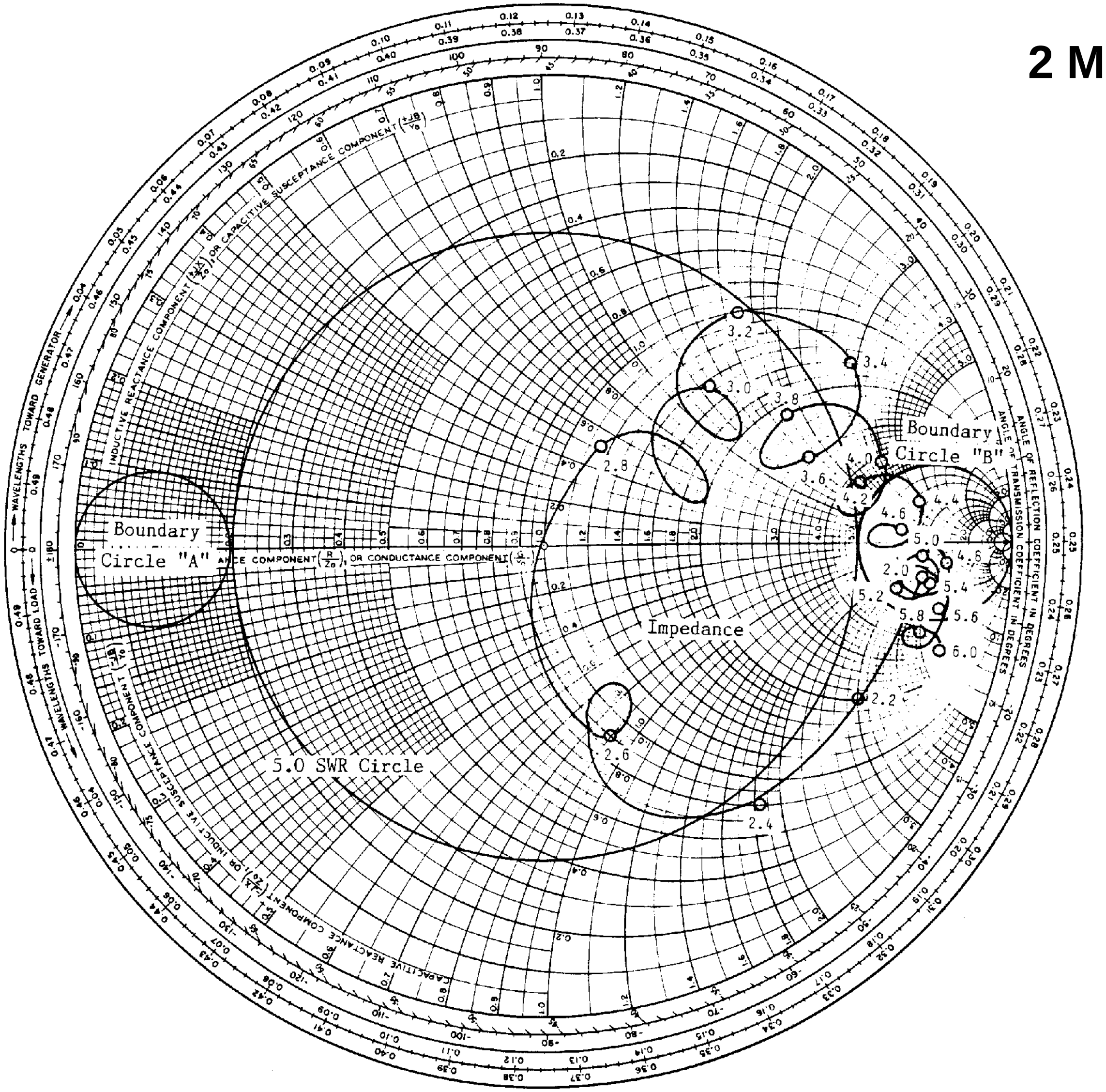


Case Study No. 2

Wilfred Caron’s Infamous Example 11

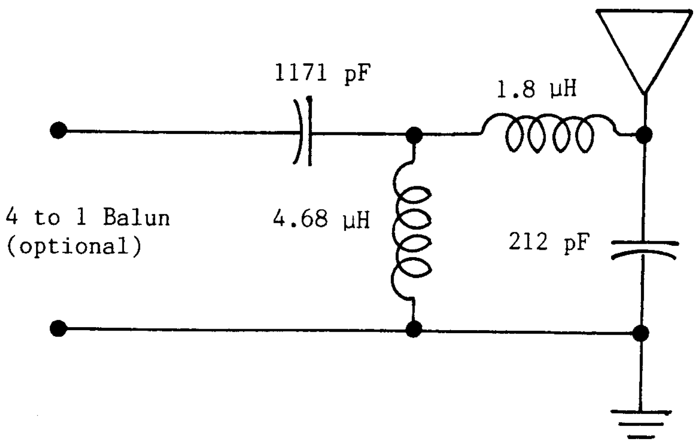
Caron's Example 11: Long-Wire Receiving Antenna

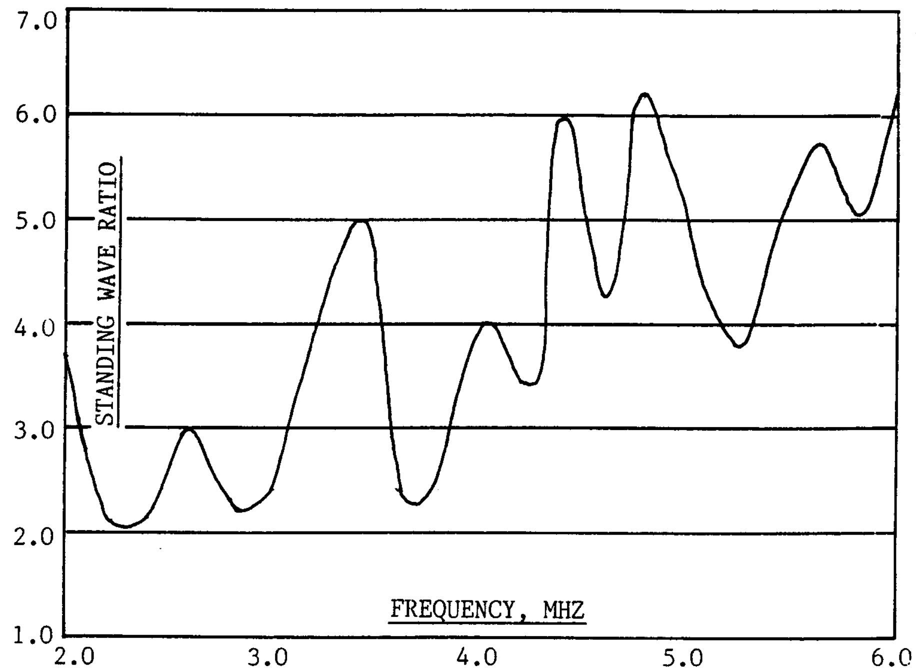
2 MHz to 6 MHz



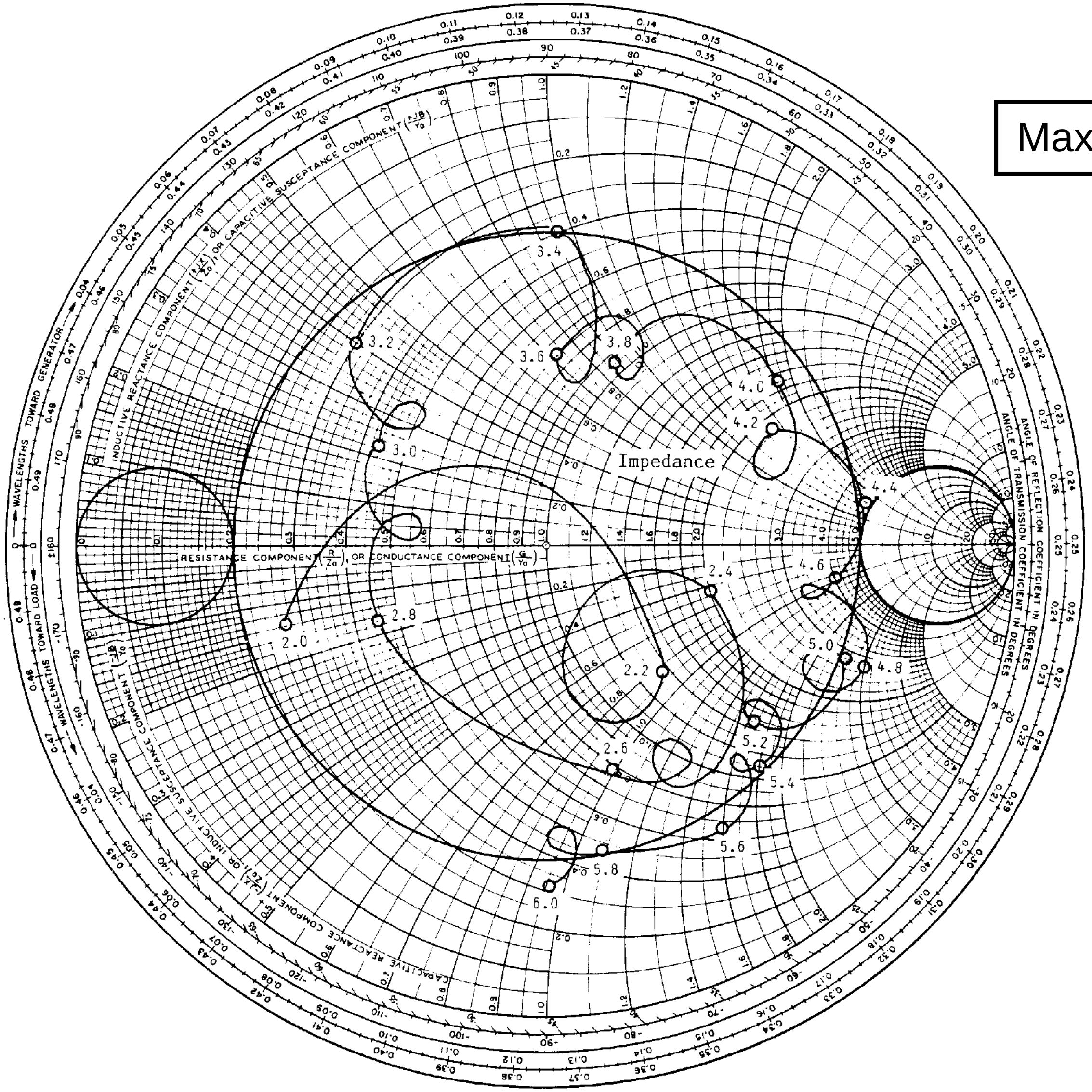
Caron's Solution

Max SWR=6.2



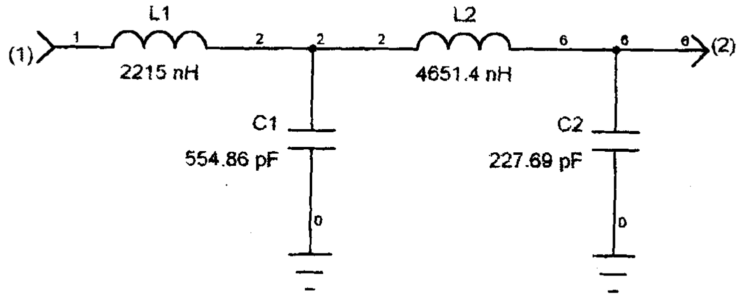


Caron's Solution

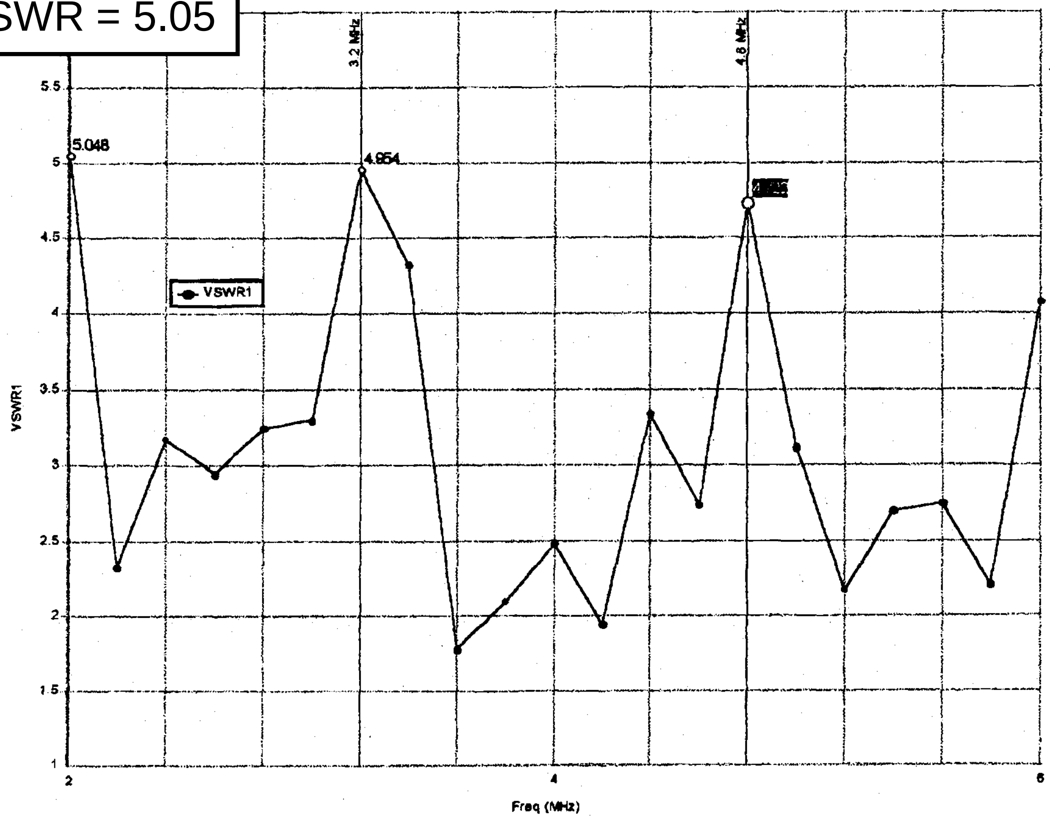


Max SWR = 6.2

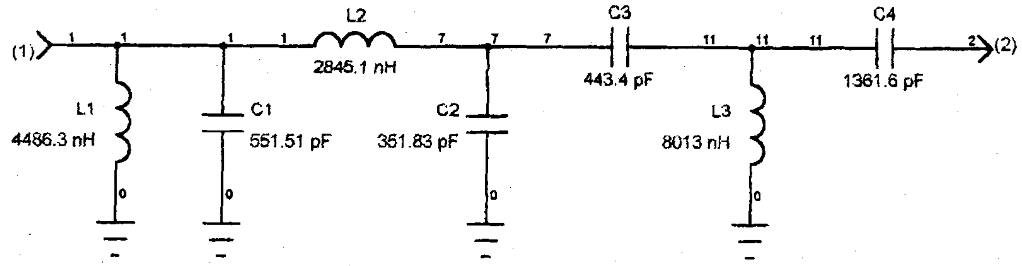
Eagleware's Solution 1

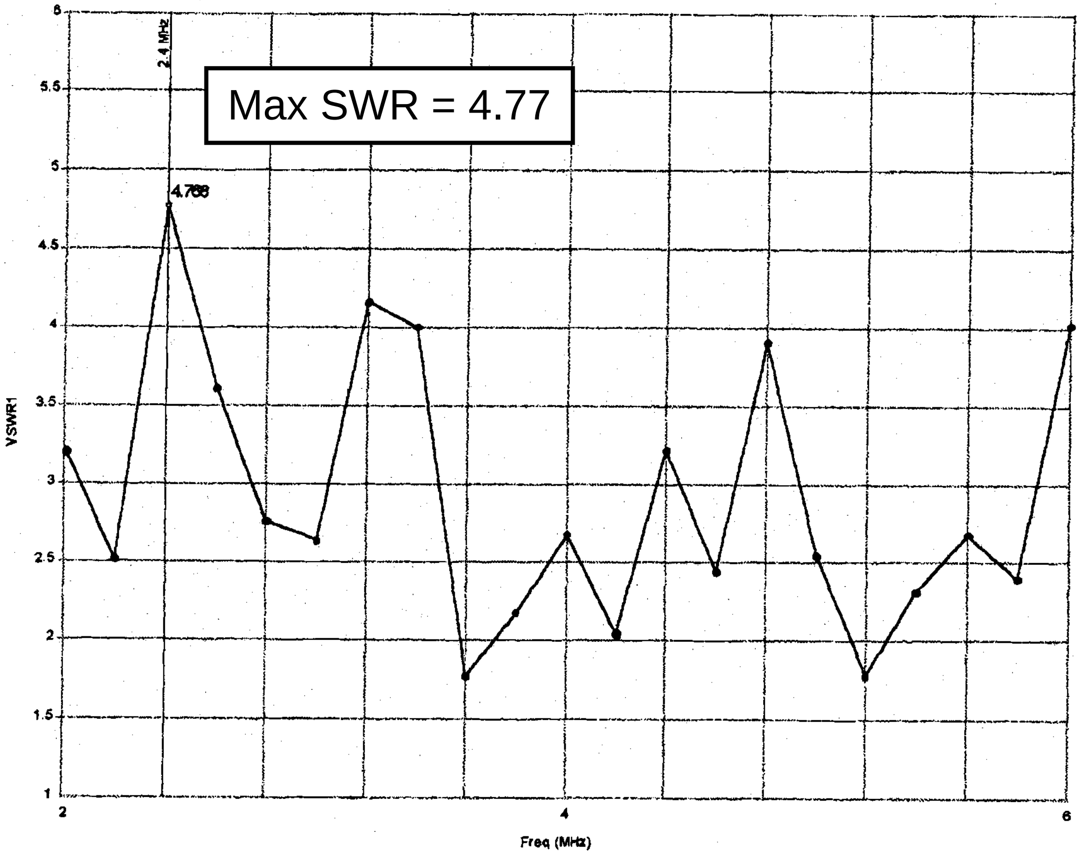


Max SWR = 5.05

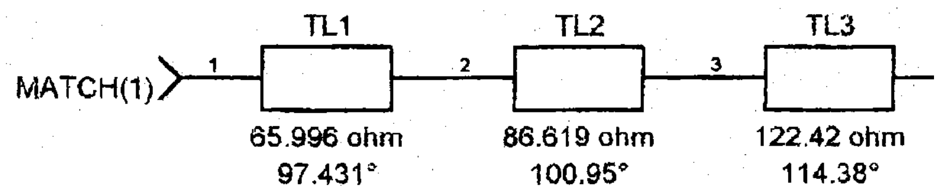


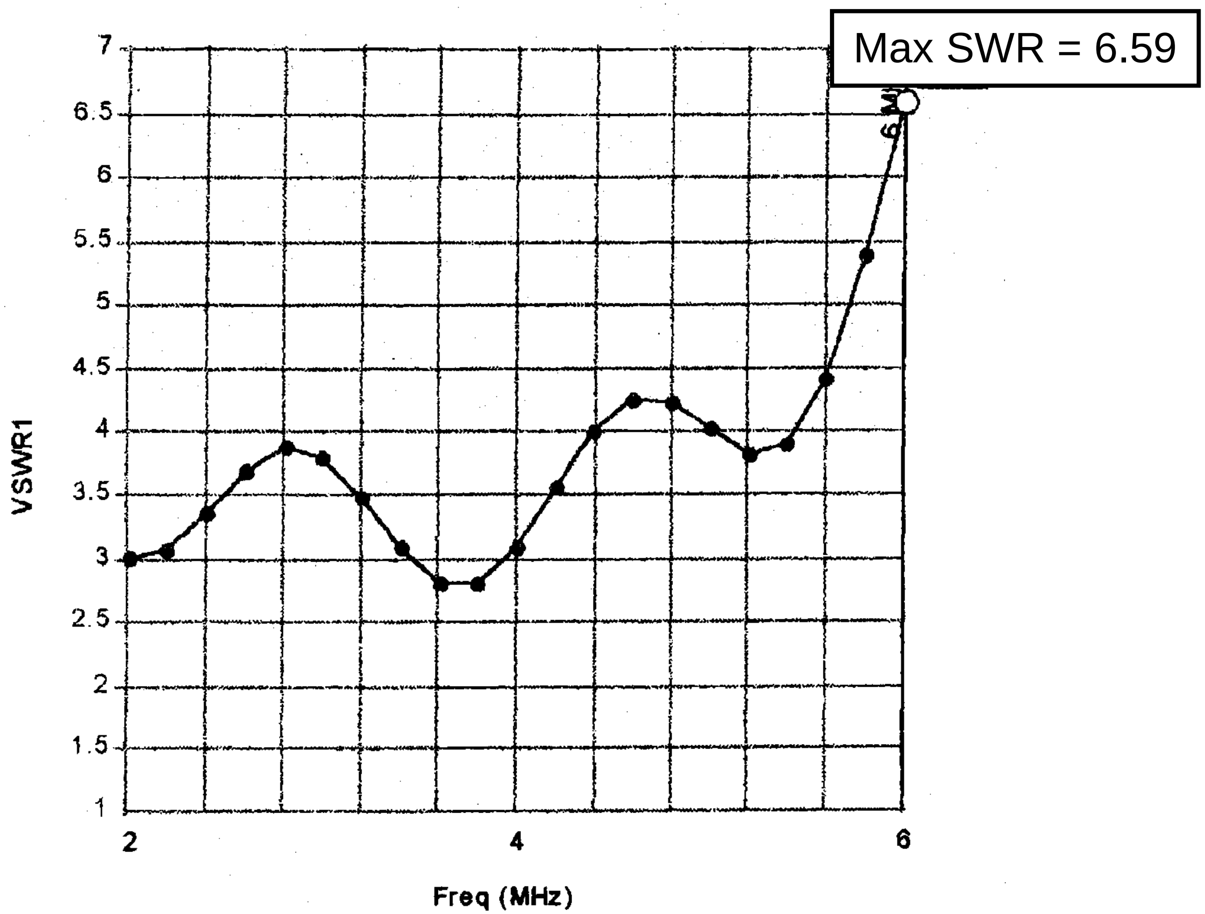
Eagleware's Solution 2



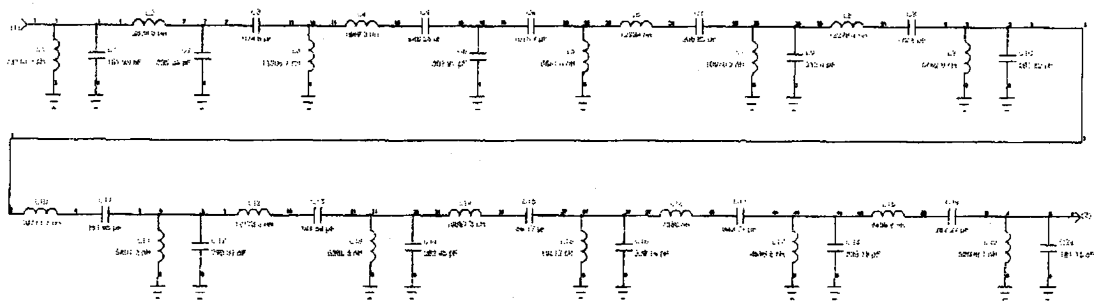


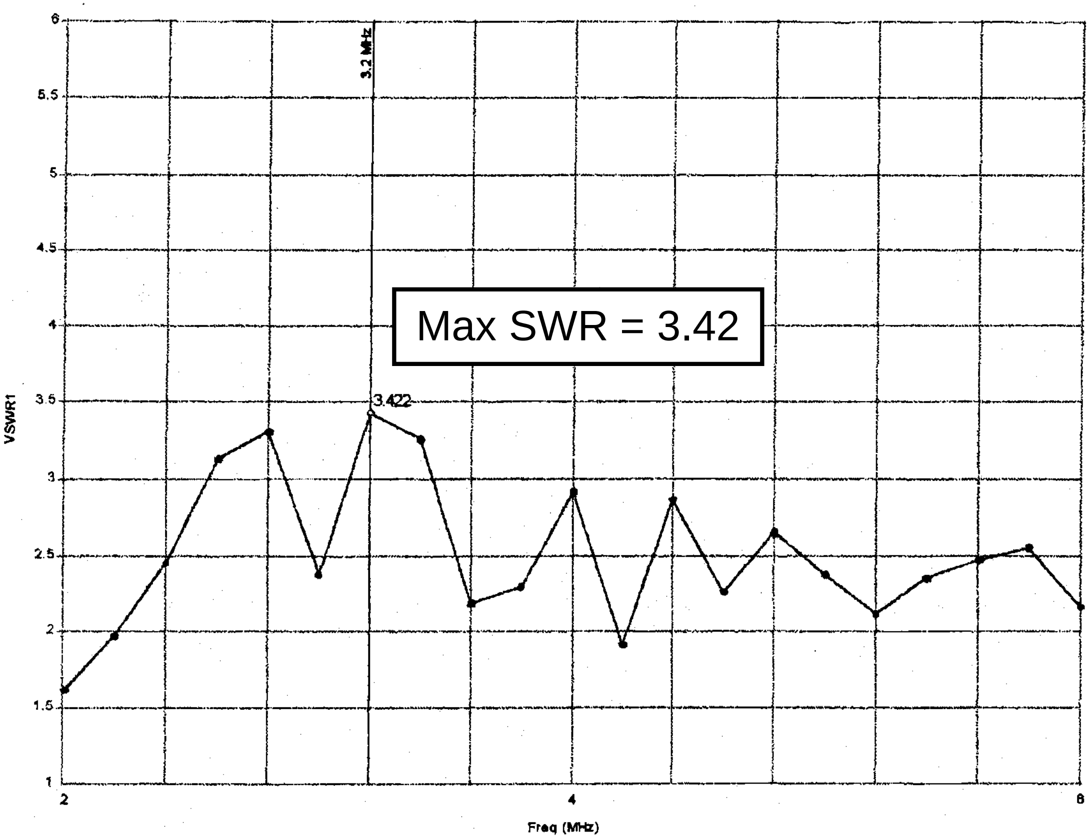
Eagleware's Solution 3: Cascaded Transmission Lines





Eagleware's Solution 4: Fano Limit





Fano Bound on Match Bandwidth

Robert Mario Fano, 1917-2016

R.M. Fano's Bound (1947)

- **Proved in Fano's Ph.D. dissertation at MIT in 1947**
- **Published in summary form in the *Journal of the Franklin Institute*, 1950**
- **Applies to passive lossless impedance-matching networks**
- **Limits how well an arbitrary impedance can be matched by a passive lossless network of any complexity – even infinite**
- **Bounds the return loss bandwidth product**
  - Bounds the achievable SWR for a given bandwidth
  - Bounds the achievable match bandwidth for a given SWR

The Fano bound is fundamental, but it can be bypassed by an ingenious designer.

Fano's Bound (1947)

- **Bounds the area under the return loss curve of all lossless impedance-matching networks**

$$\int_0^\infty \log\left(\frac{1}{\rho(\omega)}\right) d\omega \le \min\{A_1, A_2, \cdots, A_n\}$$

where

$$\rho(\omega) = |\Gamma(\omega)| = |s_{11}(\omega)|$$

and A_1, ... , A_n are constants that depend on the load impedance function $Z_L(f)$

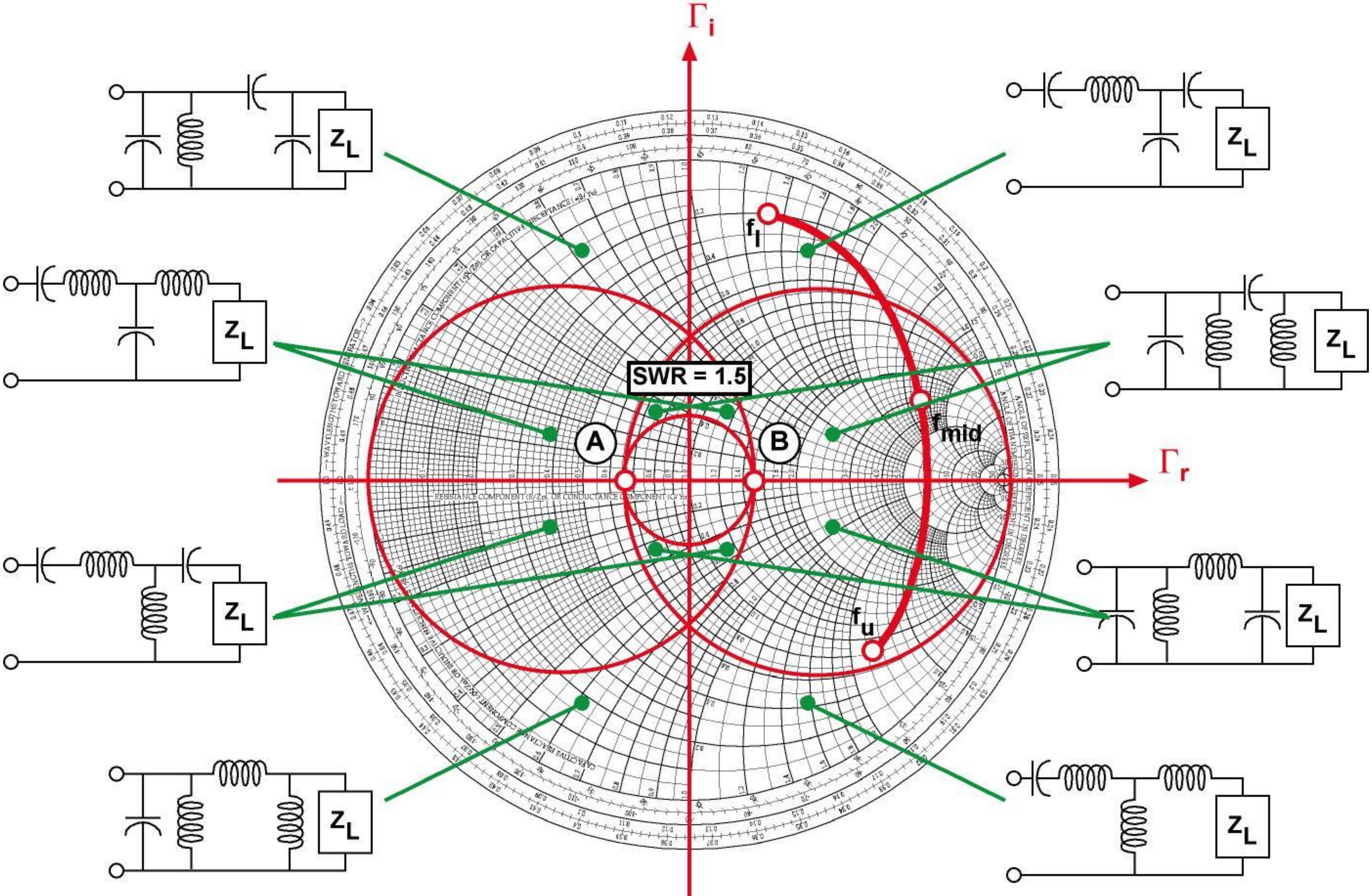
Broadband Matching Made Easy

4-element matching networks

Broadband Matching Made Easy

- **π-resonant and T-resonant network topologies**
  - Four reactive elements: an L plus a resonant circuit
  - Not to be confused with two cacaded L networks (e.g. π-L and T-L networks)
- **A simple graphical design procedure**
  - Step 1: Move mid-band impedance point to points A or B with proper orientation (curving around origin) with an L network
    - To get maximum match bandwidth, choose the minimum reactance or susceptance L-network
  - Step 2: Add a series or parallel resonant circuit to network input
    - If the curve passes through point A on left, add a shunt LC
    - If the curve passes through point B on right, add series LC
  - Step 3: Adjust *L* and *C* to wrap the impedance curve into the SWR circle

π-Resonant and T-Resonant Networks For Broadband Continuous-Frequency Matching



Tips on Broadband Matching

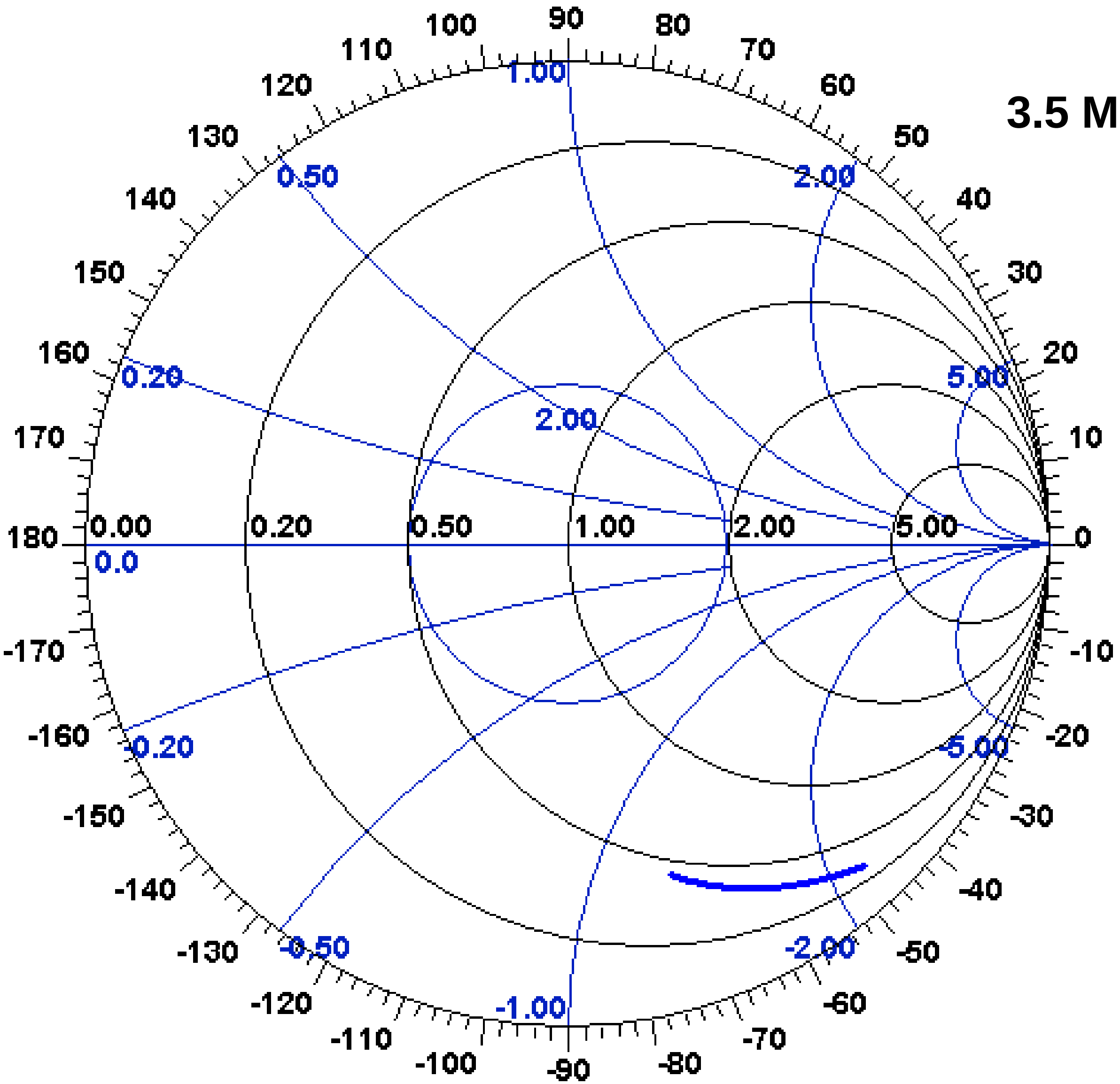
- **For maximum match bandwidth, put the matching network as close to the load (antenna feedpoint) as possible**
  - Avoid unnecessary transmission line in cascade as it stretches the curve, spreading the points to be matched farther apart
- **4-element networks give twice the match bandwidth as two-element L networks**
- **This game can be played only once. Adding four more elements will not double the match bandwidth a second time**
- **Four-element match networks achieve roughly 60% of the Fano bandwidth (at 2:1 SWR)**
- **Six-element match networks achieve roughly 75% of the Fano bandwidth (at 2:1 SWR)**

Two Examples

KM5KG monopole revisited
Inverted Half-Square Yagi

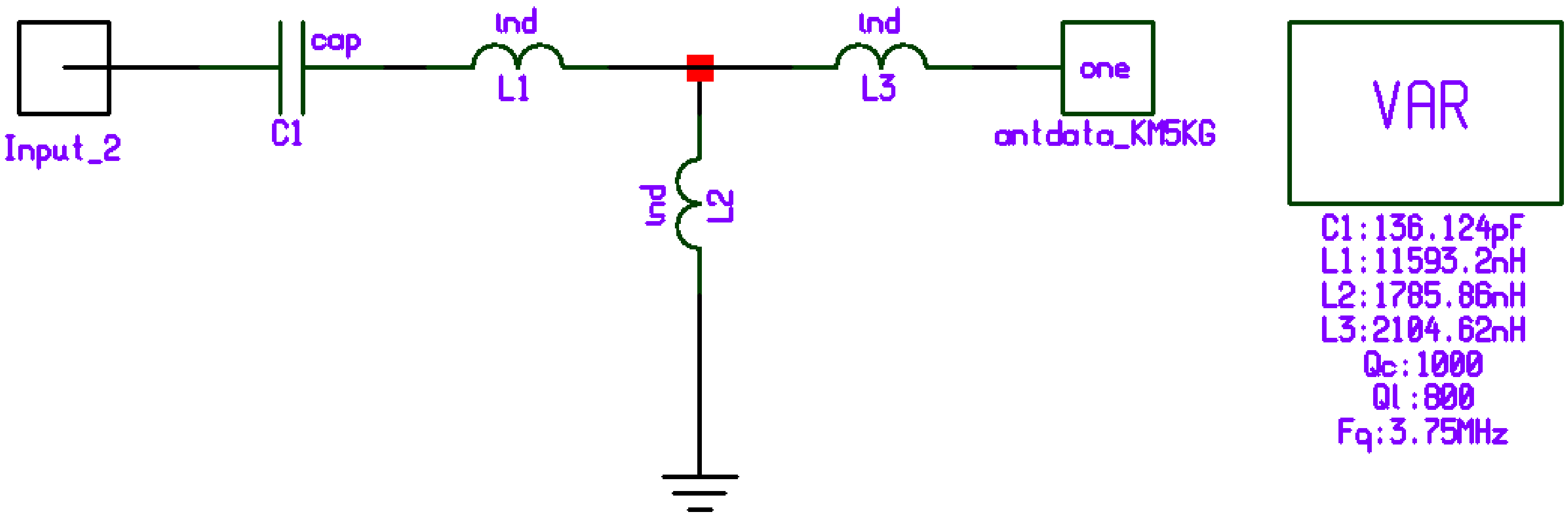
Impedance of KM5KG Vertical Monopole

3.5 MHz to 4 MHz

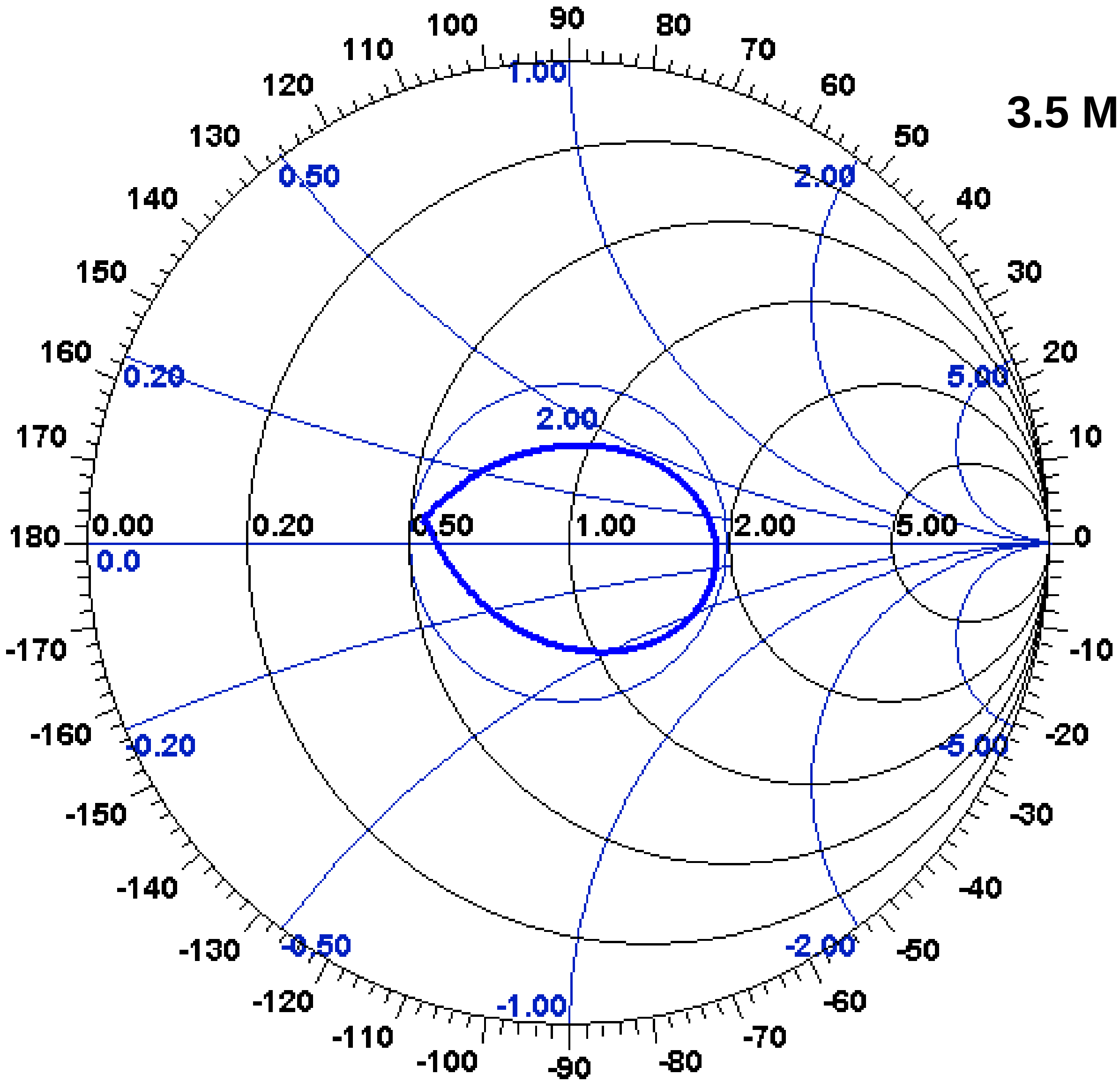


K6OIK's Simple 4-Element Network

- **Designed to minimize the maximum SWR across the band of interest**

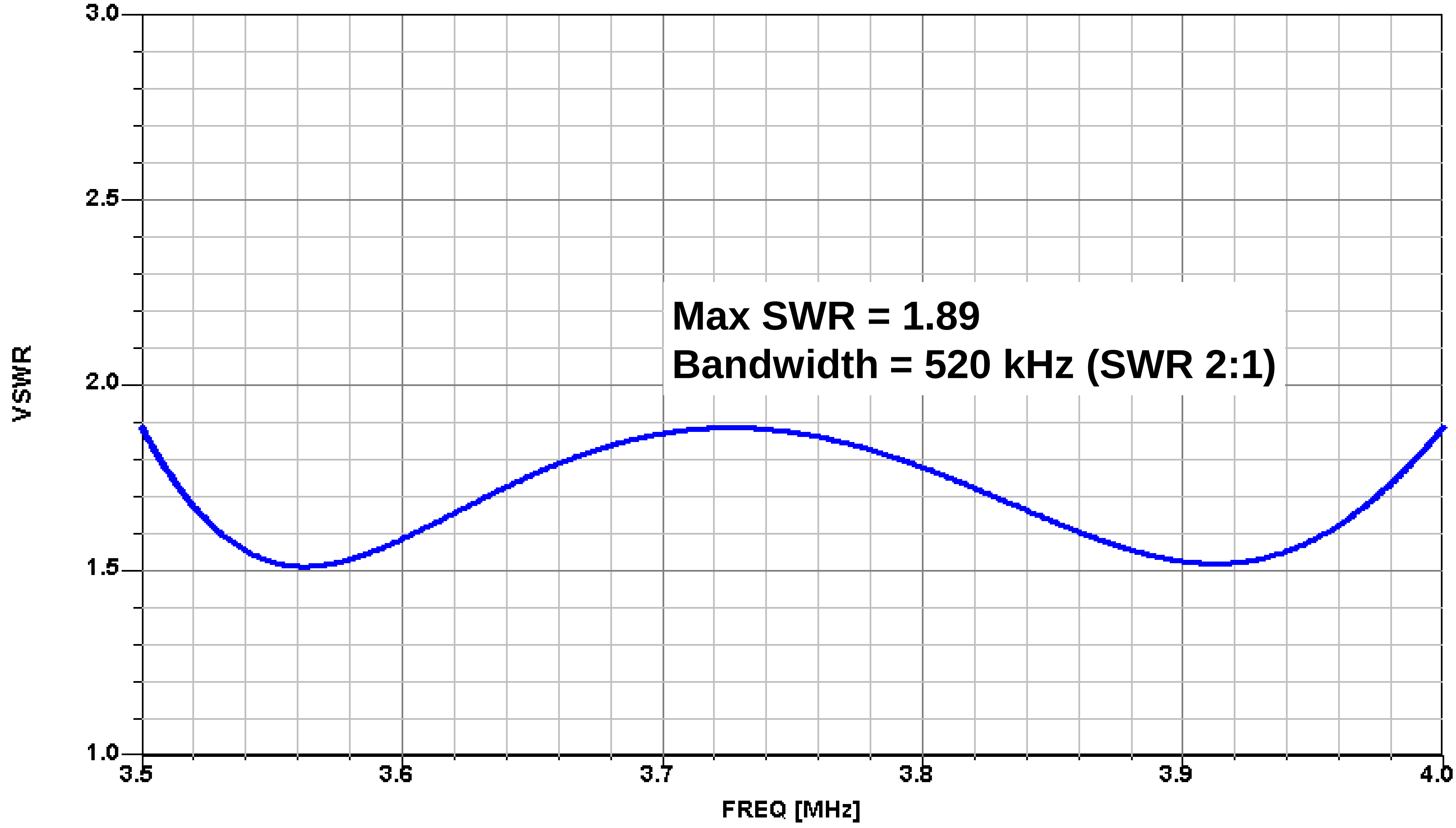


Input Impedance of 4-Element Matching Network

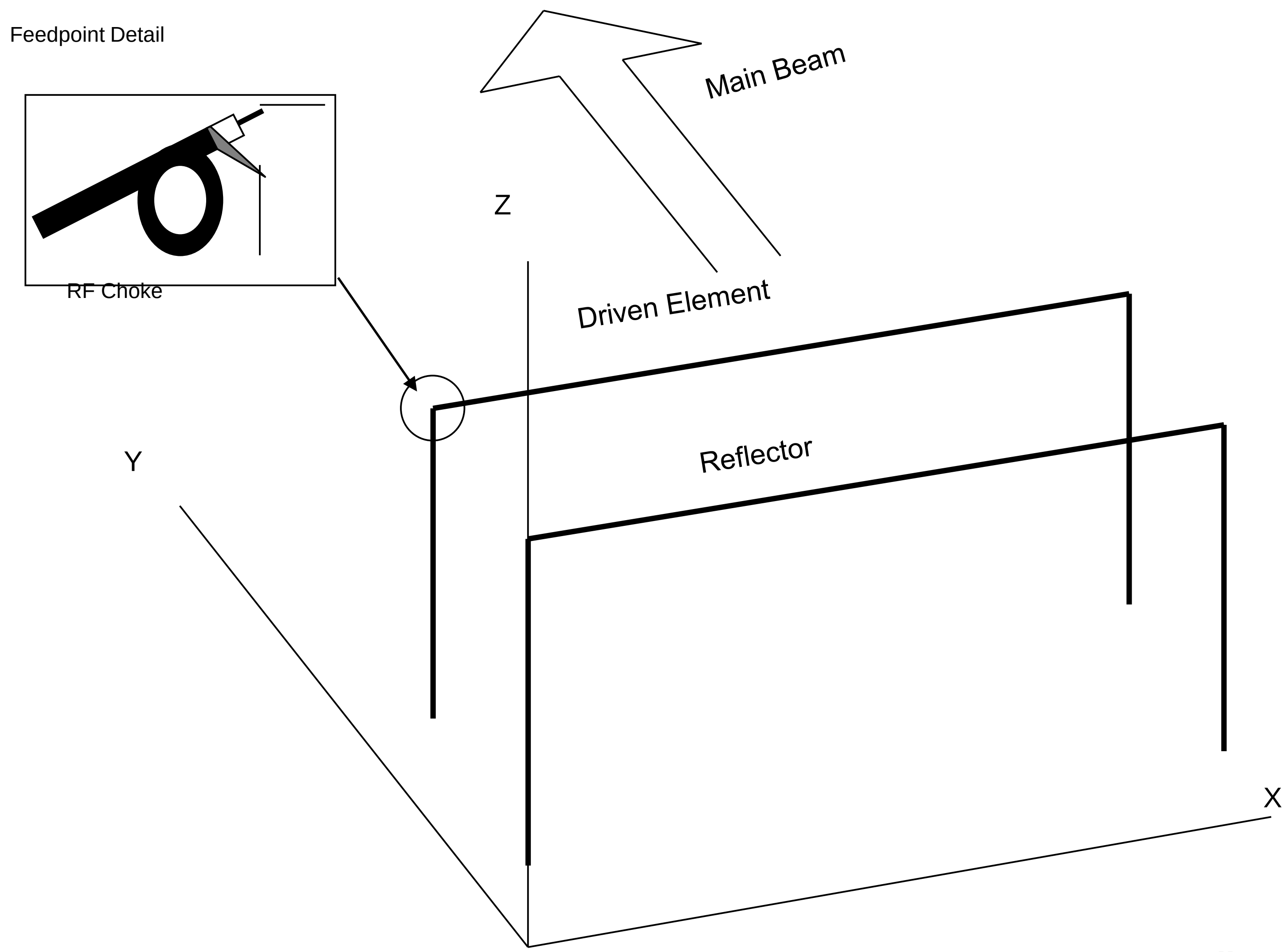


3.5 MHz to 4 MHz

Input SWR of 4-Element Matching Network

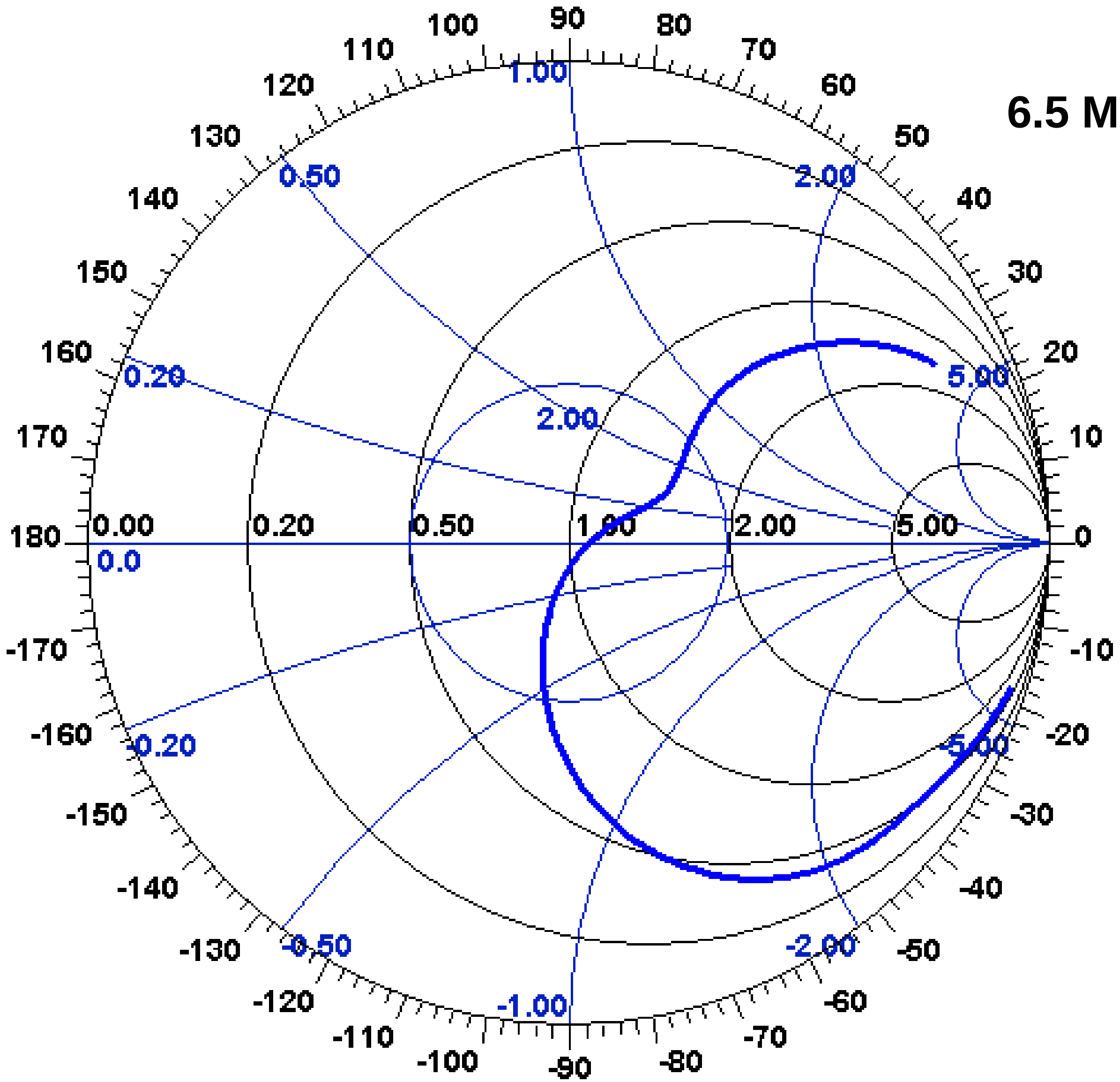


K6EI's Inverted Half-Square "Yagi"



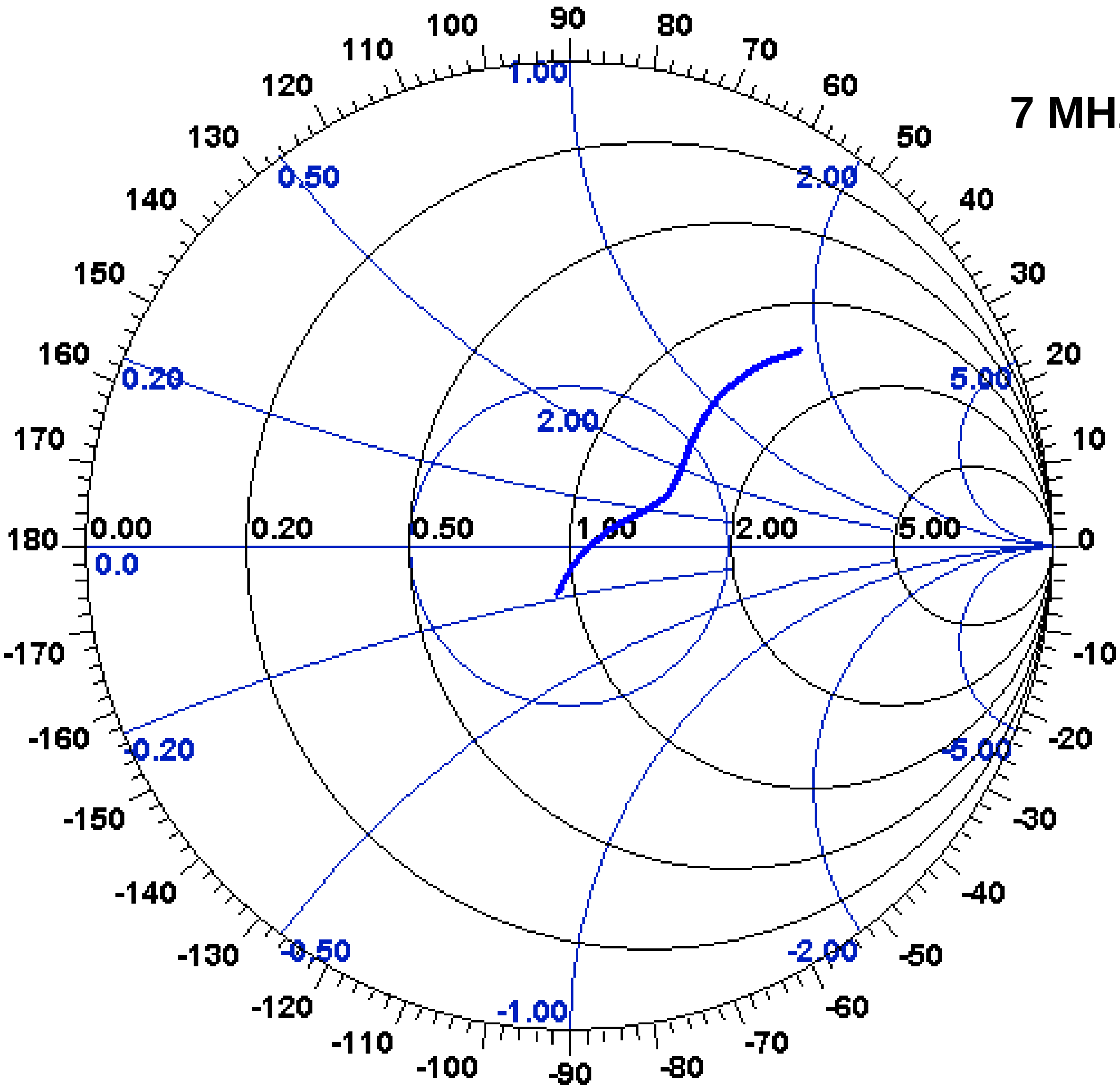
From *ARRL Antenna Compendium*, Volume 7

Inverted Half-Square Yagi Feedpoint Impedance

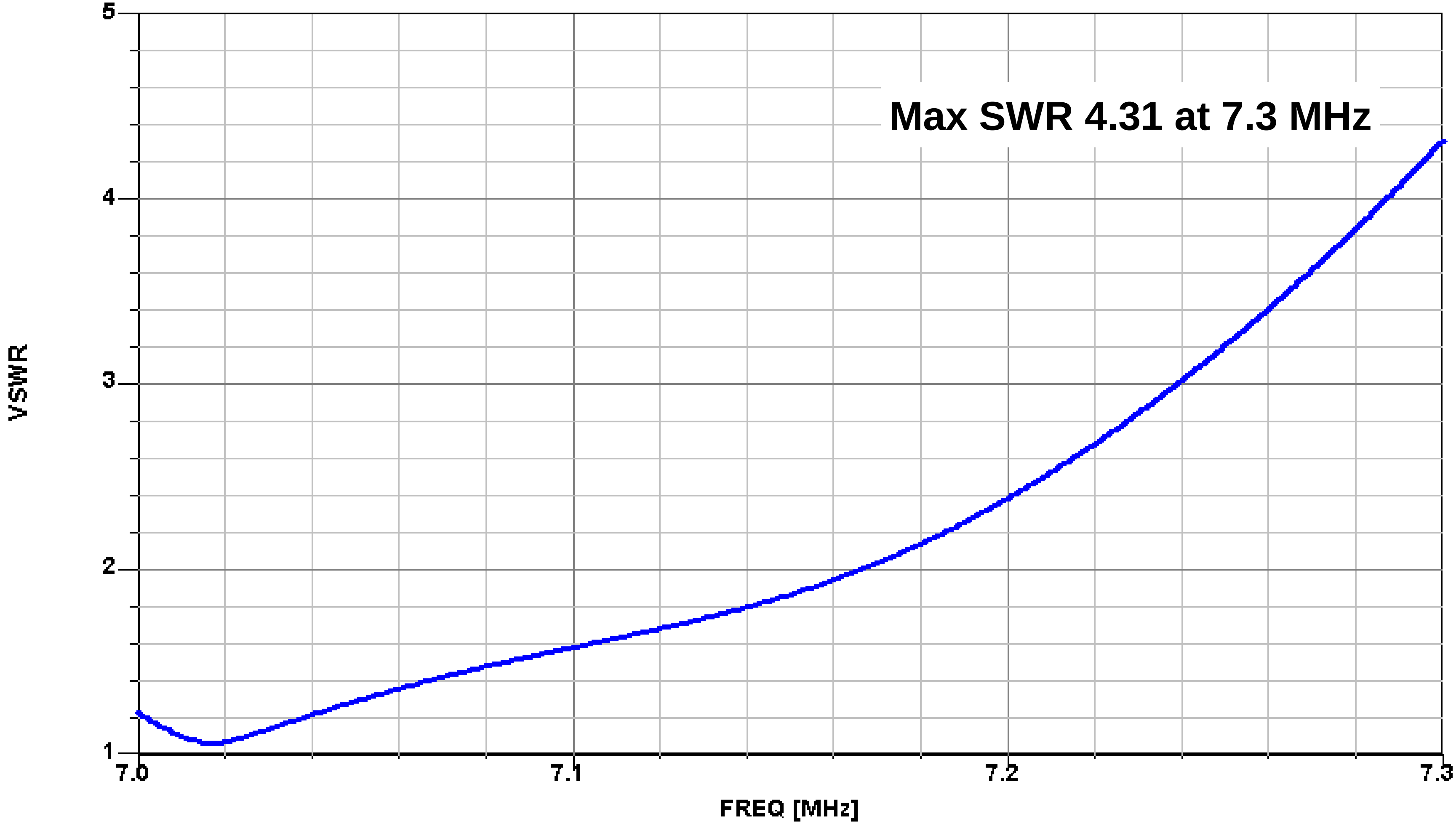


Feedpoint Impedance in 40-Meter Band

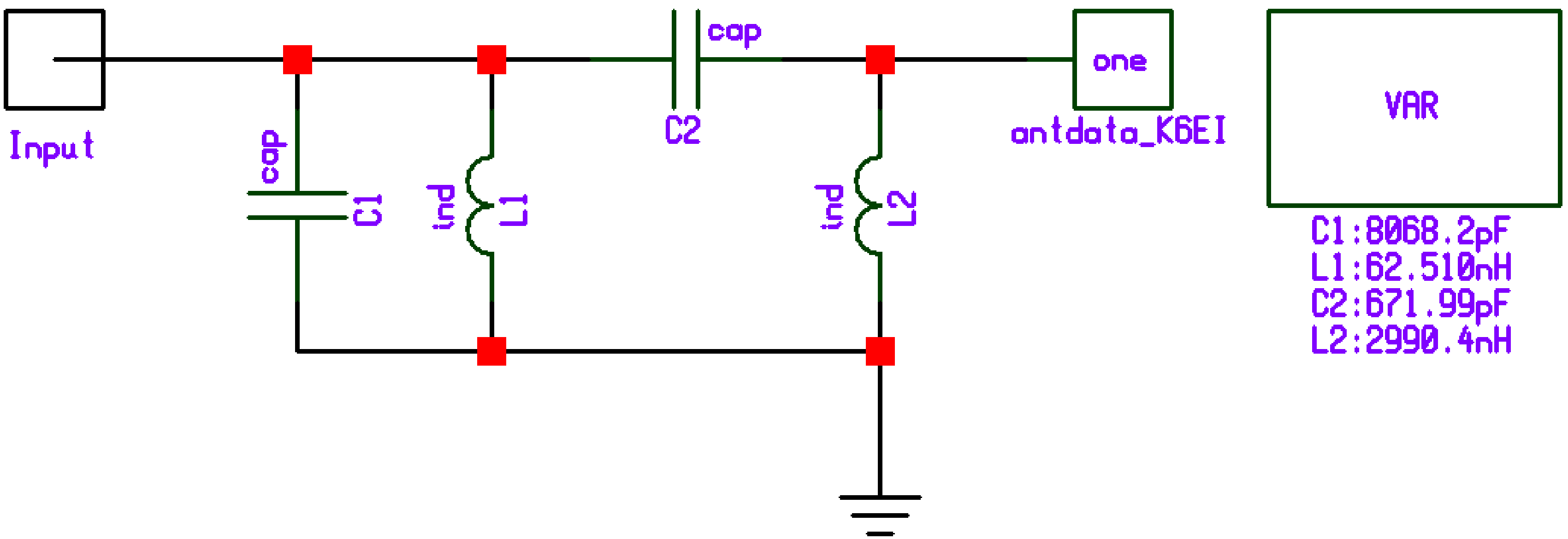
7 MHz to 7.3 MHz



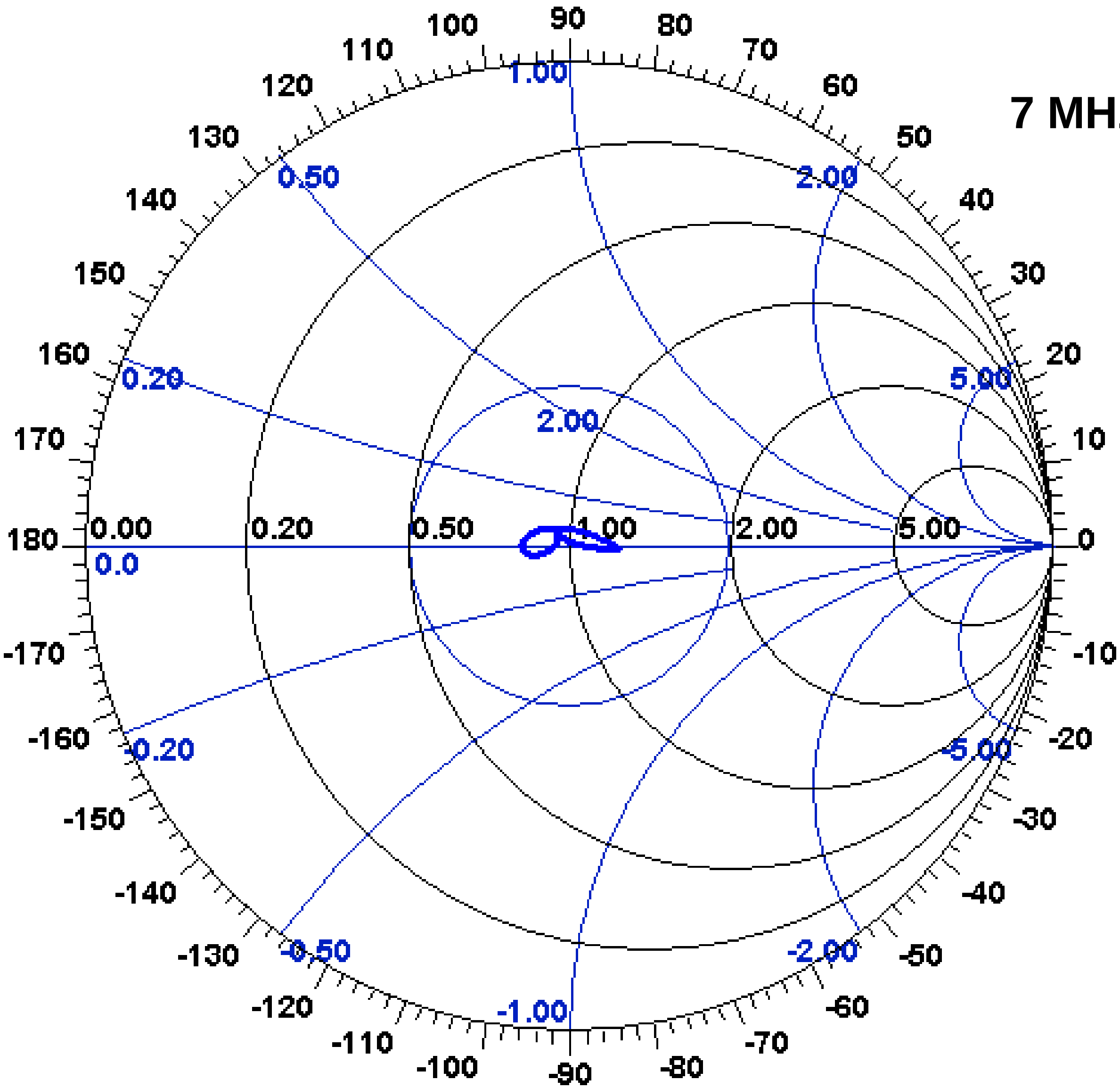
SWR of Half-Square Yagi Designed for 7.020 MHz



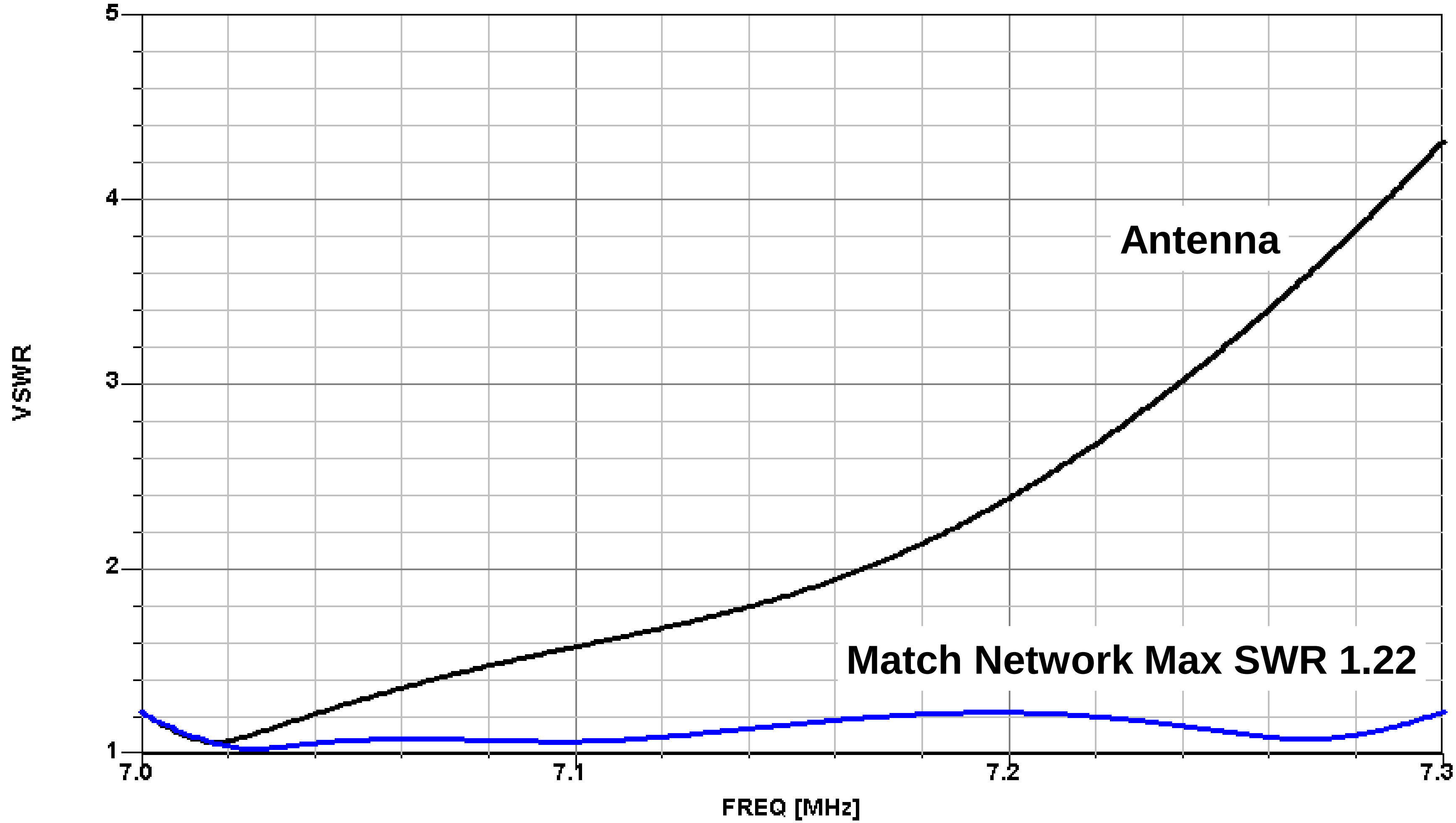
Author's Optimized 4-Element π-Resonant Network



Input Impedance of Matching Network

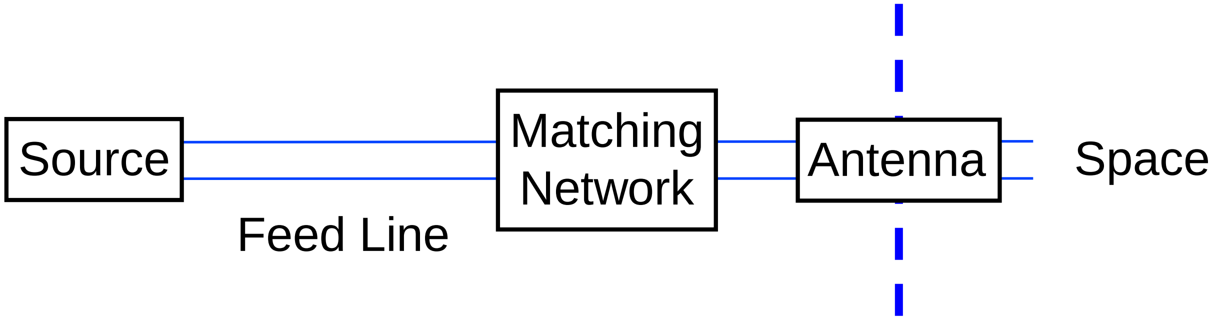


Input SWR of Matching Network



Passive Reflectionless Matching Networks

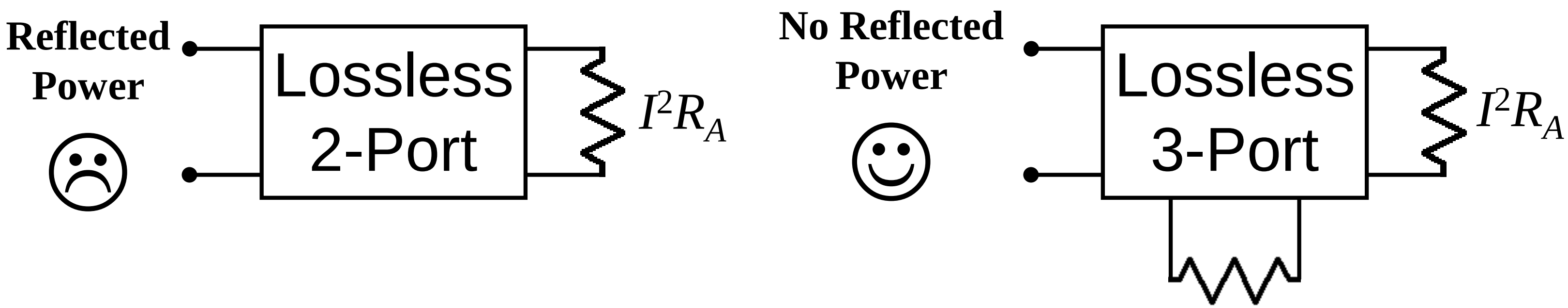
Question



- **Can a fixed (non-tunable) impedance-matching network provide unity SWR at all frequencies?**

Edward Lawry Norton, 1898-1983, photo 1925

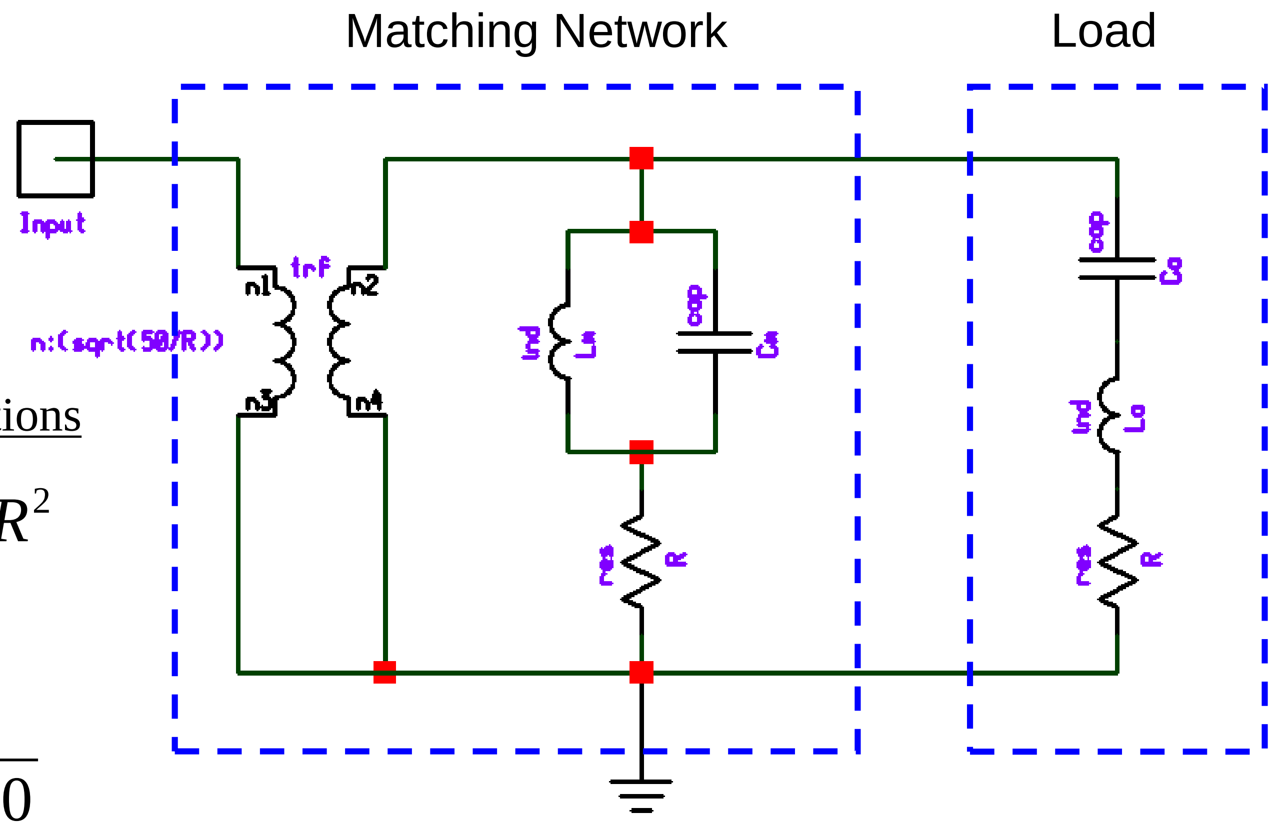
The Idea



From this…… **To this!**

- **Traditional impedance-matching networks are "reflection" filters that create reflected waves**
  - Subject to the Fano bound
- **Constant-resistance reflectionless networks**
  - Developed by O.J. Zobel (1928) and E.L. Norton (1937)
  - Have unity SWR over a wide band
  - Are "diplexers" that divide power between two loads
  - Rated by insertion-loss bandwidth instead of SWR bandwidth
  - Side-step the Fano bound but are subject to the Carlin-LaRosa bound

Constant Resistance Network for a Series RLC Load



Design Equations

$$L_M = C_A R^2$$

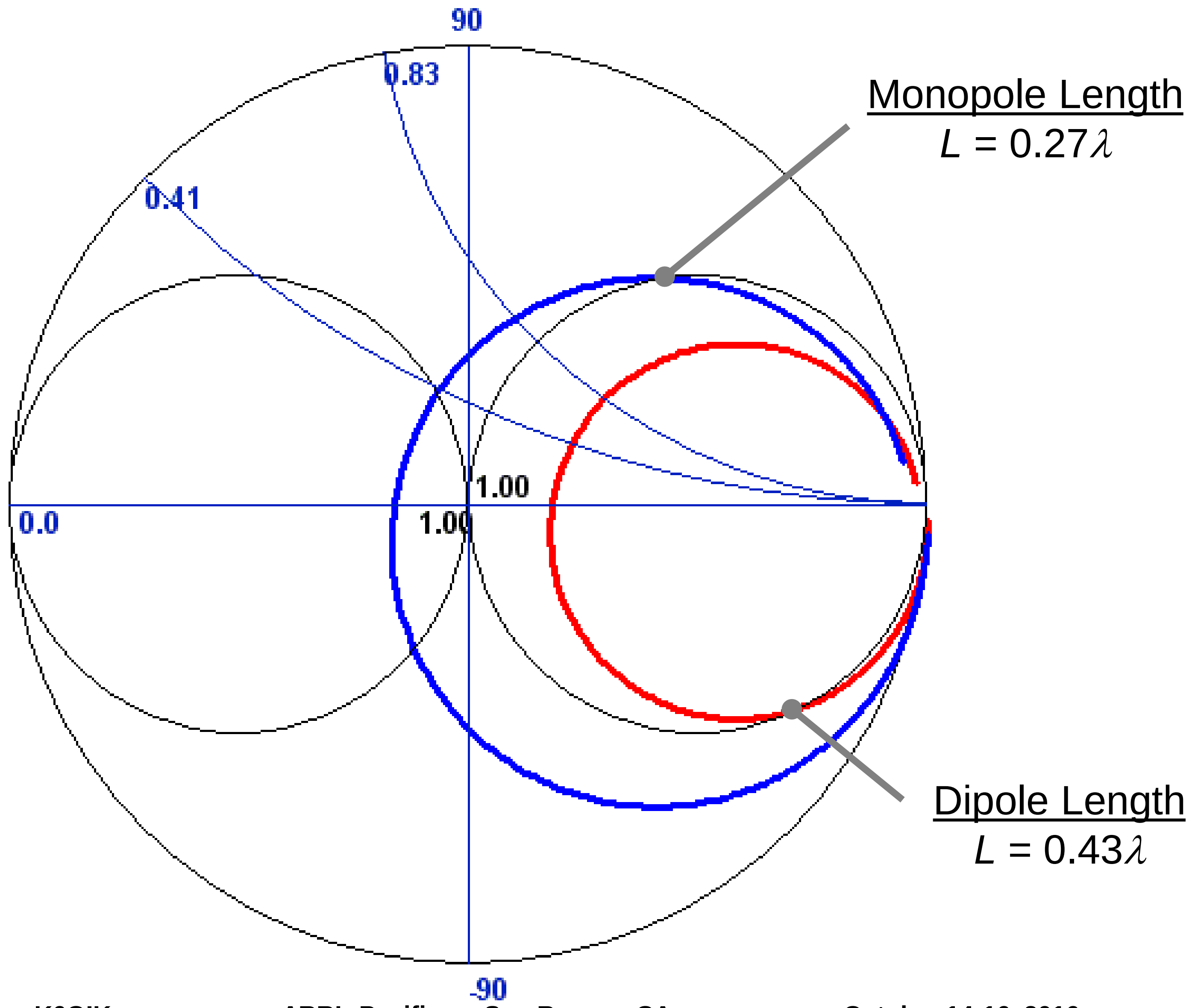
$$C_M = \frac{L_A}{R^2}$$

$$N = \sqrt{\frac{50}{R}}$$

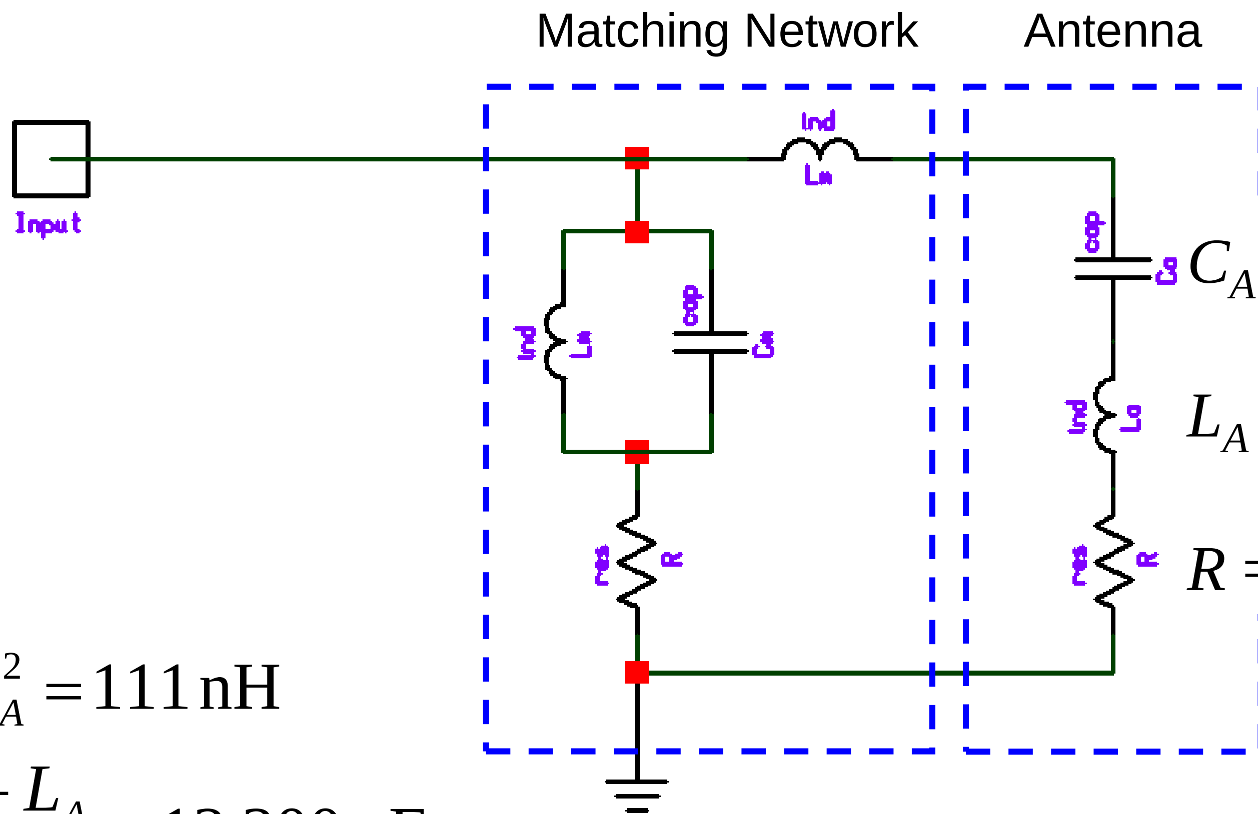
A Simple Transformerless Design

- **Step 1: Shorten "half-wave" dipoles or lengthen "quarter-wave" monopoles so that R_A = 50 ohms**
  - Dipoles: Use $K \approx 0.86$, or $L \approx 0.43\lambda$
  - Monopoles: Use $K \approx 1.07$, or $L \approx 0.27\lambda$
- **Step 2: Insert a series reactance to cancel feedpoint reactance**
  - Dipoles: Add a series inductor
  - Monopoles: Add a series capacitor
- **Step 3: Insert a shunt network to yield a 50-ohm constant-resistance network**
- **Note: transformerless designs are possible which use a few more elements, but don't require antenna modification**

Eliminating the Transformer



Example: Reflectionless Match to 0.43λ Dipole



$$C_A = 44.5\ \text{pF}$$

$$L_A = 23.8\ \mu\text{H}$$

$$R = 50\ \Omega$$

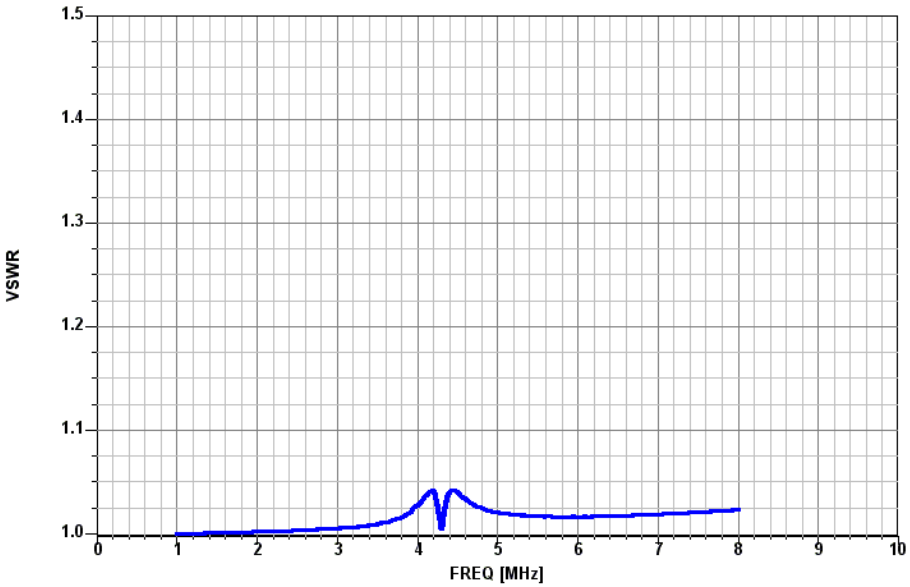
$$L_S = 7\ \mu\text{H}$$

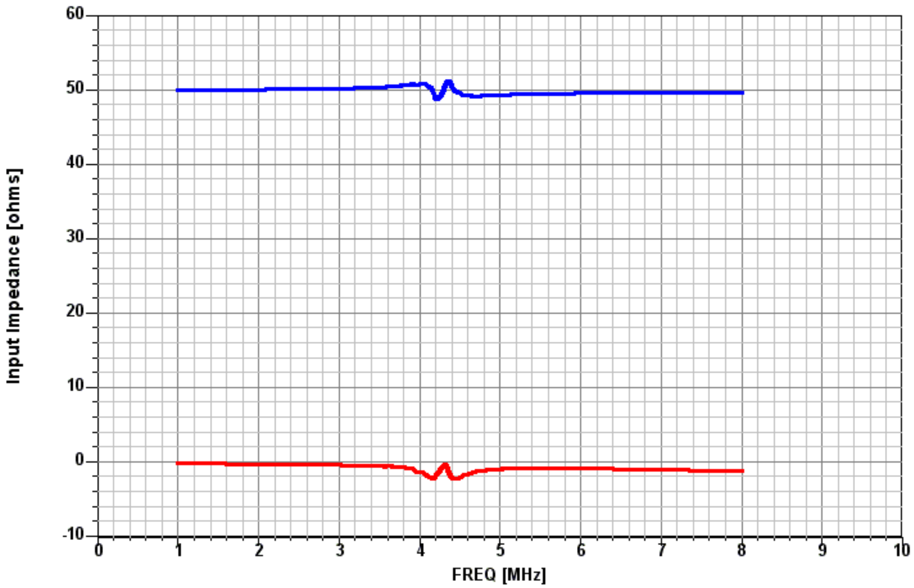
$$L_M = C_A R_A^2 = 111\ \text{nH}$$

$$C_M = \frac{L_S + L_A}{R_A^2} = 12{,}300\ \text{pF}$$

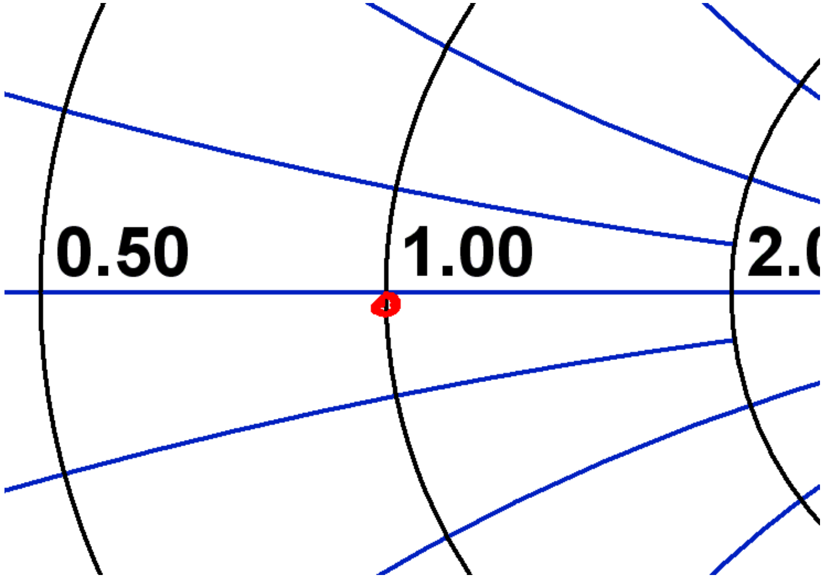
$$R = R_A = 50\ \Omega$$

Network Performance on Dipole Impedance Data

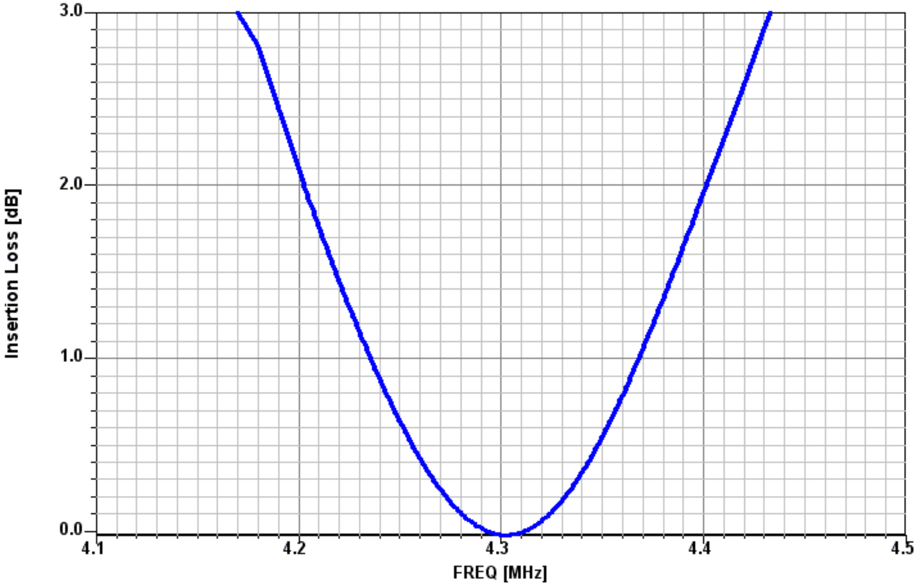




- Frequency sweep 1 to 8 MHz
- Maximum VSWR = 1.04
- Input resistance: 48.8 to 51.2 ohms
- Input reactance: -2.1 to 0 ohms



Power Delivered to the 0.43λ Dipole



- Pattern gain = - 0.11 dBd
- Minimum insertion loss = 0 dB
- 100% power delivery at 4.3 MHz
- 3-dB Bandwidth = 259 kHz (6.0%)
- 0.51-dB Bandwidth = 91 kHz (2.1%)

Bandwidths to Compare	
Lossless Networks	Reflectionless Networks
$BW_{SWR\ 5.83:1}$ ⟷	$BW_{IL\ 3\text{-}dB}$
$BW_{SWR\ 2:1}$ ⟷	$BW_{IL\ 0.51\text{-}dB}$

Reflectionless Matching Using Distributed Elements

- **The impedance of short dipoles and monopoles can be modeled as an open circuited stub in series with a frequency dependent resistor**
  - C.T. Tai, *Antenna Engineering Handbook*, 1961, p. 3-2
    - Square-law frequency dependent resistance

$$Z_{Ant} = 20(kl)^2 - jZ_0 \cot(kl)$$

  - Frank Witt, AI1H, *Antenna Compendium*, Vol. 4, 1995, p. 32
    - Linear frequency dependent resistance

$$Z_{Ant} = R_0\left[1 + K_R\left(\frac{f - f_0}{f_0}\right)\right] - jZ_0 \cot(kl)$$

  - where

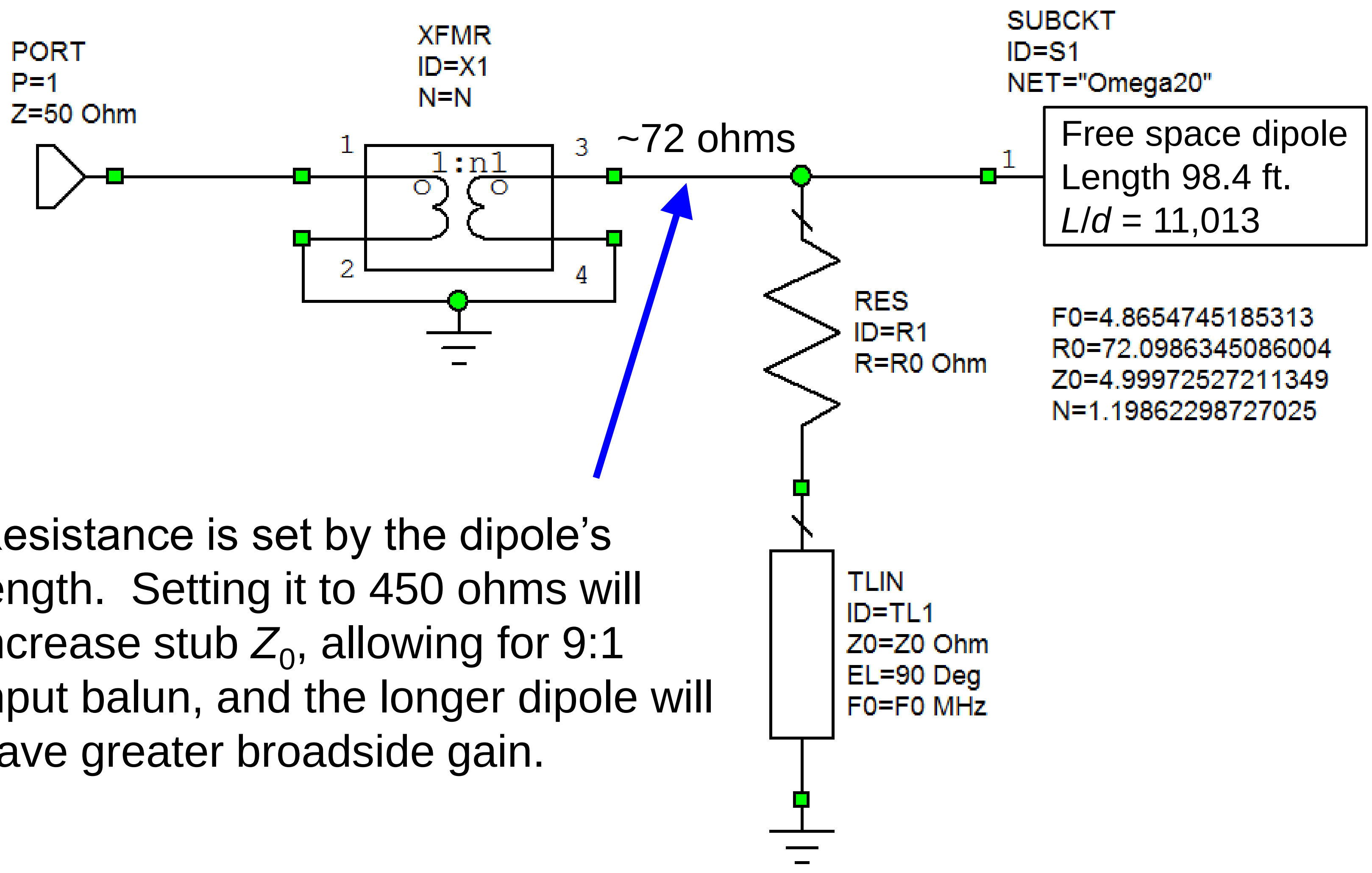
$$Z_{0,Tai} = 120\left[\ln\left(\frac{2L}{d}\right) - 1\right] = 60\left(\Omega' - 2\right) \text{ and } Z_{0,Witt} = \frac{4R_0 Q_0}{\pi}$$

- **These models do not have equivalent circuits because they violate Kramers Kronig but are useful nonetheless**

Reflectionless Matching Using a Complementary Stub

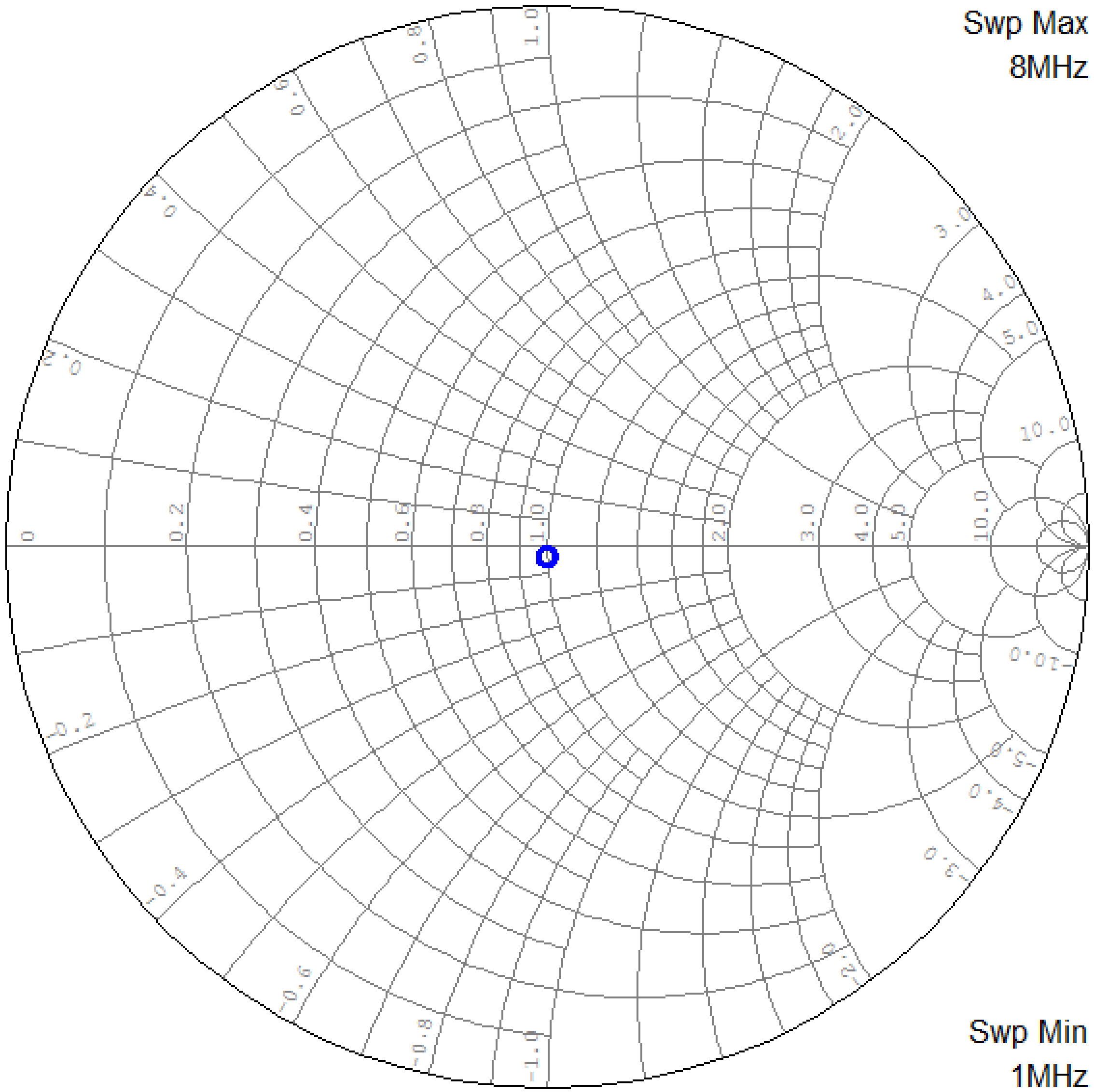
- **Assume the resistance varies slowly across a band**
- **A reflectionless match network is obtained by putting a complementary admittance in parallel with the dipole or monopole**
- **We obtain a constant-resistance (CR) match network made from a stub and a resistor**

Parallel Connection of Resistor-Stub and Dipole

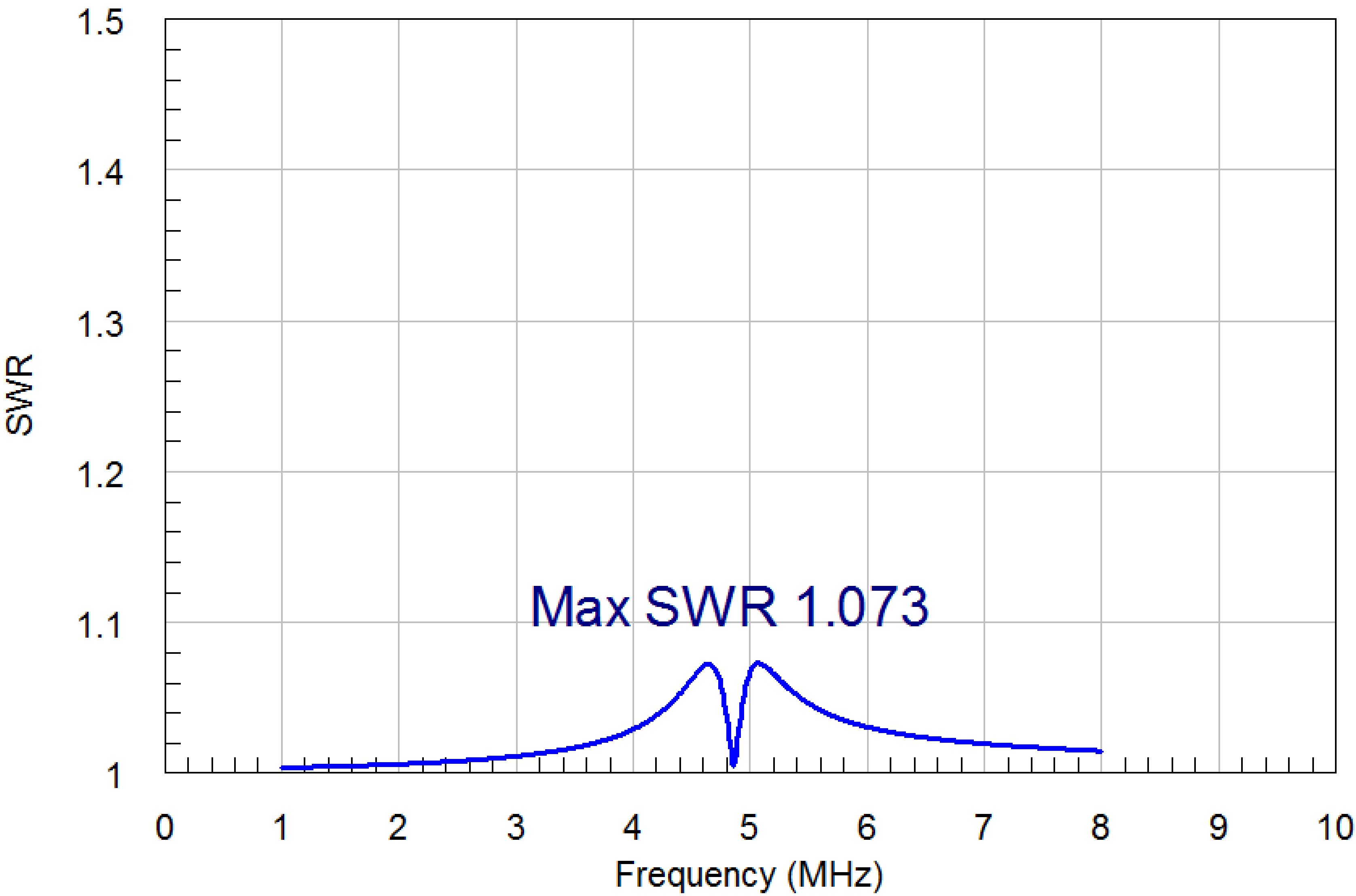


Resistance is set by the dipole's length. Setting it to 450 ohms will increase stub Z_0, allowing for 9:1 input balun, and the longer dipole will have greater broadside gain.

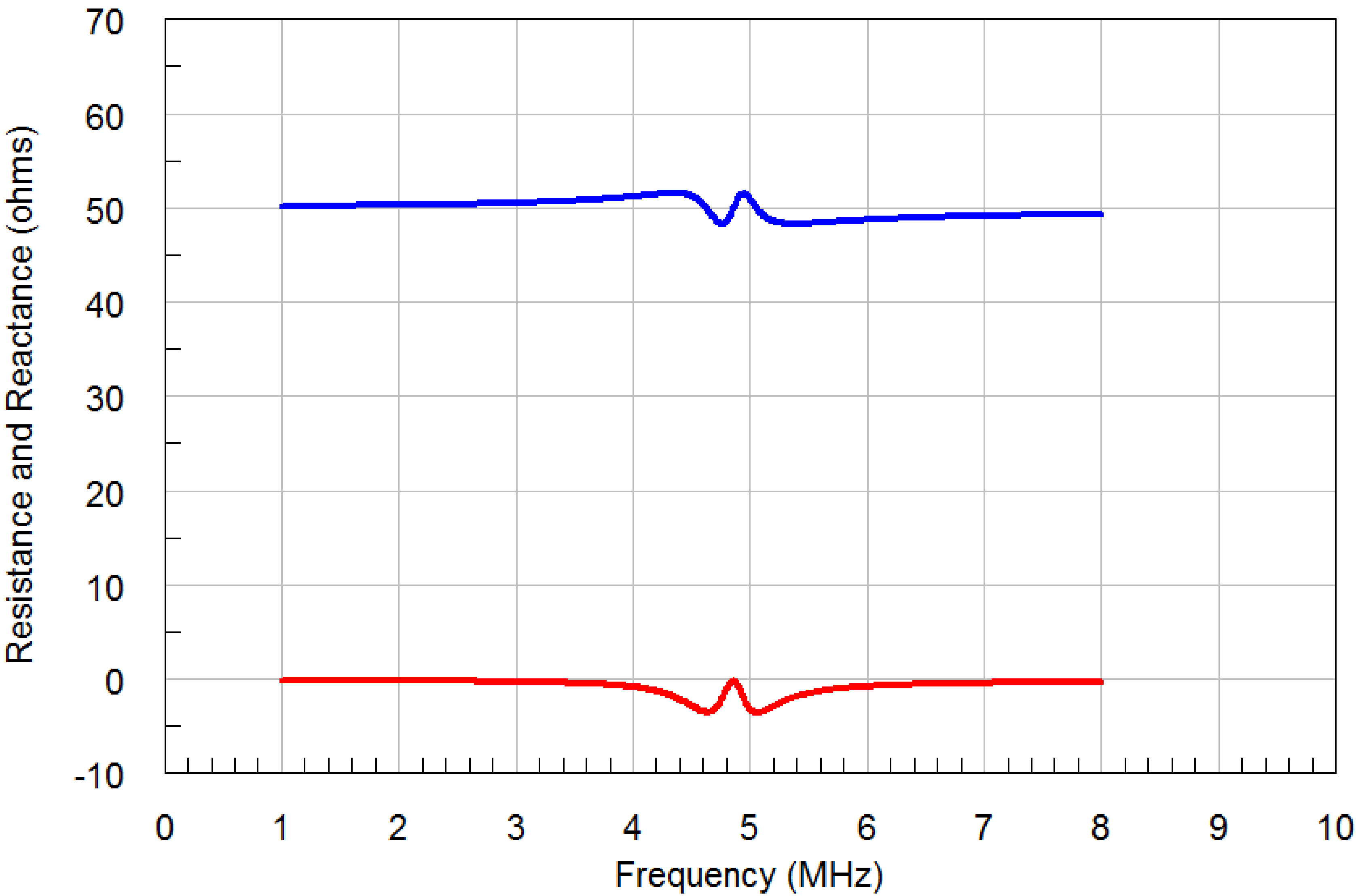
Multi-Octave Match from 1 MHz to 8 MHz



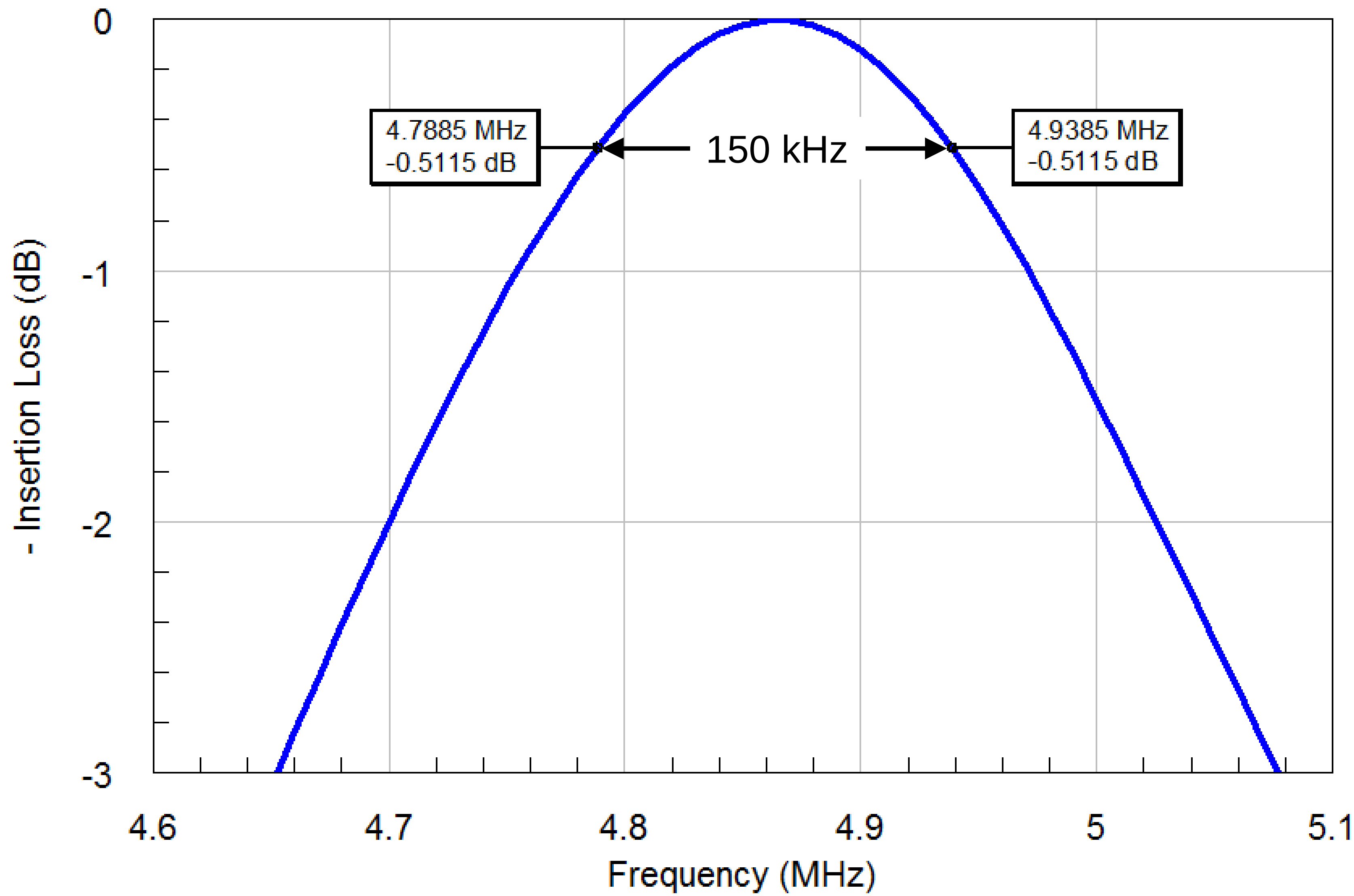
SWR



Resistance and Reactance



– Insertion Loss



General Design Procedure for Dipoles or Loops

- **This design procedure creates transformerless, reflectionless match networks using 6 to 8 elements**
- **Step 1: Move the impedance curve so the desired operating frequency point is centered on the Smith chart by using an L network (2 elements)**
- **Step 2: Rotate the impedance curve so that it faces left or right by using a 50-ohm transmission line segment (1 element)**
- **Step 3: Optionally, bend the curve onto the unit resistance (conductance) circle by adding a parallel (series) LC (2 elements)**
- **Step 4: If the impedance curve faces right (left), insert a shunt (series) network to complete a 50-ohm constant resistance network (3 elements)**
  - Shunt network is a parallel LC resonant circuit in series with a 50-ohm dummy load, all in parallel with the antenna
  - Series network is a series LC resonant circuit paralleled by a 50-ohm dummy load, all in series with the antenna

H.J. Carlin and R. LaRosa’s Bound (1952)

- **Extends Fano’s bound to passive networks that use loss to prevent reflection**
- **Covers passive reflectionless impedance-matching networks**
- **Just as Fano bounded the return loss bandwidth product of lossless networks, Carlin-LaRosa bound the insertion loss bandwidth product of reflectionless networks**
  - Bounds the achievable insertion loss for a given bandwidth
  - Bounds the achievable bandwidth for a given insertion loss
- **The bound is fundamental like the Fano bound**
- **Covers one of the two paths that bypass Fano**

A Surprising Implication for Antenna Q Formulas

- **Put a reflectionless match network at an antenna's feedpoint; network being**
  - Part of the antenna
  - Nonradiating
- **The feedpoint impedance is a constant real number independent of frequency**

$$Z_A = R_A + j0 \qquad \frac{\partial R_A}{\partial \omega} = 0 \qquad X_A = 0 \qquad \frac{\partial X_A}{\partial \omega} = 0$$

- ***Q* formulas based on feedpoint *Z* yield zero**
  - *Q* by formula is zero
  - Antenna *Q* is not zero

No general formula for computing *Q* from *Z* exists.
Antenna *Q* must be computed from field formulas.

Formulas for Q from Feedpoint Impedance

- **Series RLC equivalent circuit**

$$Q(f) = \frac{|X(f)|}{R(f)}$$

- **Geyi (2000, 2003)**

$$Q(f) = \frac{f}{2R(f)}\left[\frac{dX(f)}{df} \pm \frac{X(f)}{f}\right]$$

- **Yaghjian and Best (2003, 2005)**

$$Q(f) = \frac{f}{2R(f)}\sqrt{\left(\frac{dR(f)}{df}\right)^2 + \left(\frac{dX(f)}{df} + \frac{|X(f)|}{f}\right)^2}$$

- **Hansen (2007)**

$$Q(f) = \frac{f}{2R(f)}\left|\frac{dX(f)}{df}\right|$$

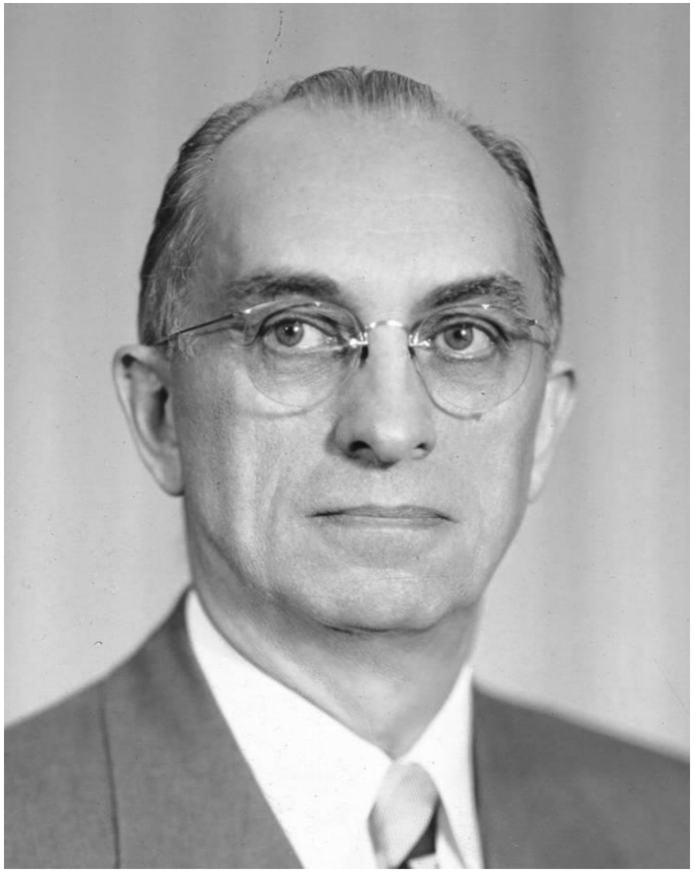
Counterexamples to these formulas exist. No general formula for computing *Q* from *Z* exists. Antenna *Q* must be computed from field formulas.

Active Non-Foster Matching Networks

Non-Foster Impedance Matching

- **Non-Foster networks contain one or more non-Foster elements or impedances**
  - Negative capacitors, negative inductors
- **Side-steps Fano and Carlin-LaRosa bandwidth bounds**
  - Active impedance matching networks that surpass known bandwidth limits
- **Implemented using negative impedance converters (NICs) or negative impedance inverters (NIVs)**
- **Disadvantages:**
  - Designing good NICs and NIVs is tricky
  - Circuit stability is problematic
  - Circuits must support high voltages or currents associated with resonance
- **Advantages:**
  - Promises great match bandwidths – a decade or more
  - Lower noise figures on receive than current active antennas
  - Bilateral $\Rightarrow$ works on transmit as well as receive
  - Potential power savings

Founders



Otto Julius Zobel
1887-1970
Courtesy of AT&T Archives

Ronald Martin Foster
1896-1998
Courtesy of IEEE

Reactance Theorems for Driving Point Functions of Lossless Networks

- **O.J. Zobel, "Theory and Design of Uniform and Composite Electric Wave-filters,"** ***Bell Syst. Tech. J.*****, vol. 2, no. 1, pp. 1-46, Jan. 1923**

$$\frac{dX(\omega)}{d\omega} > 0$$

- **R.M. Foster, "A Reactance Theorem,"** ***Bell Syst. Tech. J.*****, vol. 3, no. 2, pp. 259-267, Apr. 1924**

$$\frac{\partial X(\omega)}{\partial \omega} > 0 \quad \text{and} \quad \frac{\partial B(\omega)}{\partial \omega} > 0$$

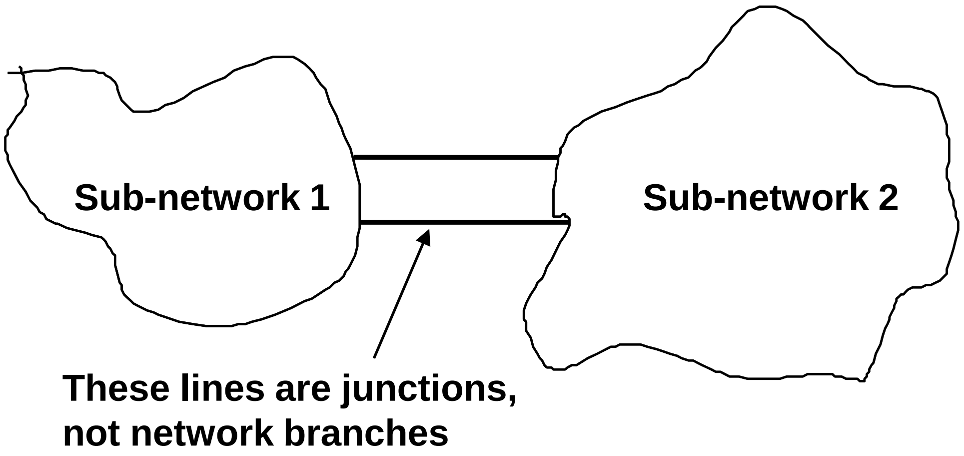
  - Poles and zeros exist only on the real frequency axis
  - Poles and zeros are simple
  - Poles and zeros have positive real residues
  - Poles alternate with zeros
  - A pole or zero exists at zero and at infinity

Key Dates

1924	R.M. Foster, Reactance Theorem
1930	B. Van der Pol, impedance transformation with –R
1931	L.C. Verman, negative circuit constants
1931	O. Brune, positive real functions, passivity, realizability
1937	S. Darlington, passive 1-port synthesis using a single resistor
1952	Emergence of active network theory
1950's	First applications of non-Foster circuits to active filters
1960	H.J. Carlin and D.C. Youla, arbitrary *N*-port synthesis using only passive elements and –R's
1964	D.C. Youla introduces the term "non-Foster"
1970's	Emergence of state-space approach to network theory
1970's	First applications of non-Foster circuits to antenna loading and matching

Junction Cuts

- Junctions connect branches to nodes
- A size-2 J-cut partitions the network into 2 or 3 disconnected sub-networks, yielding 4 terminals and 6 immittance functions



- J-cuts "de-wire" a network without deleting branches and nodes
- Non-Foster networks are a class of networks that may be defined formally by using J-cuts

Definition of a Non-Foster Network

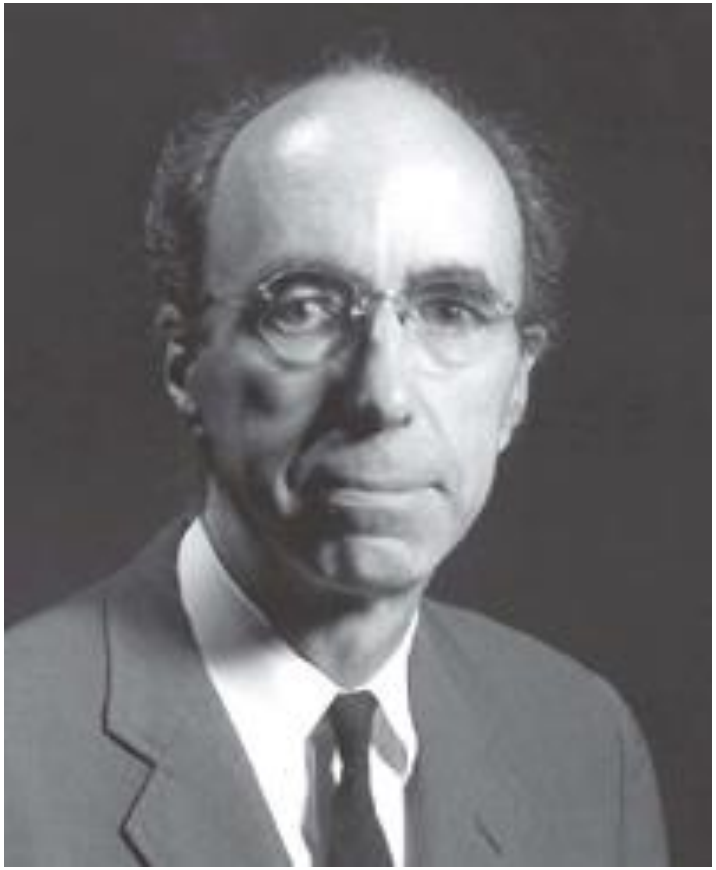
An immittance (driving point) function is "non-Foster" if its real part is zero at all real frequencies and its imaginary part has nonpositive derivative at a real frequency.

A linear network is "non-Foster" if some size-2 J-cut gives a terminal pair having a non-Foster immittance function.

- **This definition is a decision procedure for determining whether a given finite network is non-Foster**
  - Terminates in a finite number of steps

Non-Foster Impedance Matching by Formal Inversion

Sidney Darlington, 1906-1997



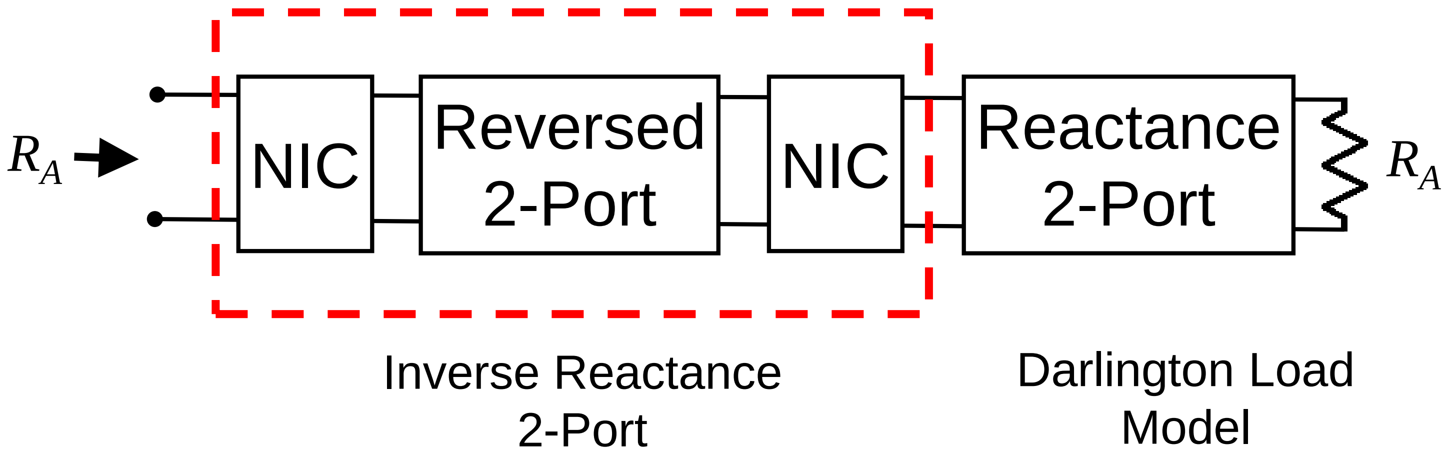
Darlington Forms (1939)

- Every positive-real, rational immittance function can be realized as a lossless lumped 2-port terminated by a resistor

- For antennas, a meromorphic antenna impedance function can be represented by a convergent sequence of equivalent circuits in Darlington form
- The Darlington form is therefore the starting point for a theory of non-Foster matching of antennas

A General Procedure (not the only one)

- **Start with load impedance model in Darlington form**
- **Formally invert the antenna's reactance 2-port**
  - Flip 2-port end-for-end, swapping input and output ports
  - Conjugate all impedances (negate all reactances)
- **NIC sandwich topology inverts the load's reactance two-port**



Formal Inversion via Two-Port Chain Matrix

- **Let**

$$T = T_M T_A = I \quad \text{and} \quad T_A = \begin{bmatrix} A(s) & B(s) \\ C(s) & D(s) \end{bmatrix}$$

- **then**

$$T_M = T_A^{-1} = \begin{bmatrix} D(s) & -B(s) \\ -C(s) & A(s) \end{bmatrix}$$

- **If T_A is a reciprocal network, then T_M is too**
  - Condition for reciprocity

$$A(s)D(s) - B(s)C(s) = 1$$

- **Corresponding impedance matrices**

$$Z_A = \begin{bmatrix} z_{11}^A & z_{12}^A \\ z_{21}^A & z_{22}^A \end{bmatrix} = \frac{1}{C(s)} \begin{bmatrix} A(s) & 1 \\ 1 & D(s) \end{bmatrix}$$

$$Z_M = \begin{bmatrix} z_{11}^M & z_{12}^M \\ z_{21}^M & z_{22}^M \end{bmatrix} = \frac{1}{C(s)} \begin{bmatrix} -D(s) & -1 \\ -1 & -A(s) \end{bmatrix}$$

Matching Network Impedance Matrix

- **Let Z_A be**

$$Z_A = \begin{bmatrix} z_{11}^A & z_{12}^A \\ z_{21}^A & z_{22}^A \end{bmatrix}$$

- **Then Z_M is found to be**

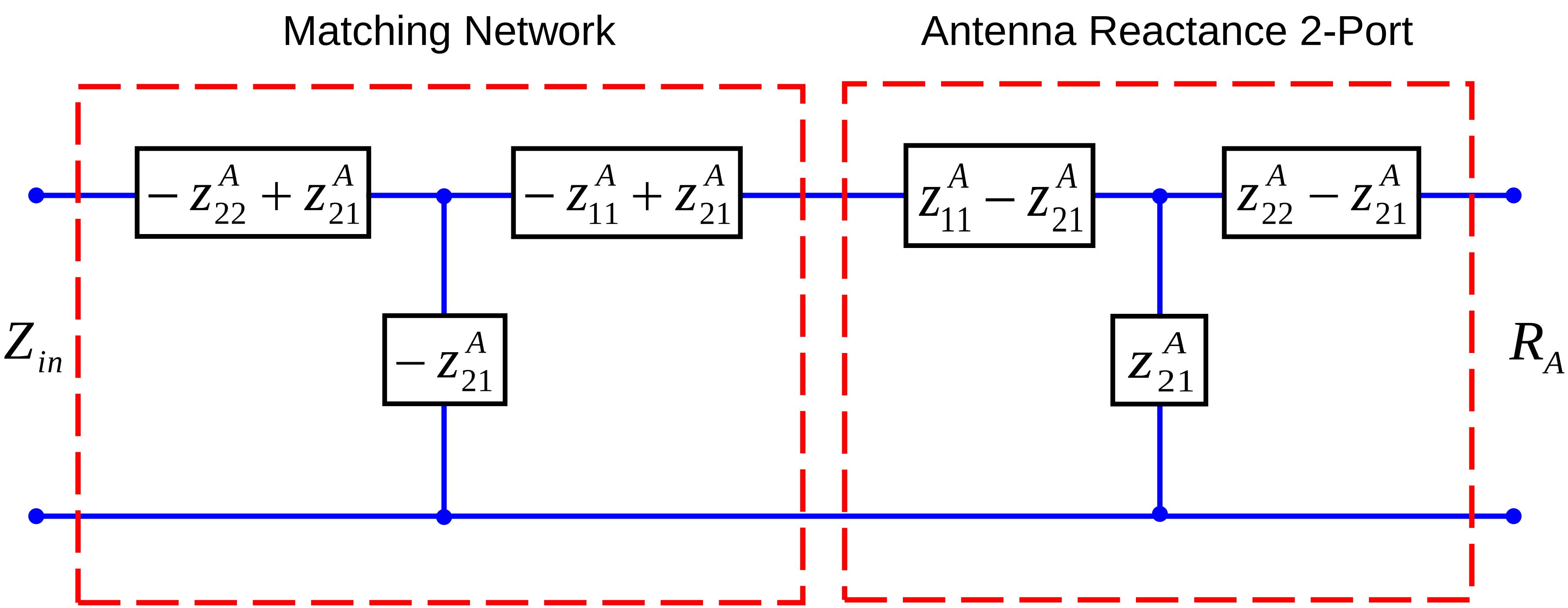
$$Z_M = \begin{bmatrix} z_{11}^M & z_{12}^M \\ z_{21}^M & z_{22}^M \end{bmatrix} = \begin{bmatrix} z_{22}^{A*} & z_{12}^{A*} \\ z_{21}^{A*} & z_{11}^{A*} \end{bmatrix} = Z_A^{\backslash H}$$

Hermitian antitranspose

- **where by reciprocity**

$$z_{12}^A = z_{21}^A \quad \text{so that} \quad z_{12}^M = z_{21}^M$$

T-Network Inversion



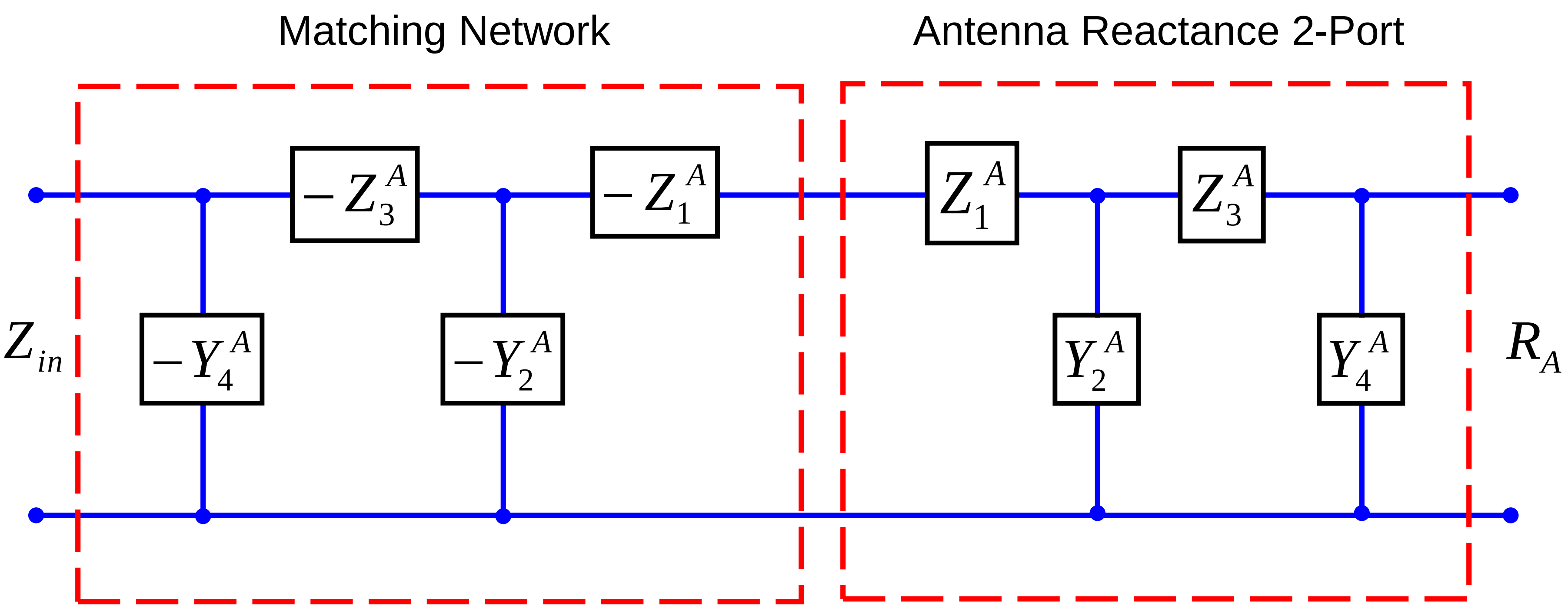
$$Z_{in} = R_A$$

Continued Fraction Expansion for Z_{in}

$$Z_{in} = \cfrac{1}{-Y_4^A + \cfrac{1}{-Z_3^A + \cfrac{1}{-Y_2^A + \cfrac{1}{-Z_1^A + Z_1^A + \cfrac{1}{Y_2^A + \cfrac{1}{Z_3^A + \cfrac{1}{Y_4^A + \cfrac{1}{R_A}}}}}}}}$$

$$Z_{in} = R_A$$

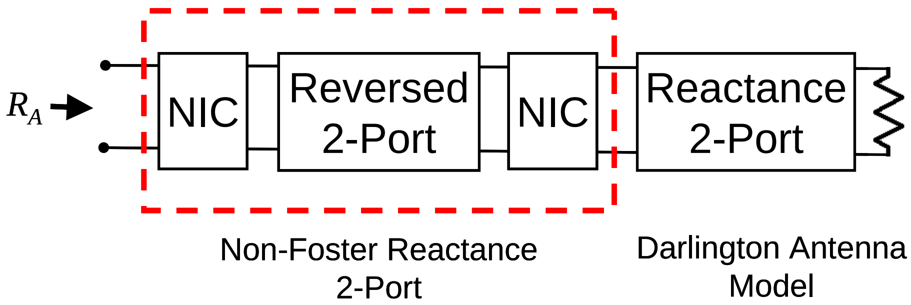
Realization as Cascaded Ladder Networks



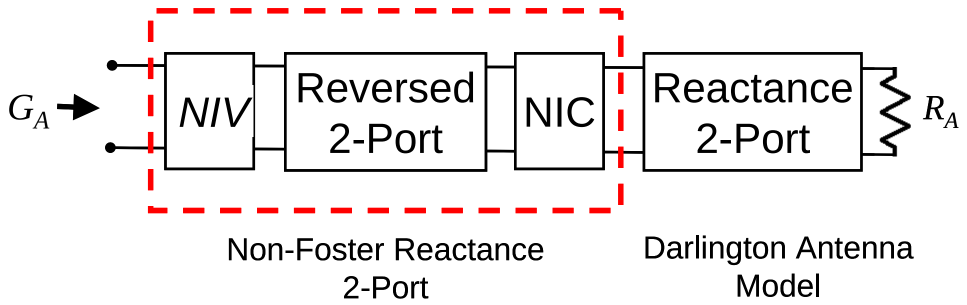
$$Z_{in} = R_A$$

Cascade Inversions Using NICs and NIVs

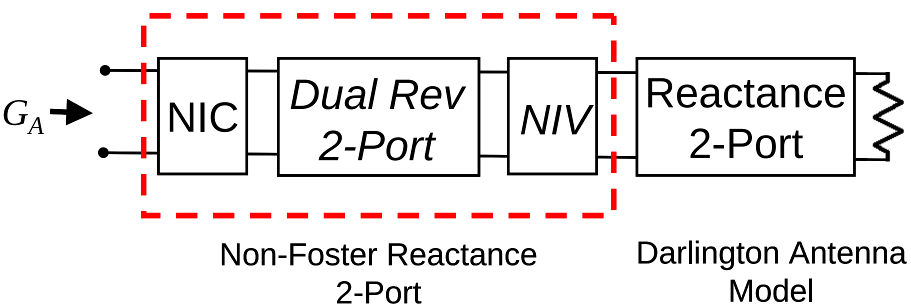
1st Canonical Form



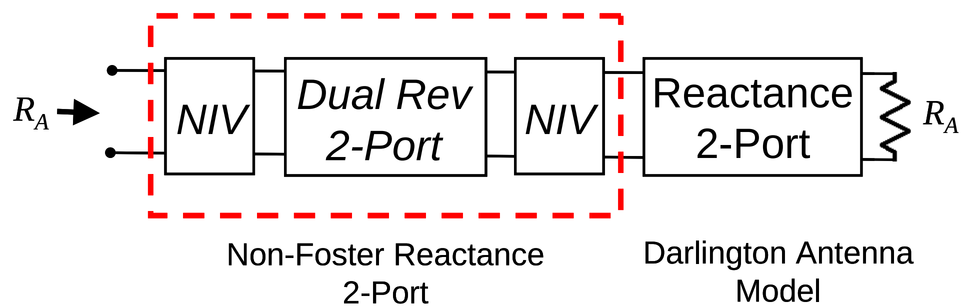
2nd Canonical Form



3rd Canonical Form



4th Canonical Form



Formal inverse match networks achieve perfect impedance matching and infinite match bandwidth.

Comments

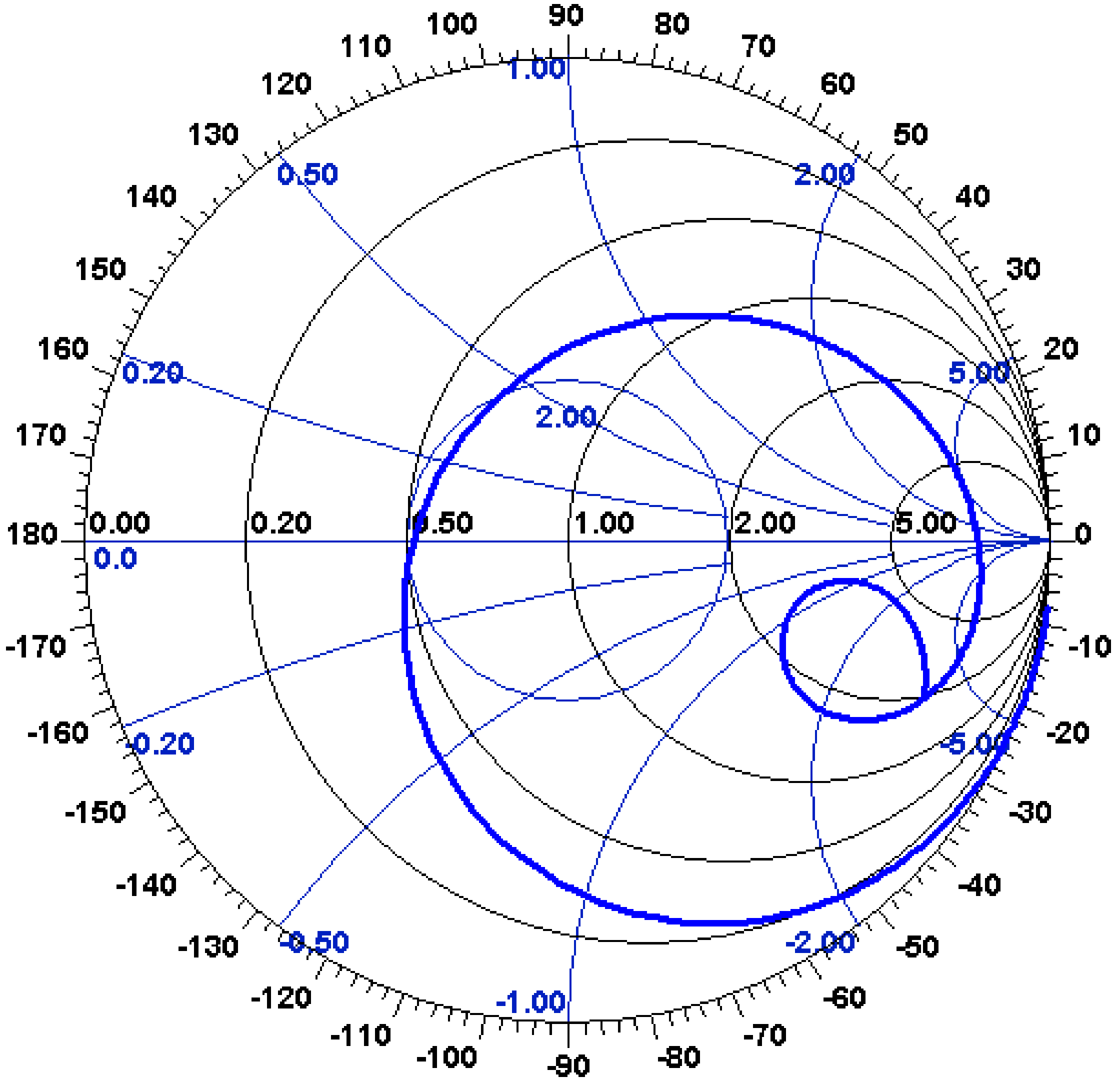
- **The canonical forms define a synthesis procedure (a recipe) for making broadband impedance matching networks for arbitrary passive loads**
- **Commutative property**

$$\frac{1}{\Delta}\begin{bmatrix} D(s) & -B(s) \\ -C(s) & A(s) \end{bmatrix}\begin{bmatrix} A(s) & B(s) \\ C(s) & D(s) \end{bmatrix} = I = \begin{bmatrix} A(s) & B(s) \\ C(s) & D(s) \end{bmatrix}\frac{1}{\Delta}\begin{bmatrix} D(s) & -B(s) \\ -C(s) & A(s) \end{bmatrix}$$

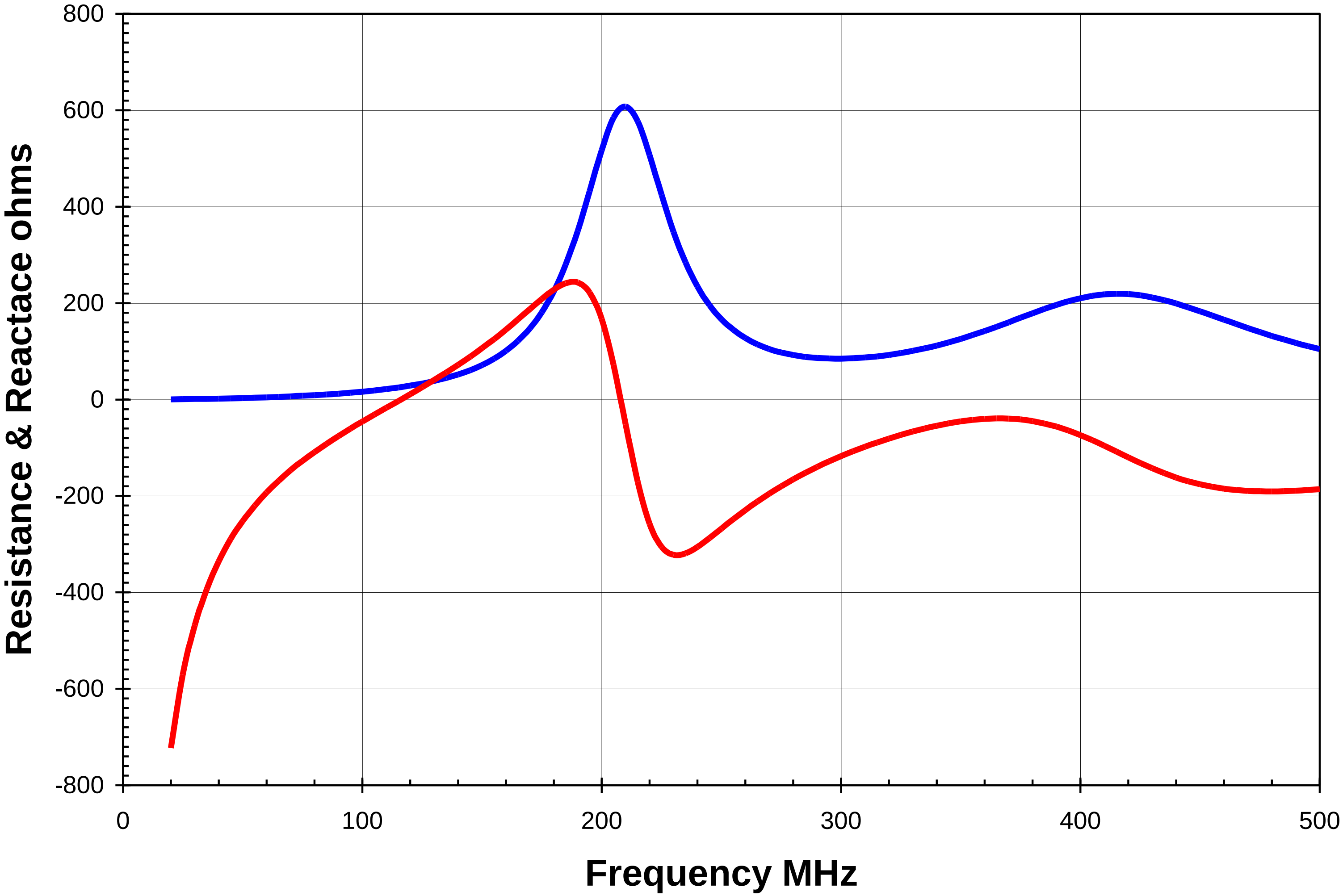
$$T^{-1}\,T = T\,T^{-1}$$

- **The formal inverse network can be placed on either side of the antenna's reactance 2-port, i.e. the matching network can be placed between it and the load resistor**
  - ➢ An antenna can be matched by using a non-Foster feedpoint network or a non-Foster shell (cap or radome)

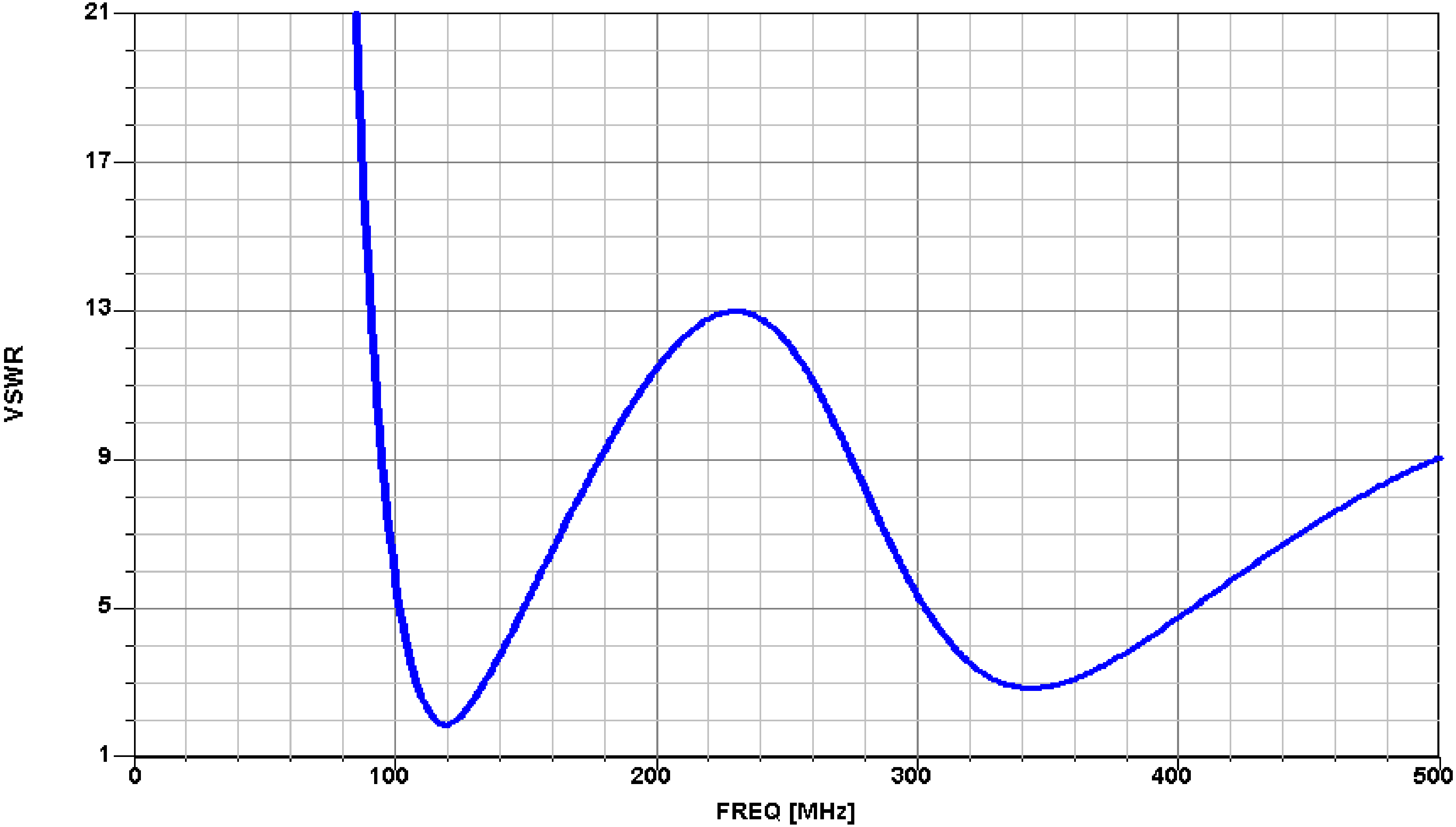
Example 1: Bent Blade Antenna



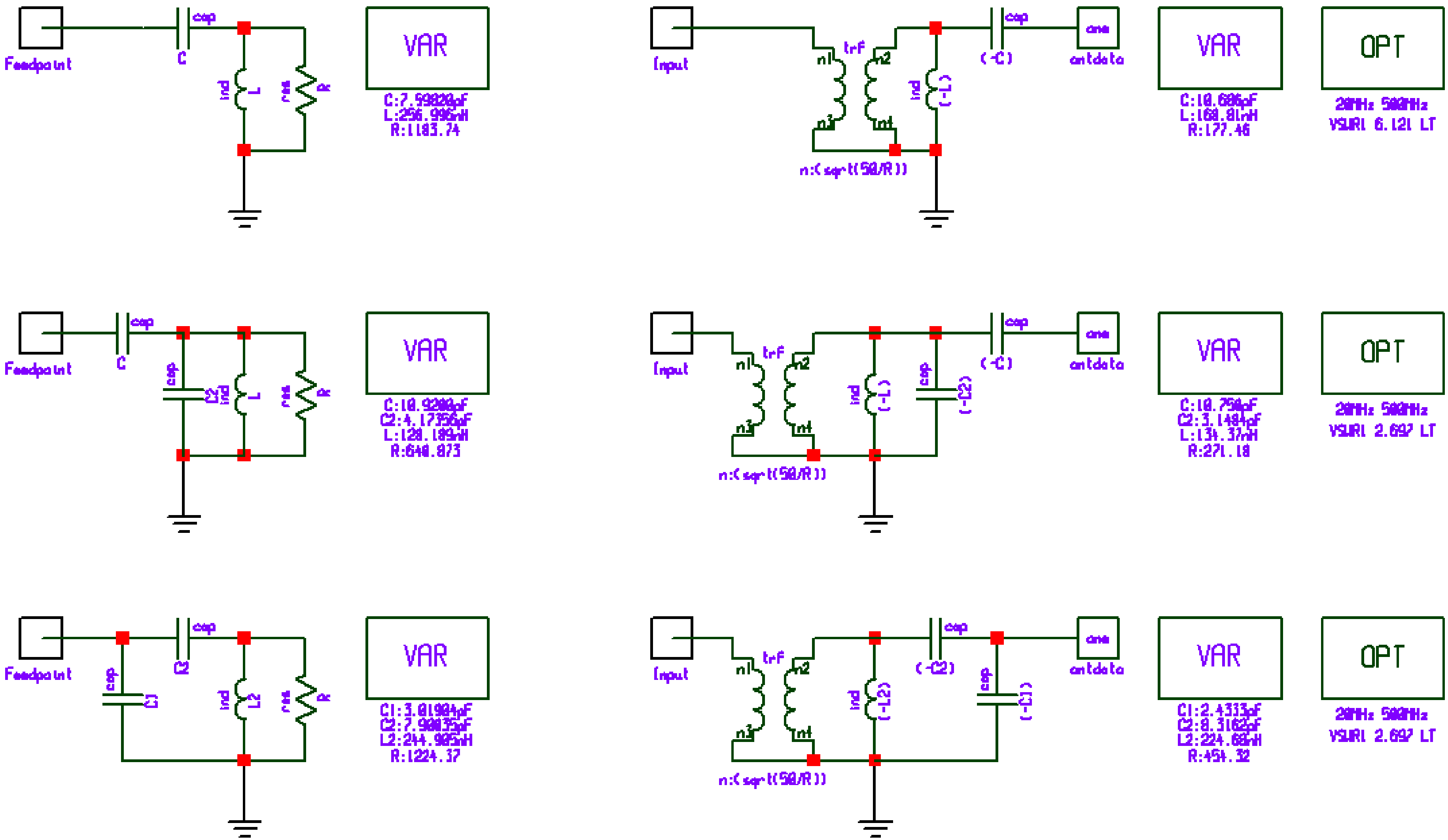
Bent Blade Antenna Feedpoint Impedance



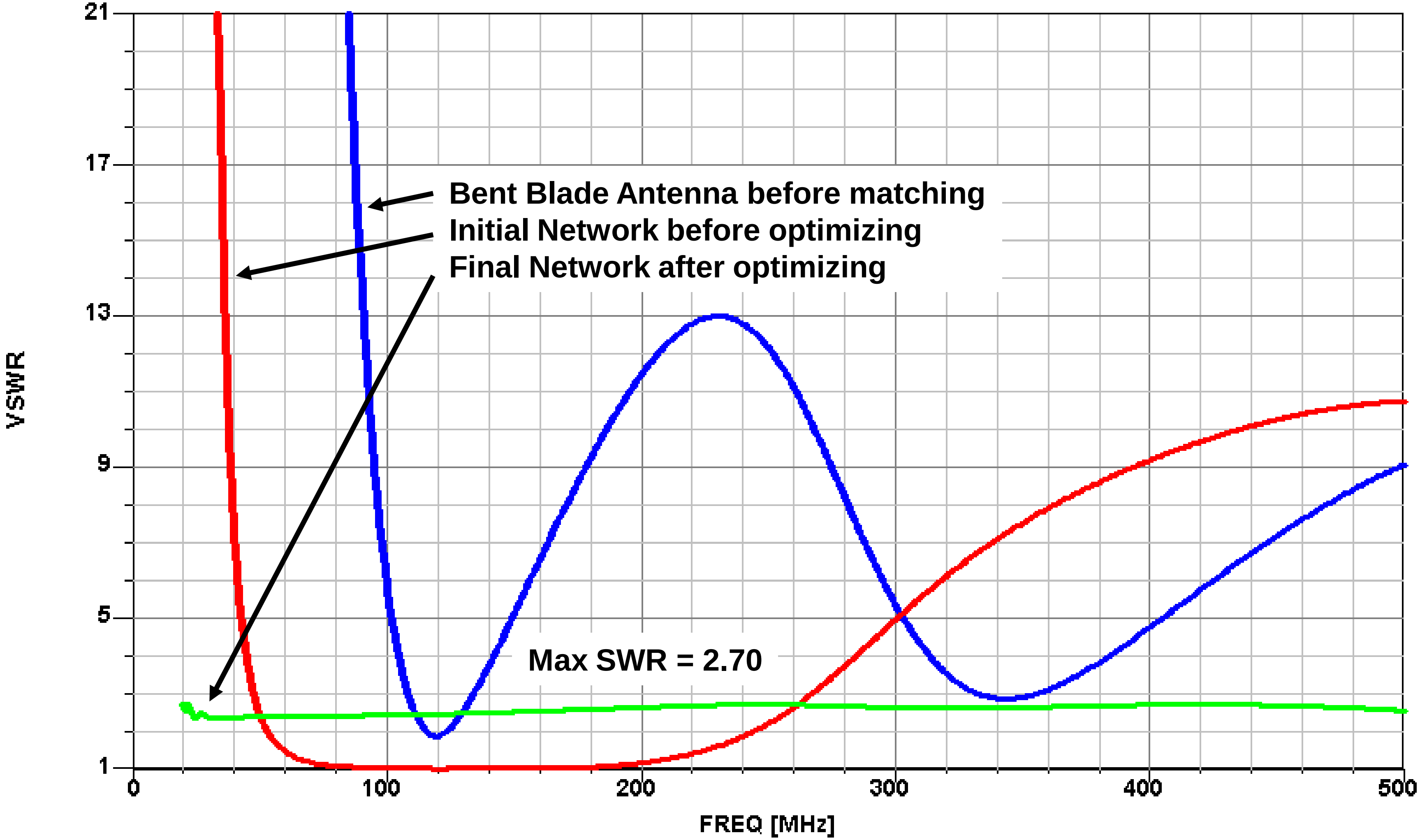
Antenna SWR Before Matching



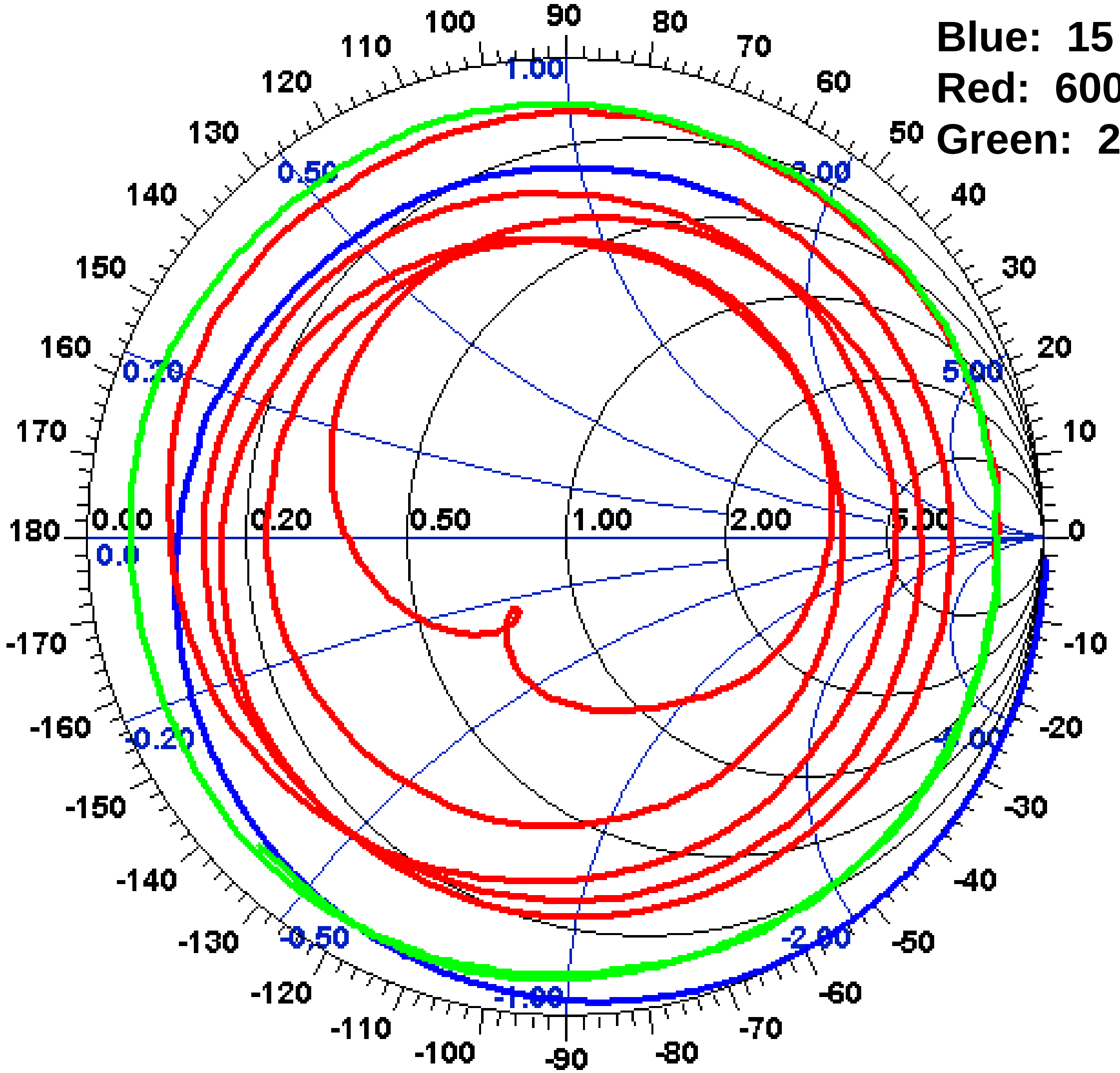
Bent Blade Equivalent Circuits and Matching Networks



Match Results Using Non-Foster Networks

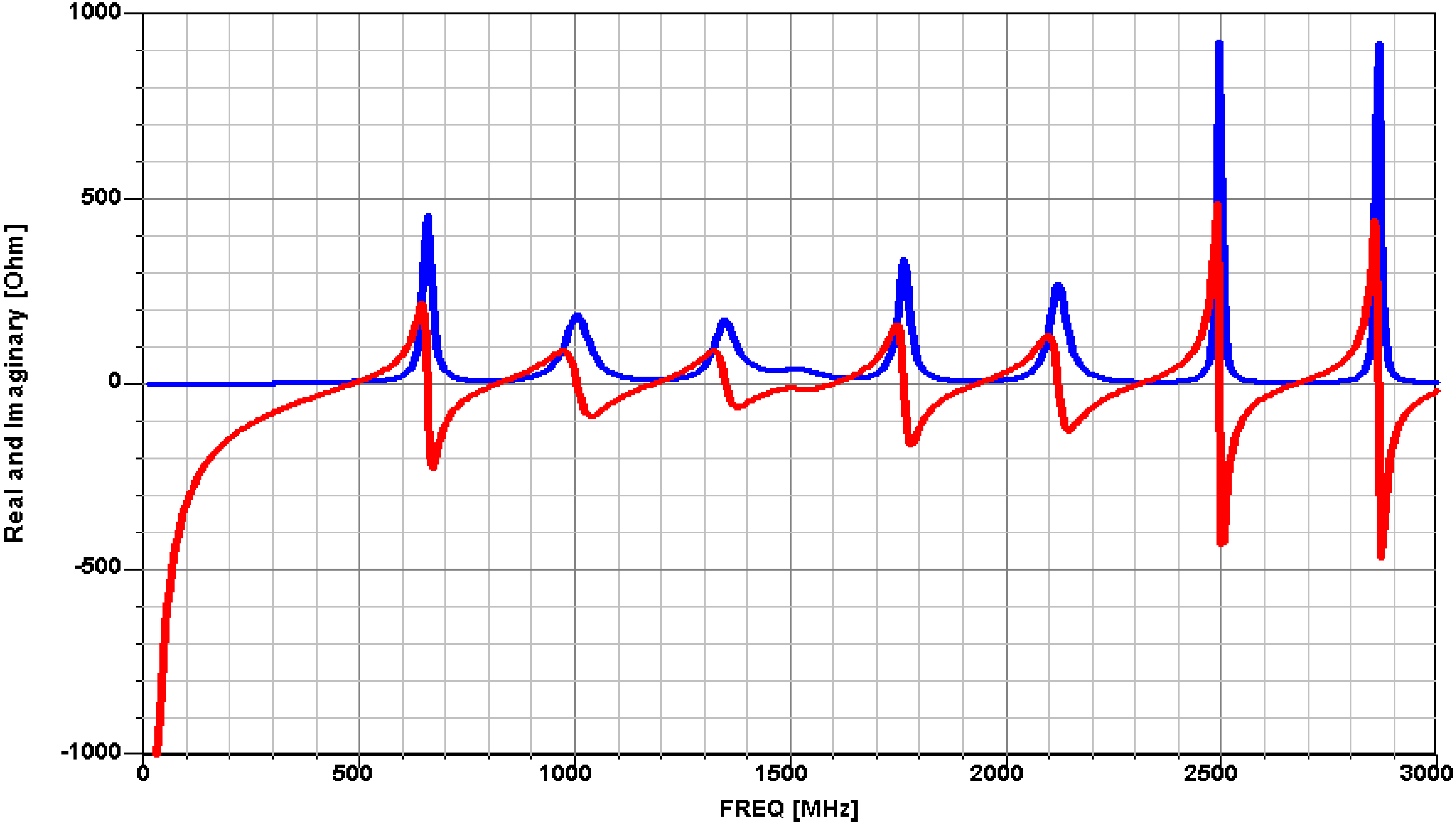


Example 2: Broadband Antenna

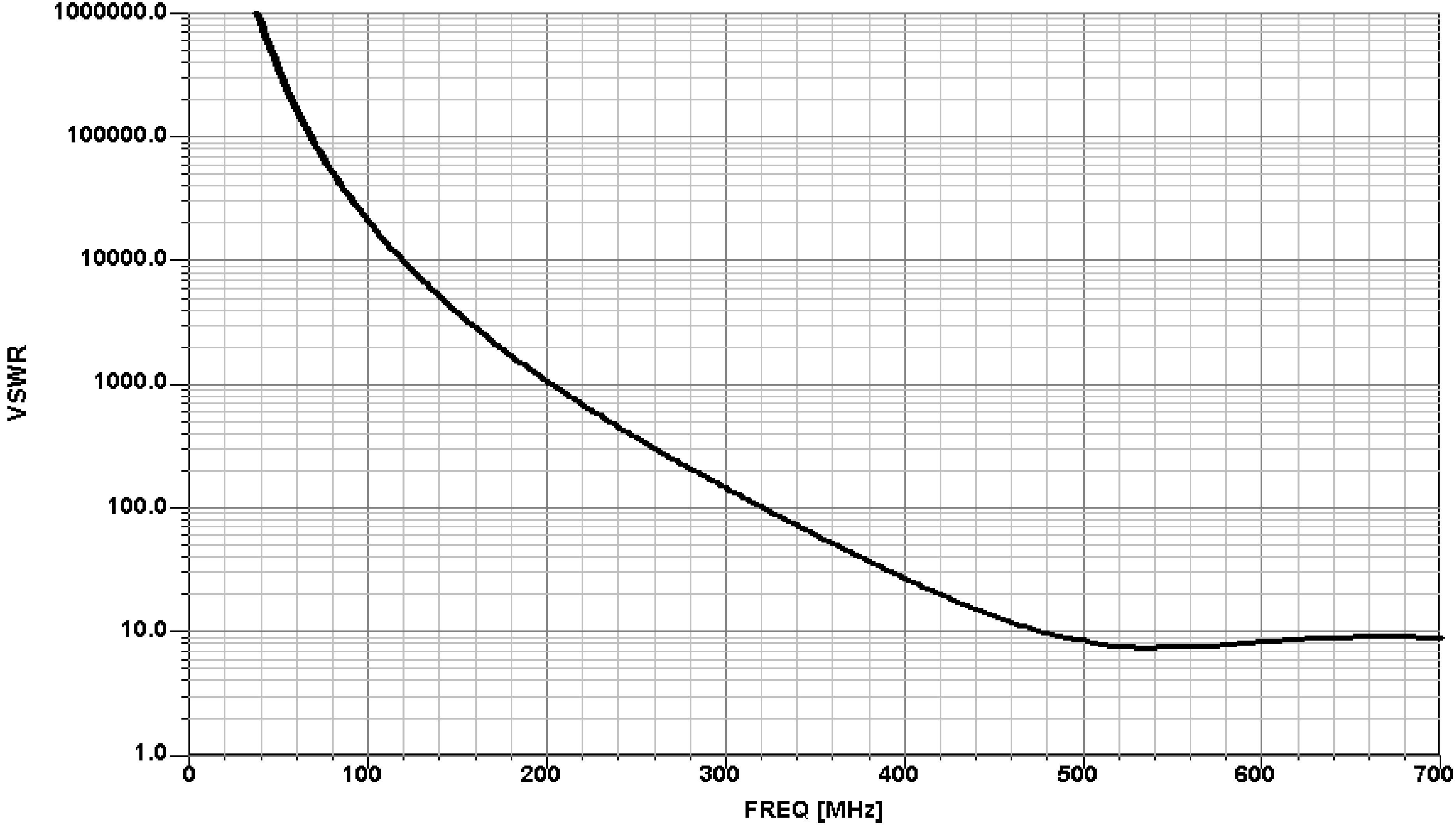


Blue: 15 to 600 MHz
Red: 600 to 2,500 MHz
Green: 2,500 to 3,000 MHz

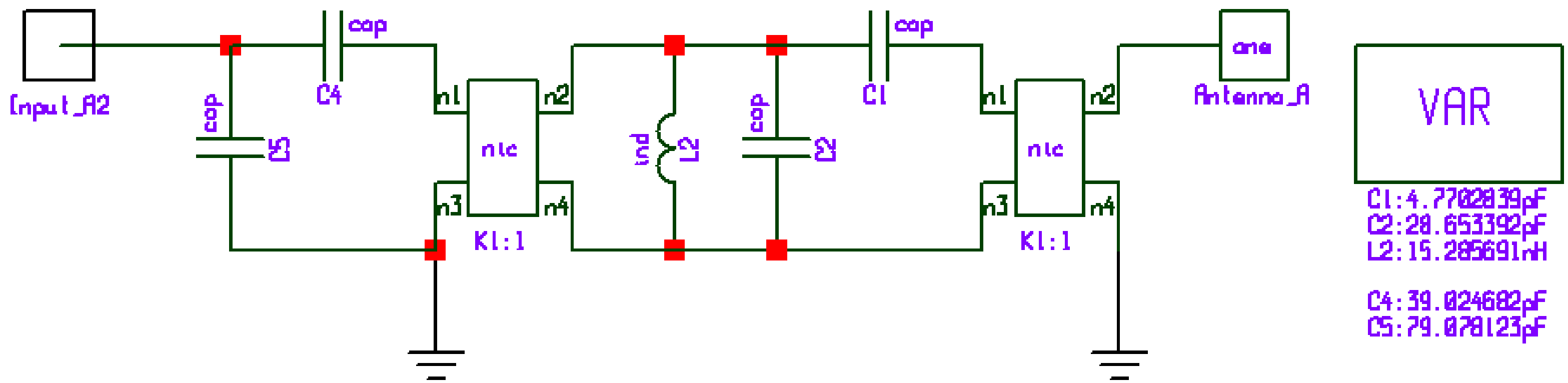
Antenna Impedance to 3 GHz

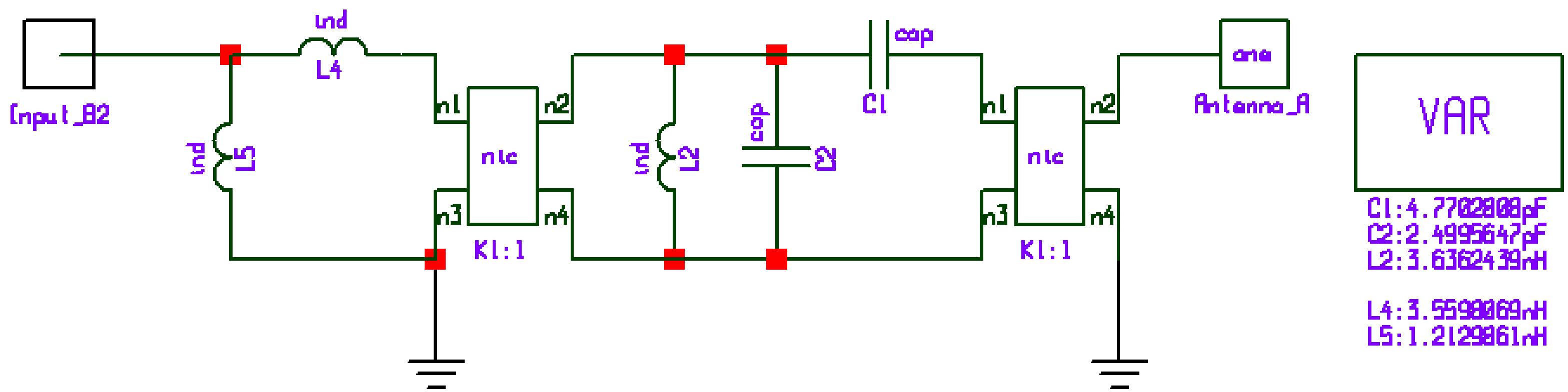


Antenna SWR to 700 MHz

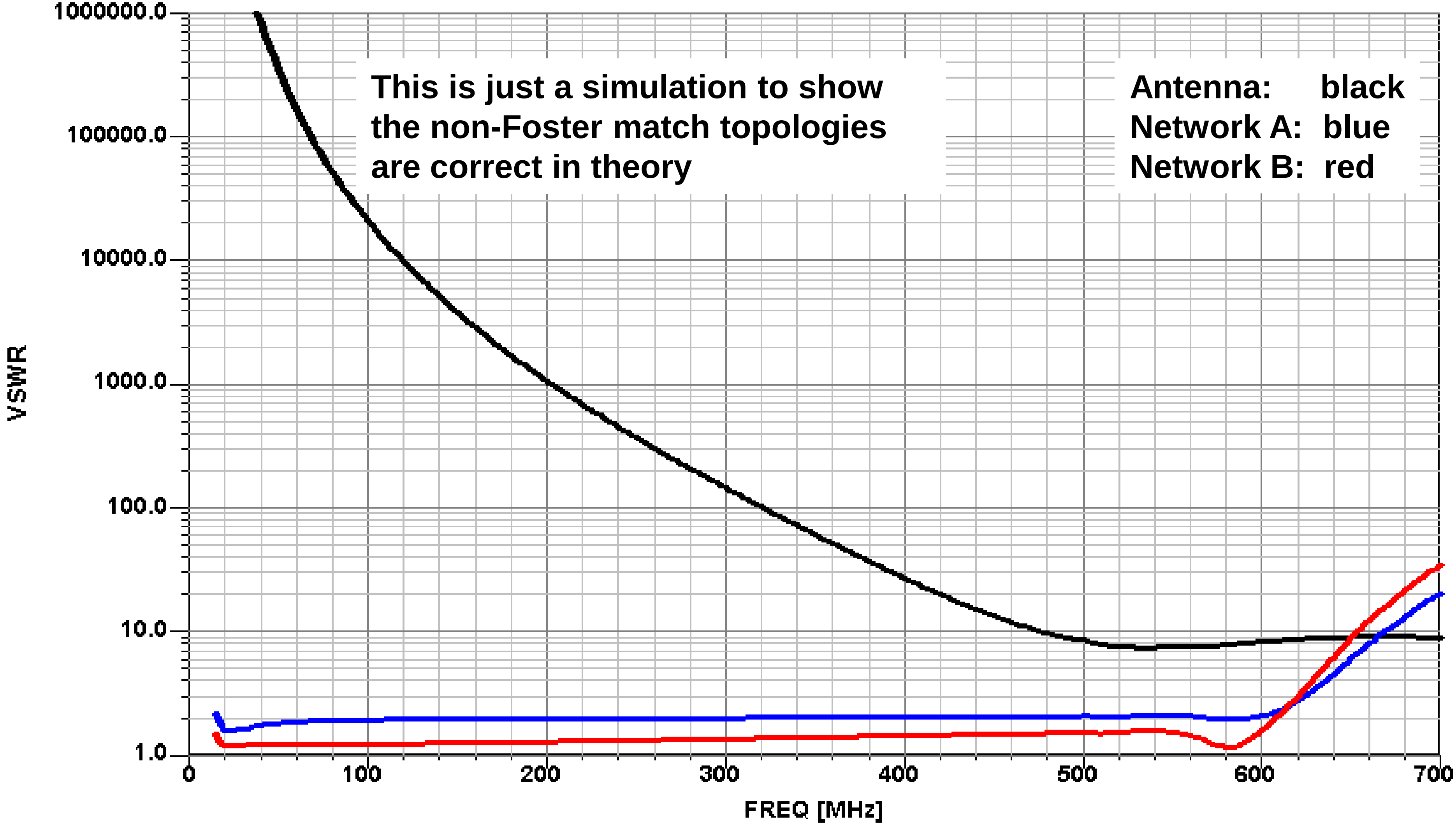


Non-Foster Match Networks for 15 to 600 MHz Band

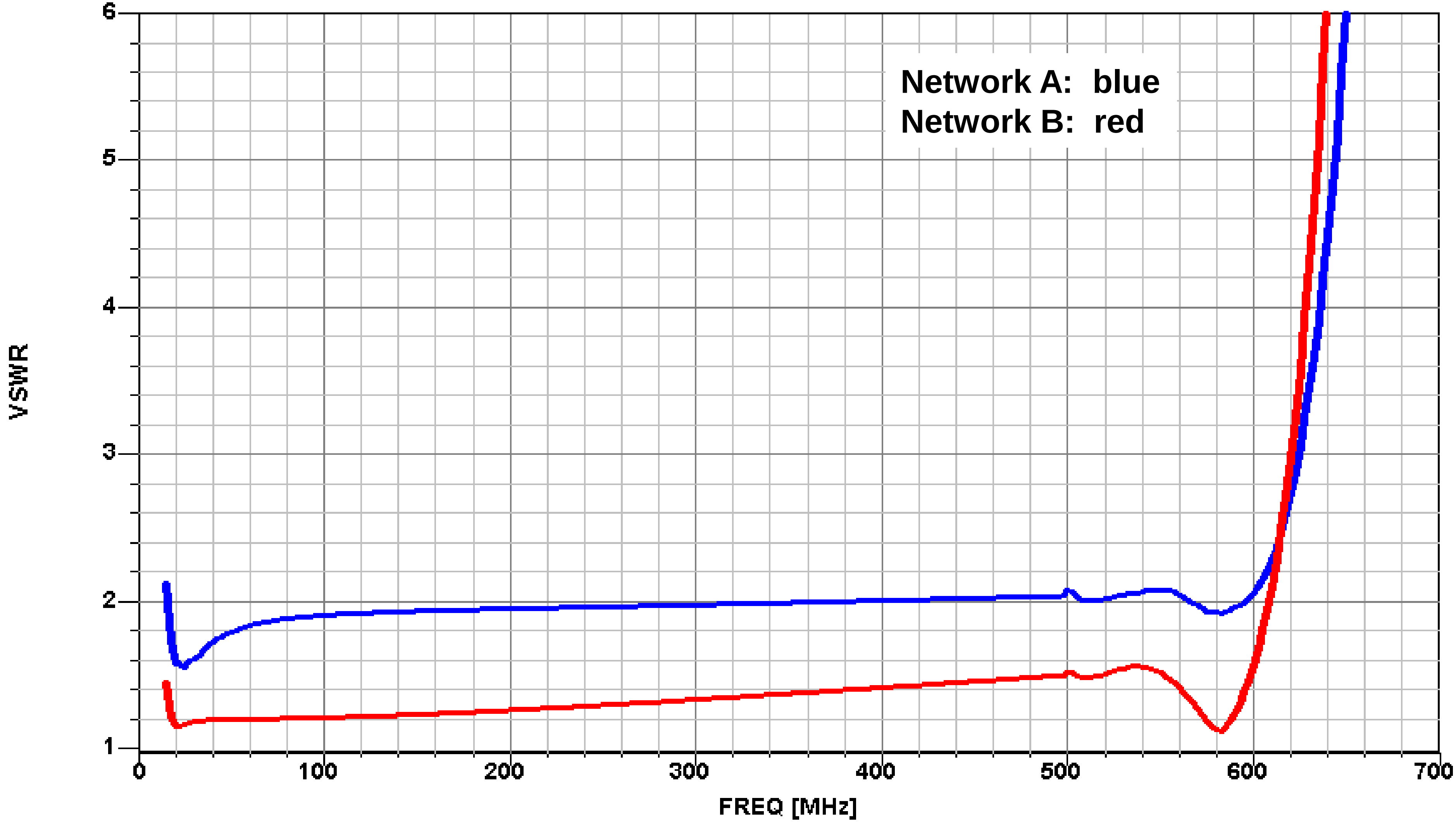




Match Results Using Non-Foster Networks A and B

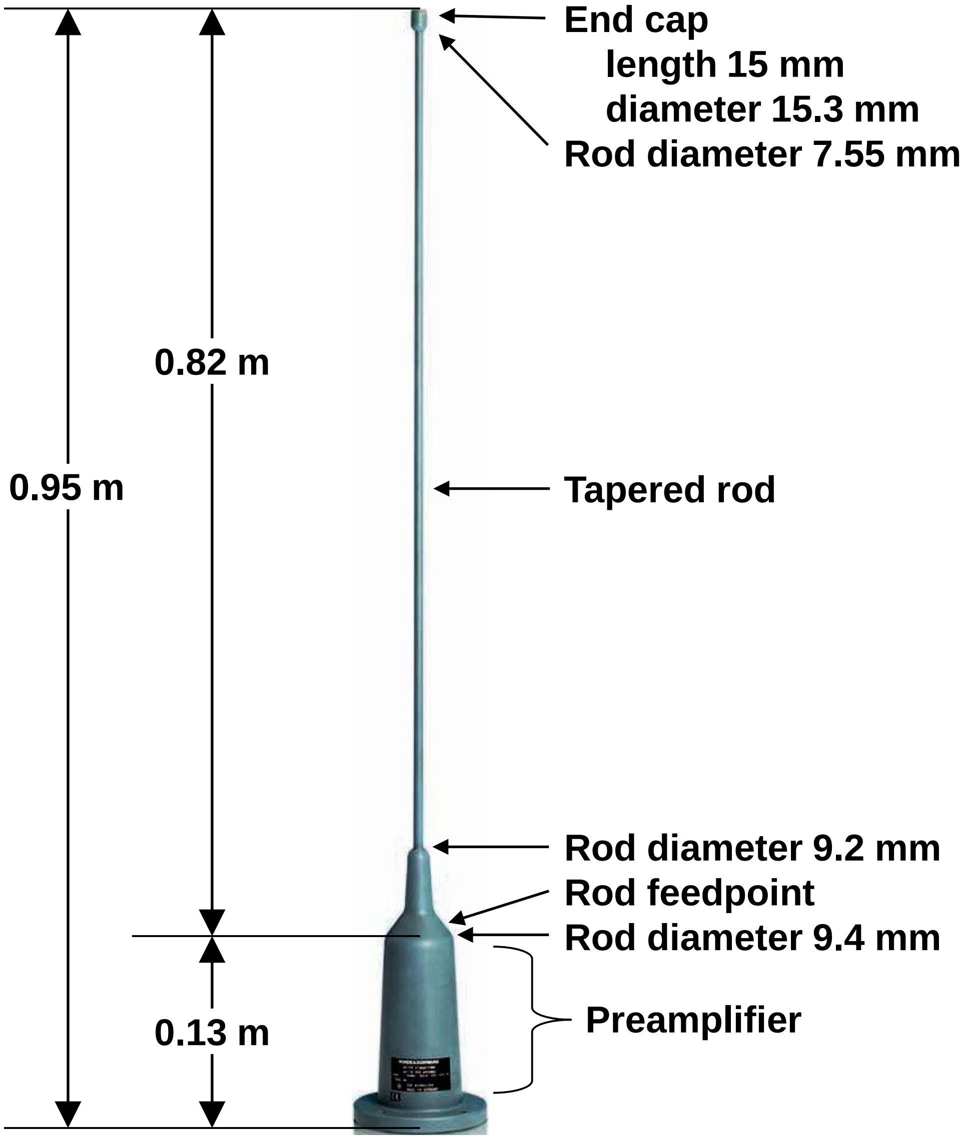


A Close-Up View: SWR After Matching

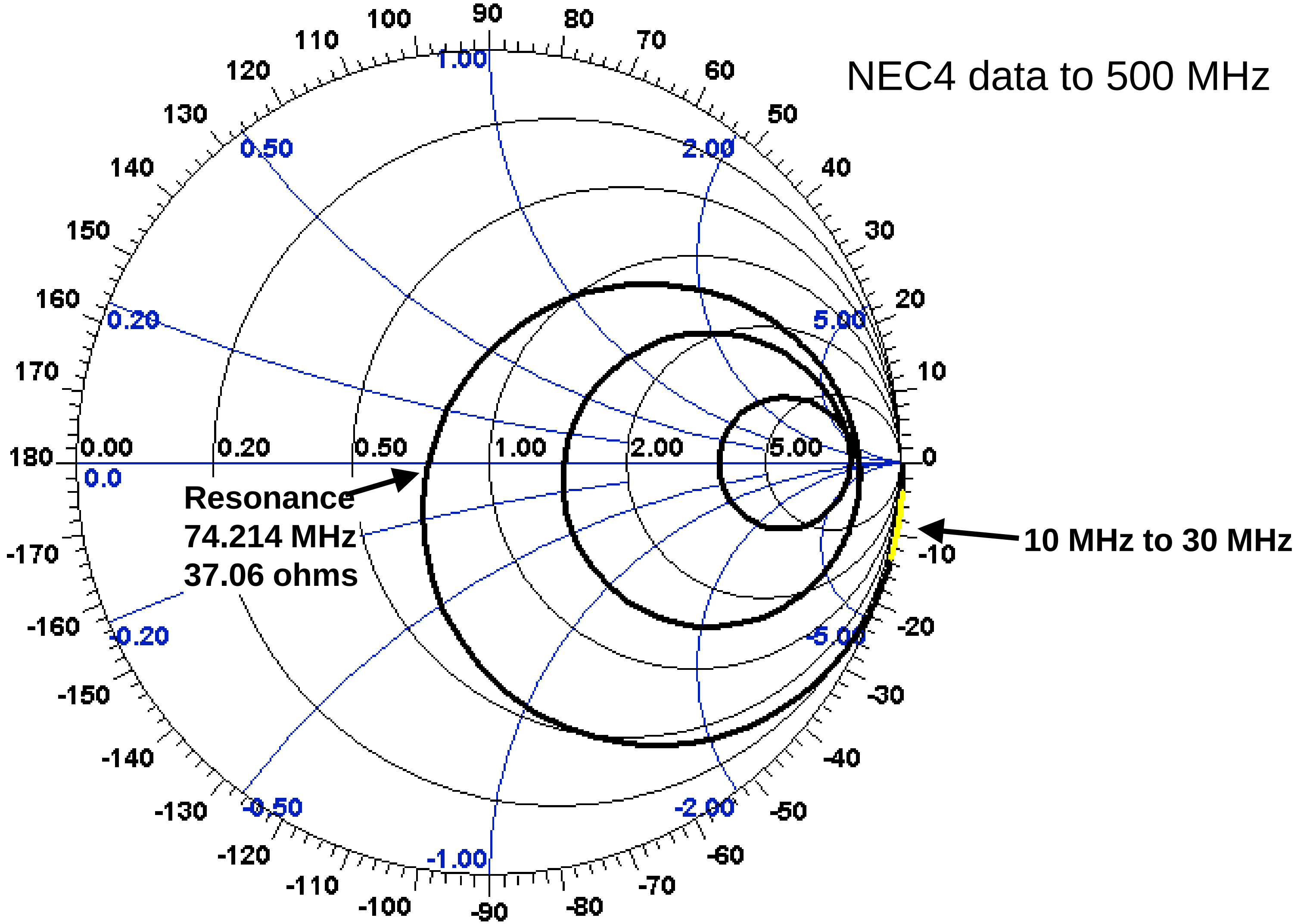


Stable Non-Foster Match Network Designs

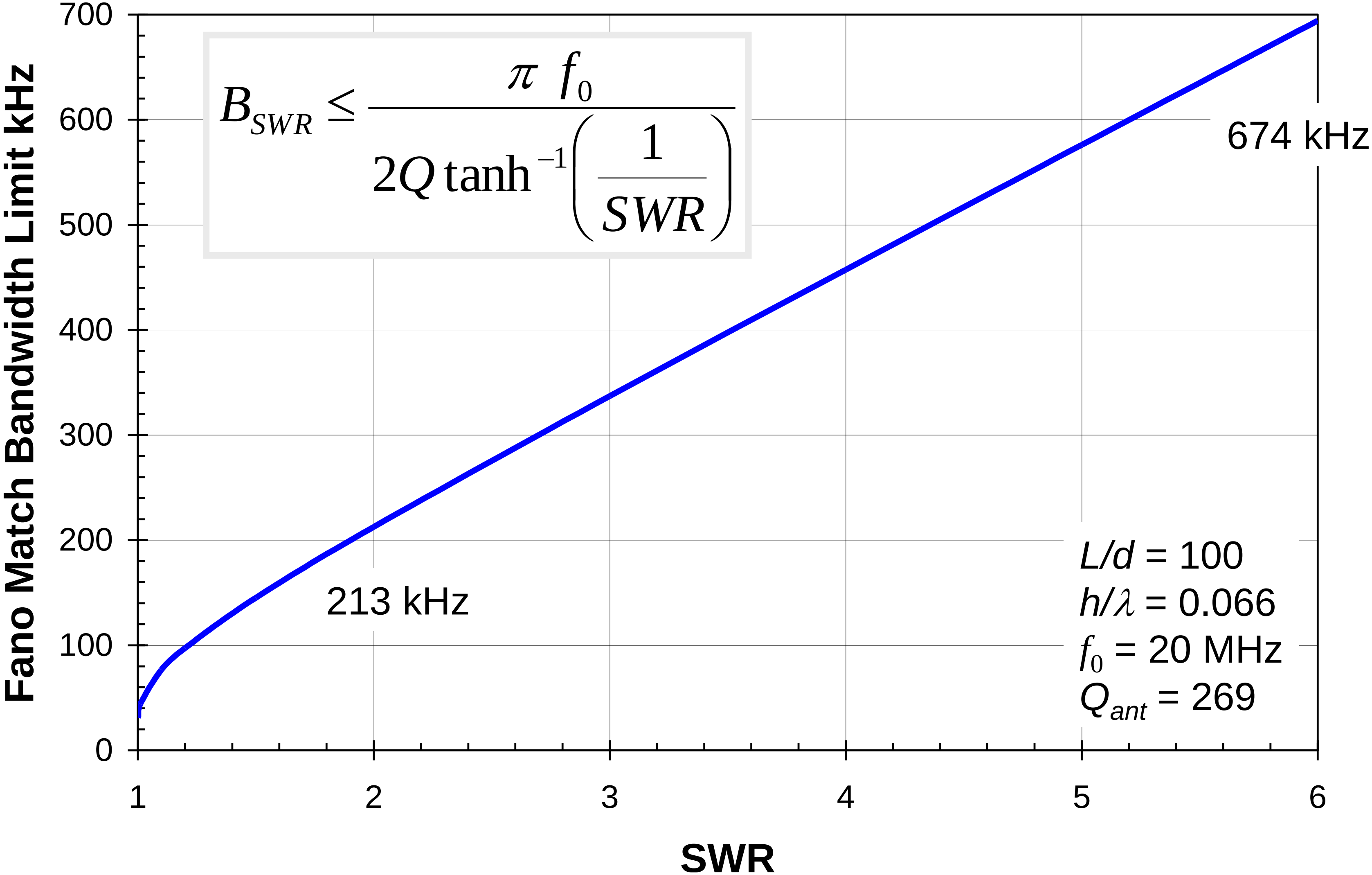
Rohde & Schwarz HE010 Active Monopole



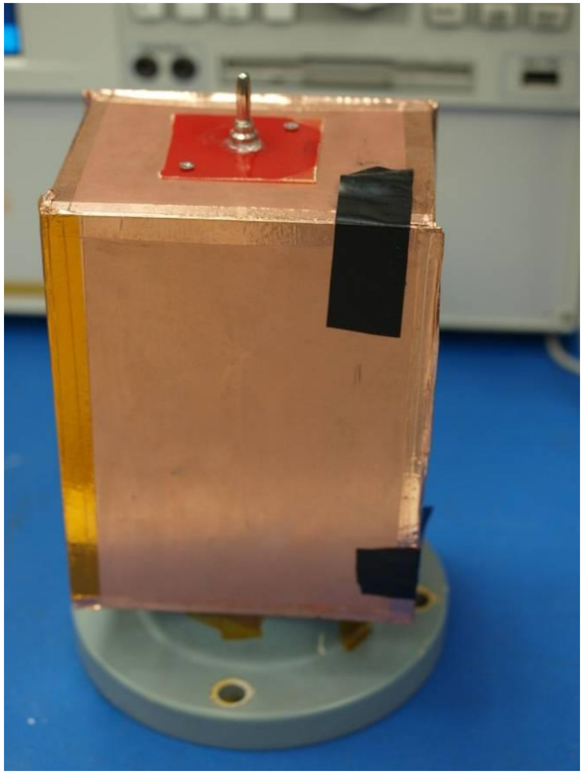
Computed Monopole Impedance

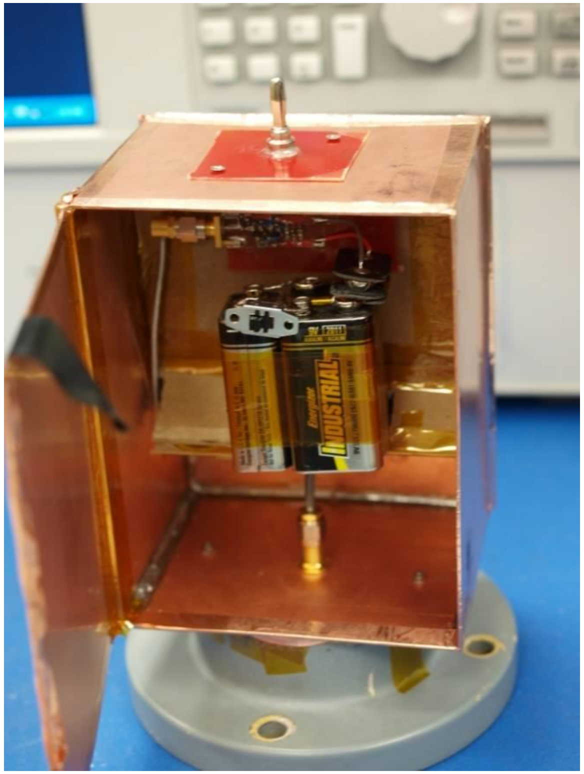


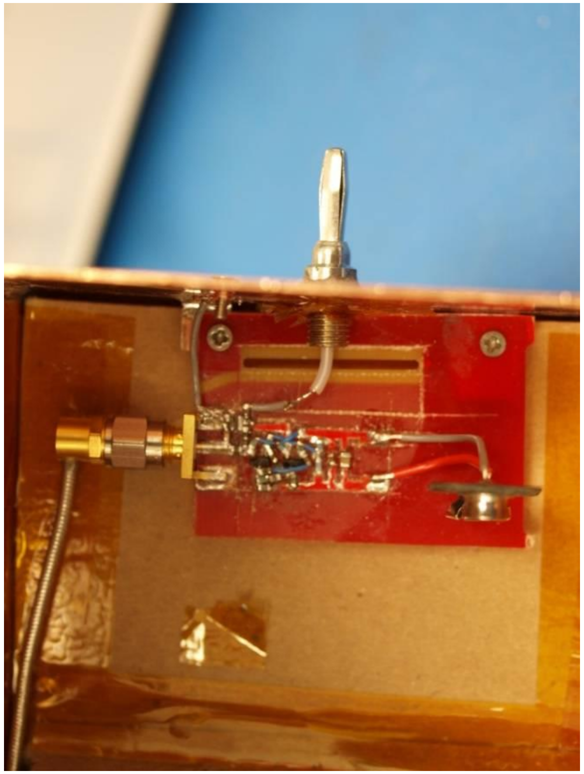
Fano Match Bandwidth Limit vs Match SWR



The Non-Foster Match Circuit





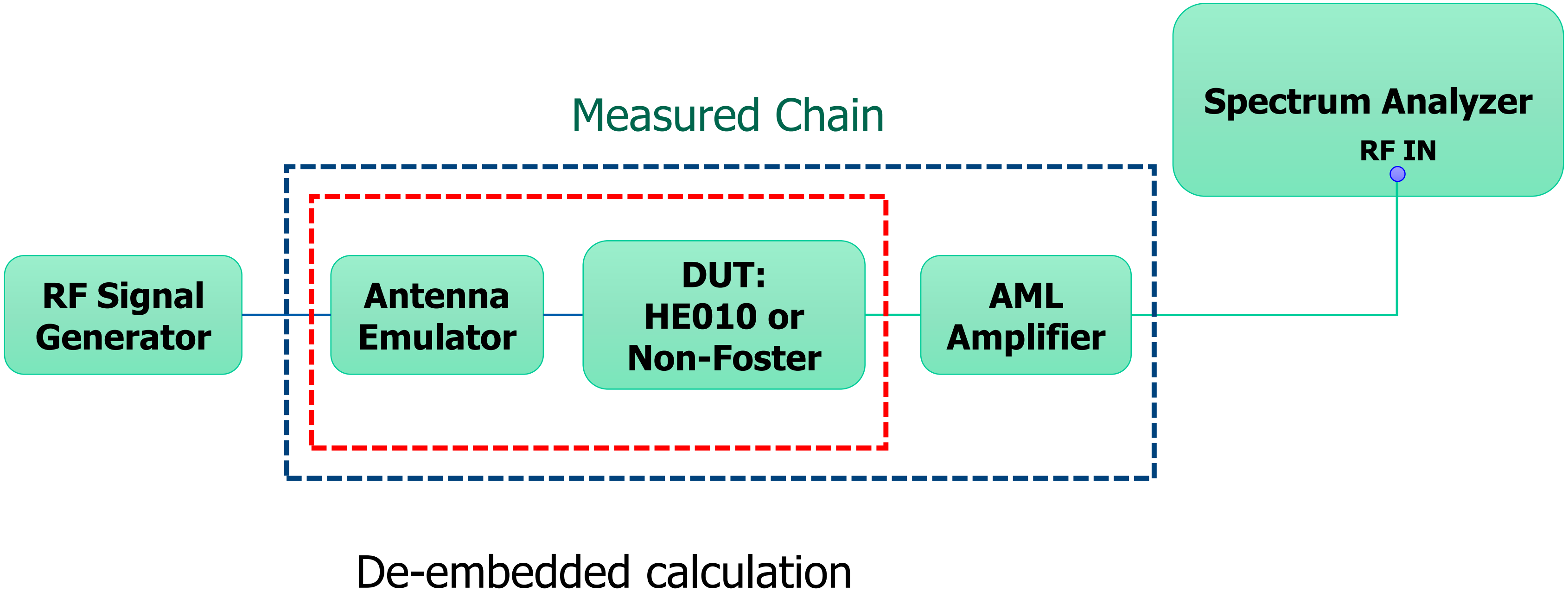


- Circuit topology has 3 elements
- A series negative capacitor
- Followed by a positive L network
- No transformers or “dualizers”

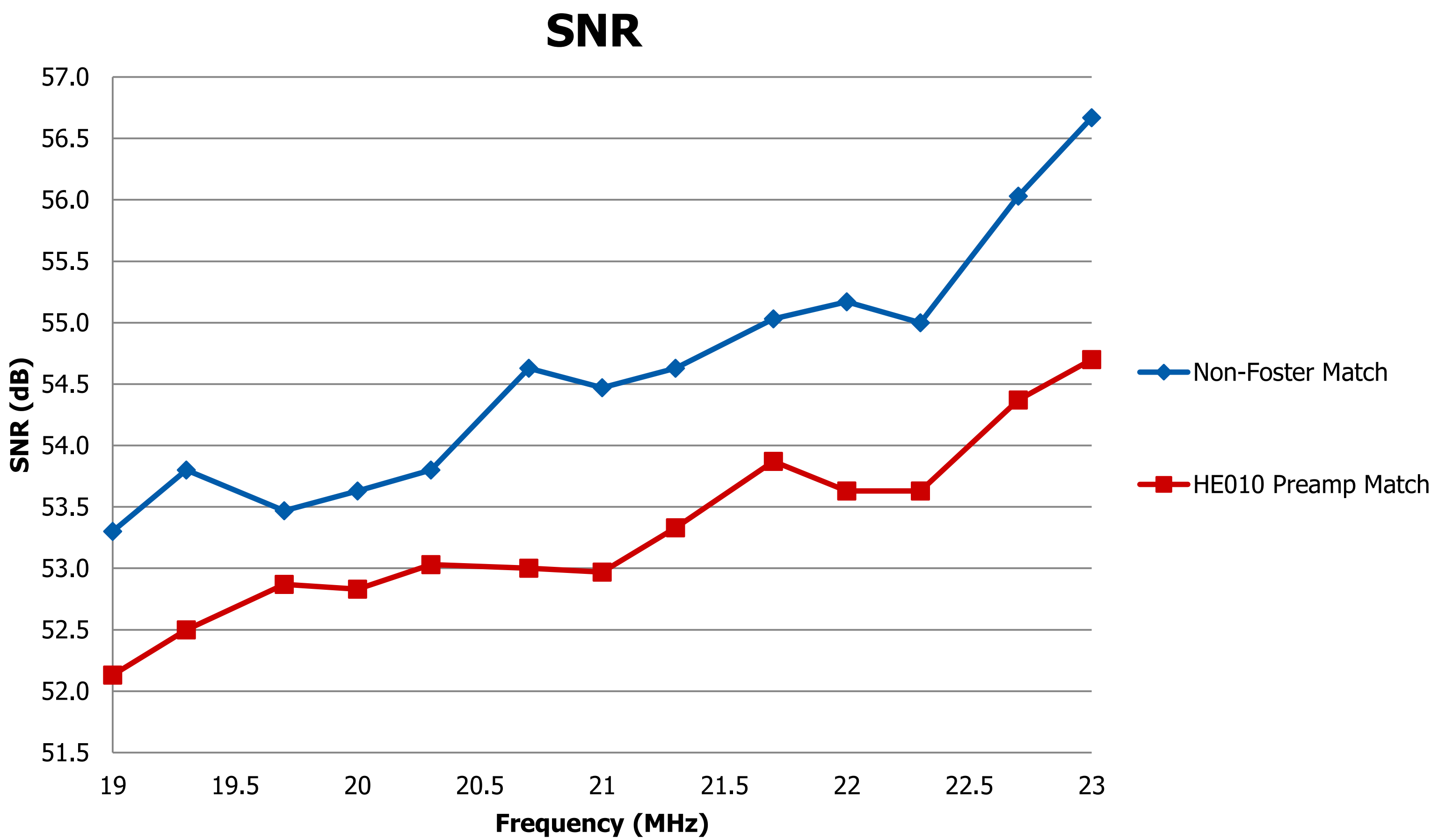
Receive Noise Tests

- Noise test compared RS HE010 active preamplifier against non-Foster negative capacitor match network when connected to antenna impedance emulator
- Noise tests performed by injecting tone from 50-ohm source into Port 2 of emulator and measuring SNR at RS preamp or non-Foster network output with a spectrum analyzer
- Frequency stepped from 19 MHz to 23 MHz
- Non-Foster network's 3-dB insertion bandwidth is 3.68 MHz (18.92 to 22.60 MHz)
- Non-Foster match bandwidth exceeds the maximum theoretical Fano match bandwidth for matching this antenna using passive lossless networks

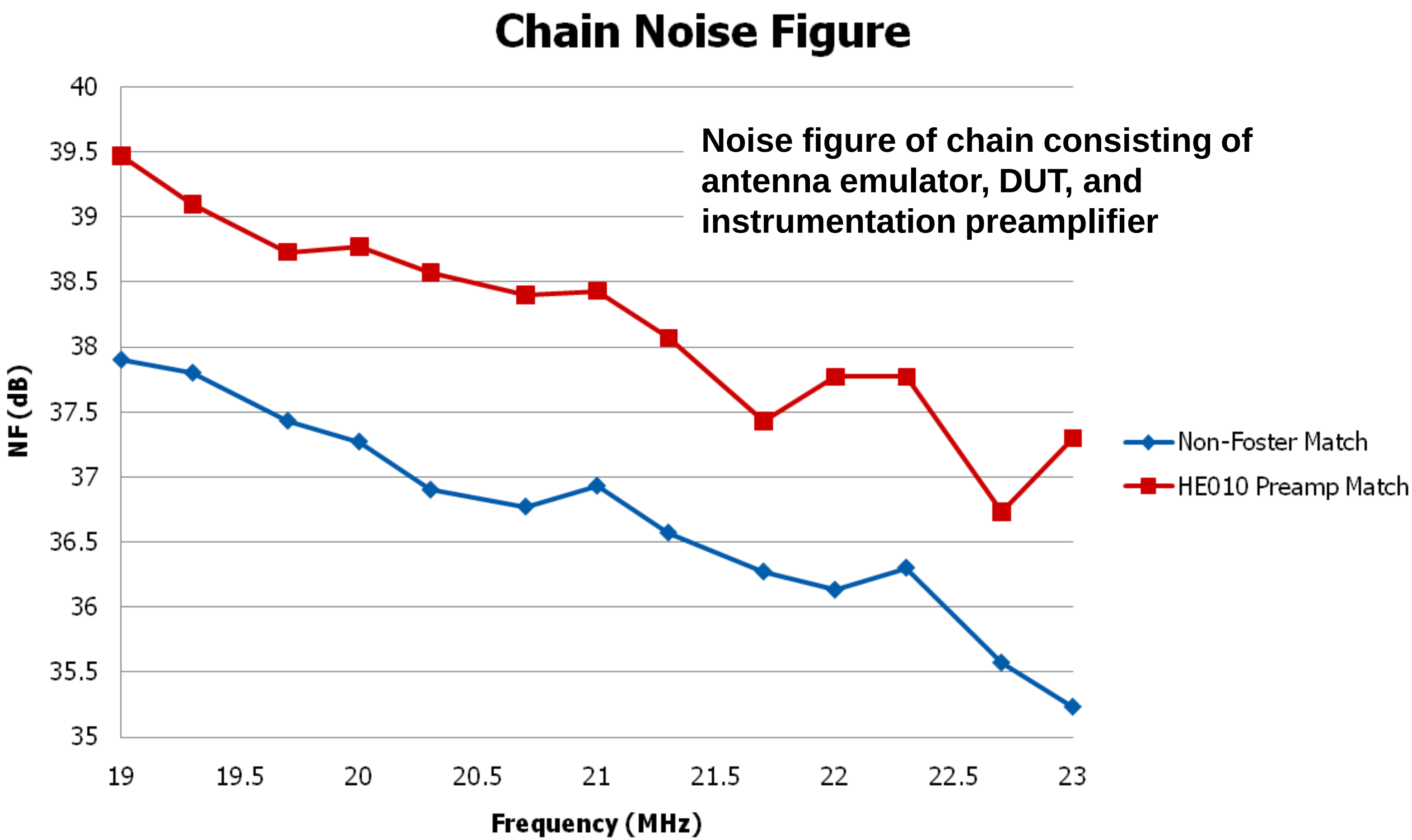
Measurement Setup



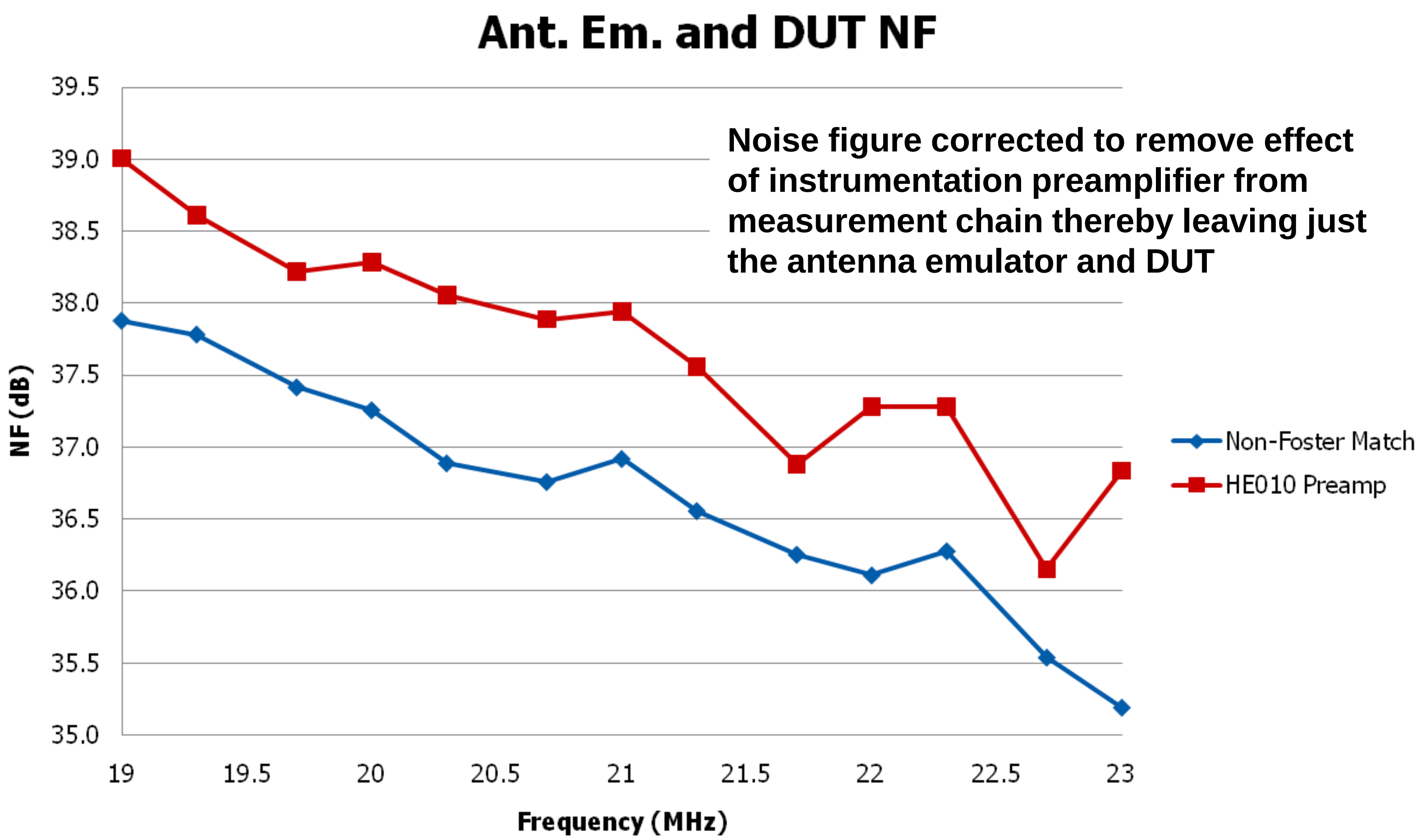
Output SNR Comparison



Noise Figure Comparison

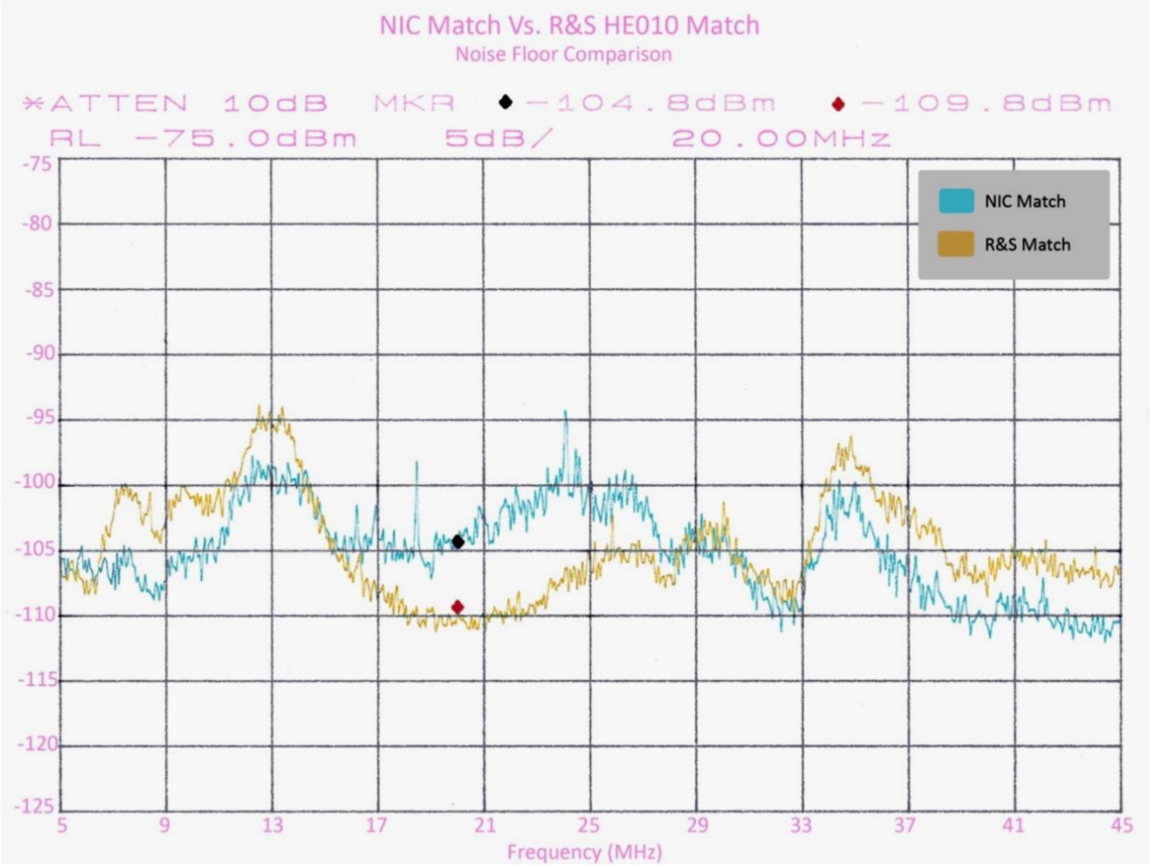


De-Embedded Noise Figure



Noise figure corrected to remove effect of instrumentation preamplifier from measurement chain thereby leaving just the antenna emulator and DUT

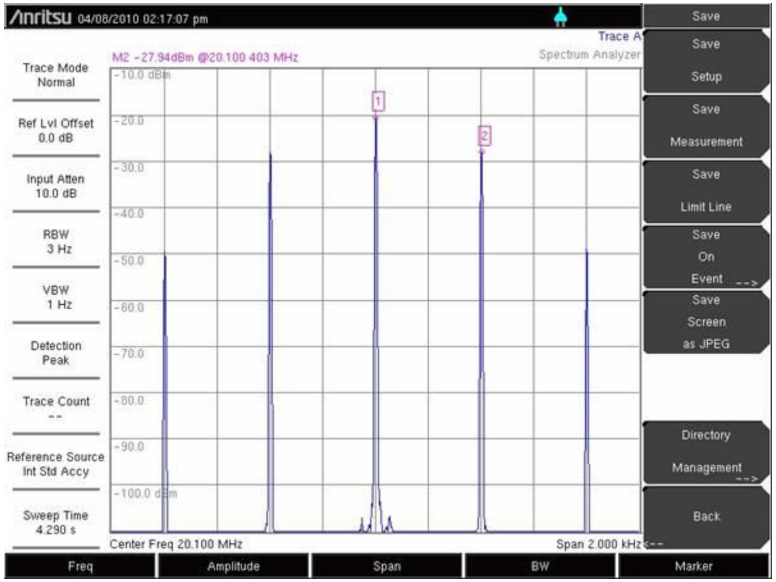
Receive Noise Comparison



- **The pass band of each match circuit is evident from its noise floor**
- **Non-Foster network exceeds R&S preamplifier from 15 to 28 MHz**

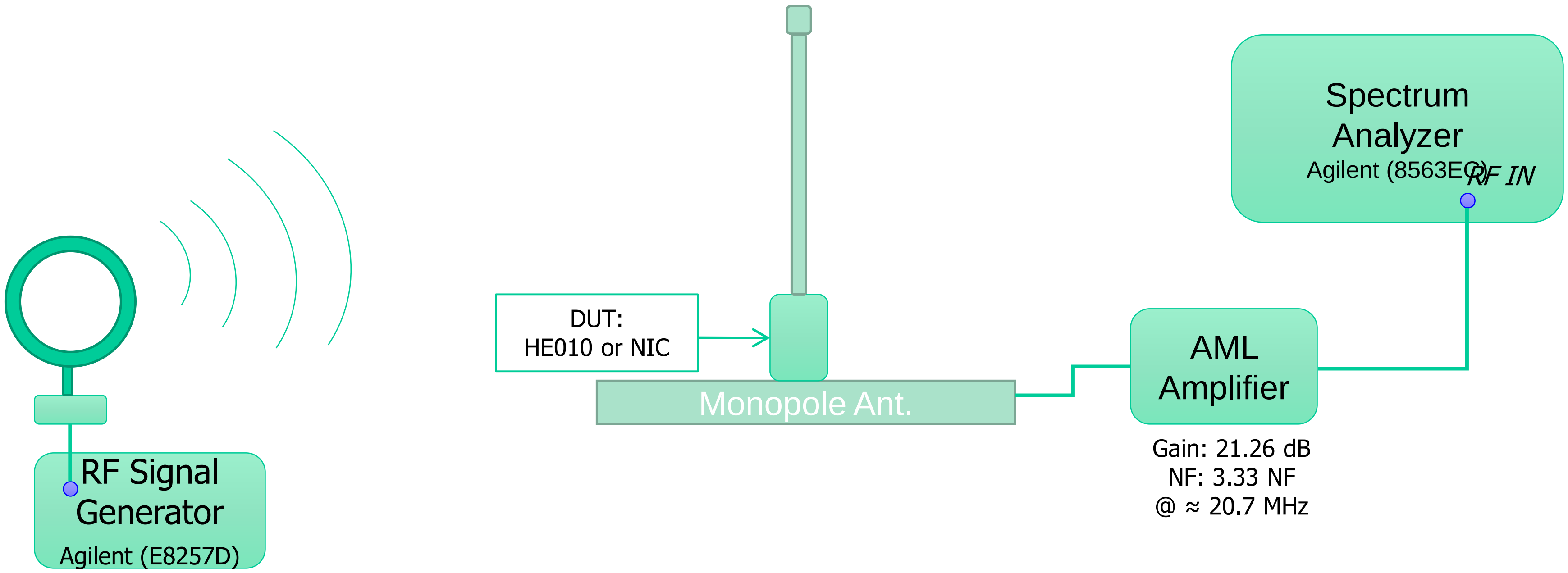
AM Modulation Test 2

- **Test signal from signal generator**
  - Carrier: 20.1 MHz
  - Modulation: AM 80%
  - Modulation signal: 400 Hz sine wave
  - Signal generator: Agilent E8257D
    - Some harmonics of the modulation tone are evident but do not affect matching circuit performance measurements
  - Signal will be transmitted through a loop antenna and received by the monopole with matching circuit

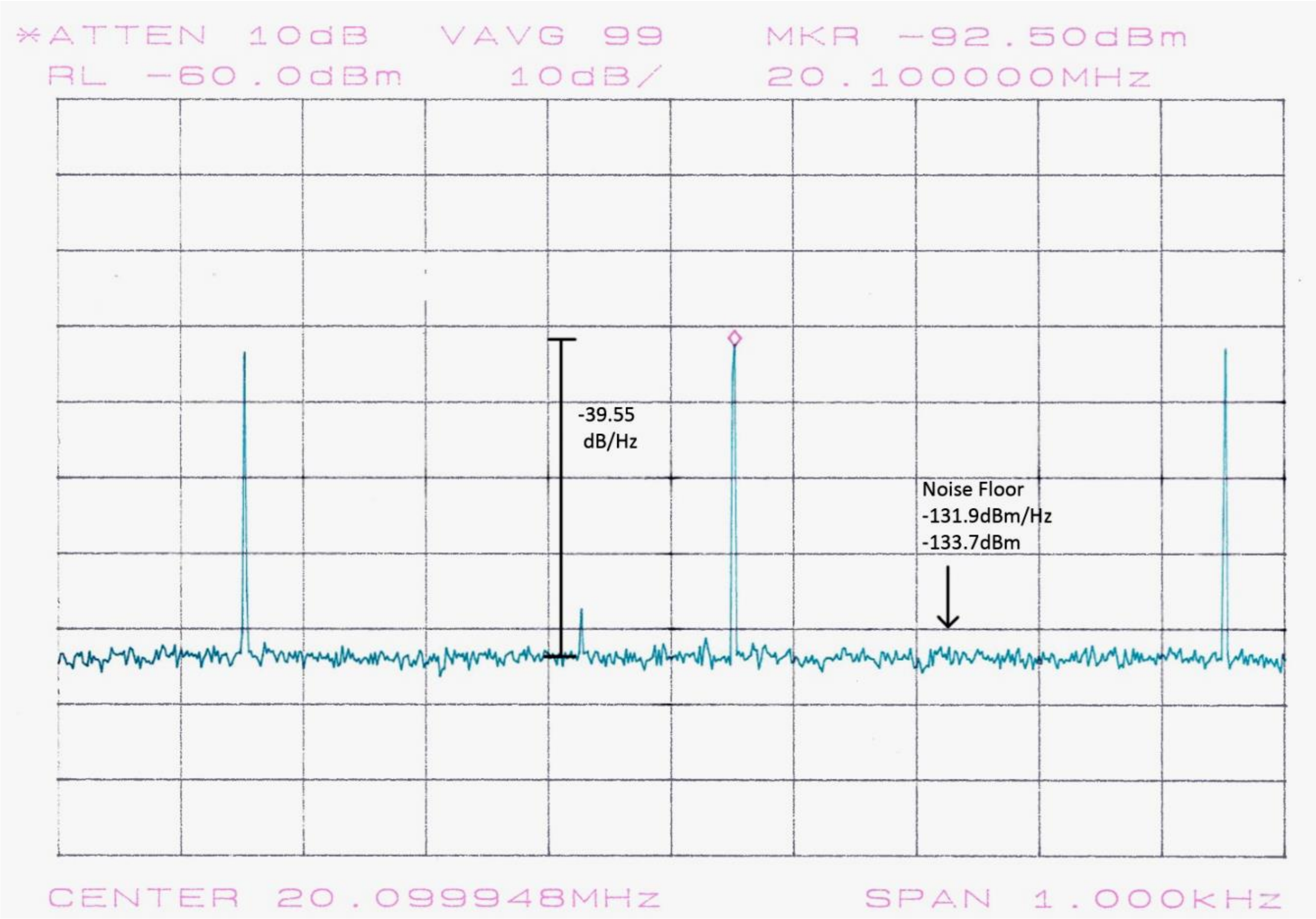


Test Setup

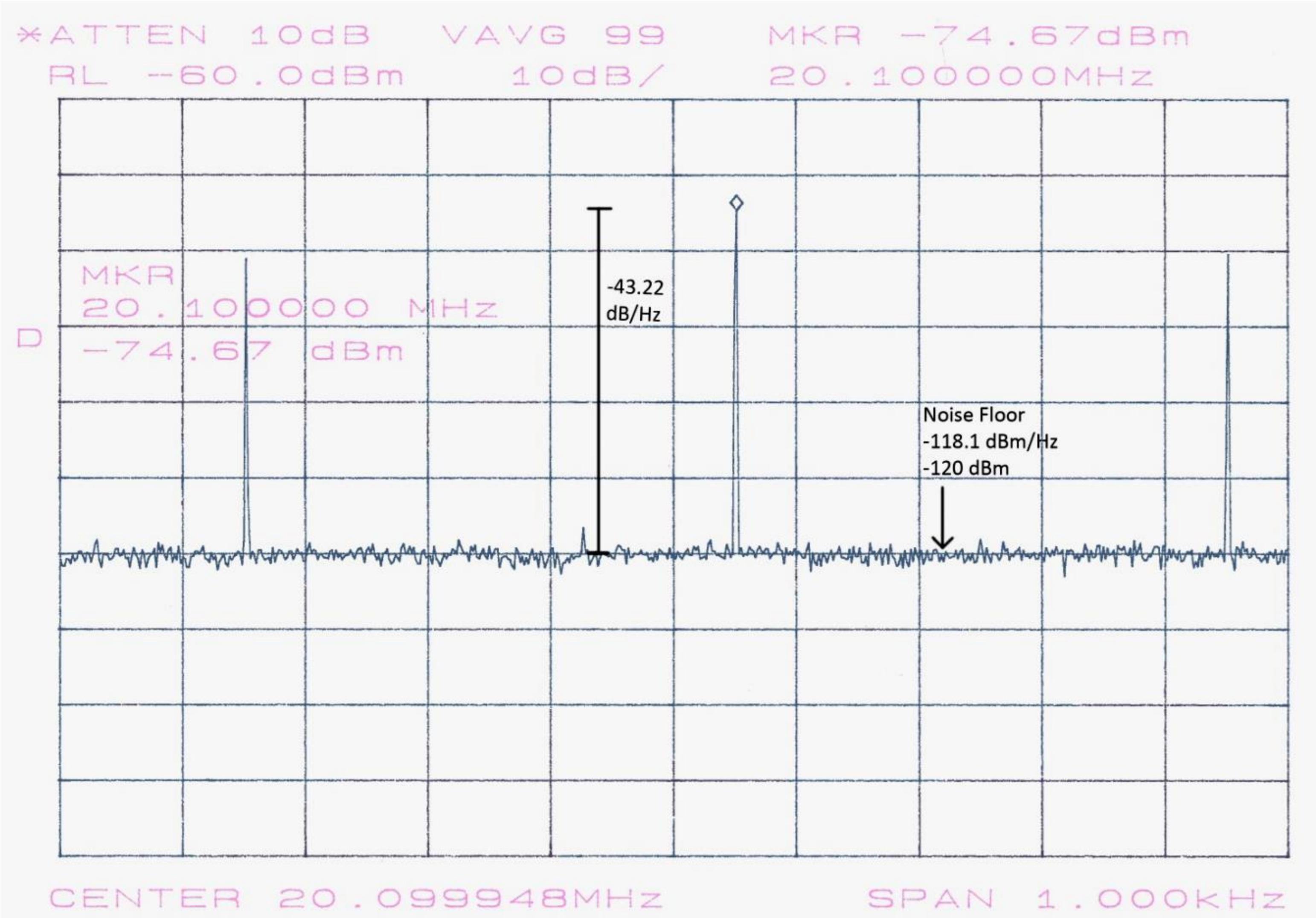
- The noise floor of R&S HE010 preamplifier is below or close to the noise floor of the spectrum analyzer
- An amplifier was inserted in the receive chain to raise the noise floor (same as for tests with Antenna Emulator).



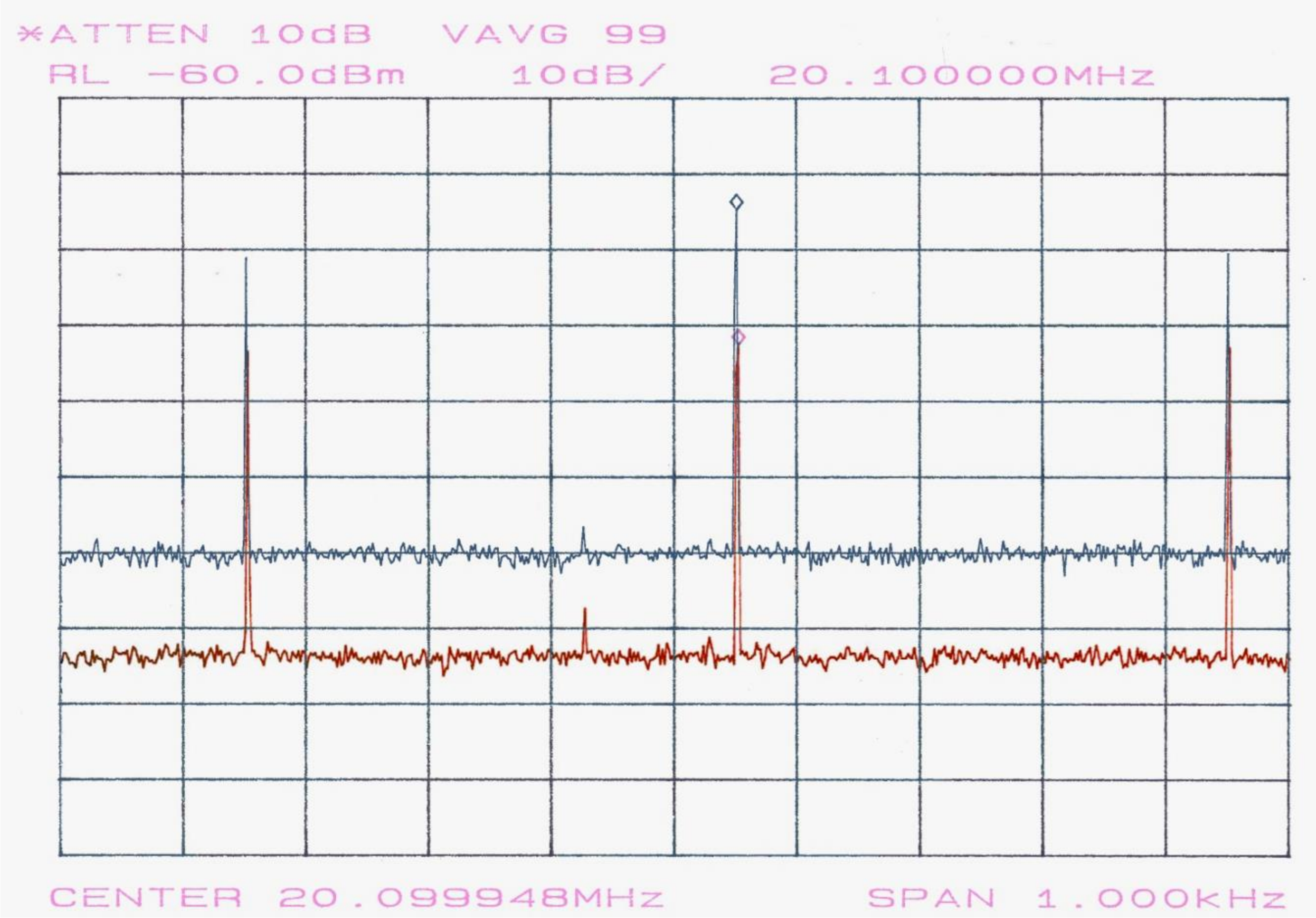
R&S HE010 Preamplifier



Non-Foster Match Network



Comparison

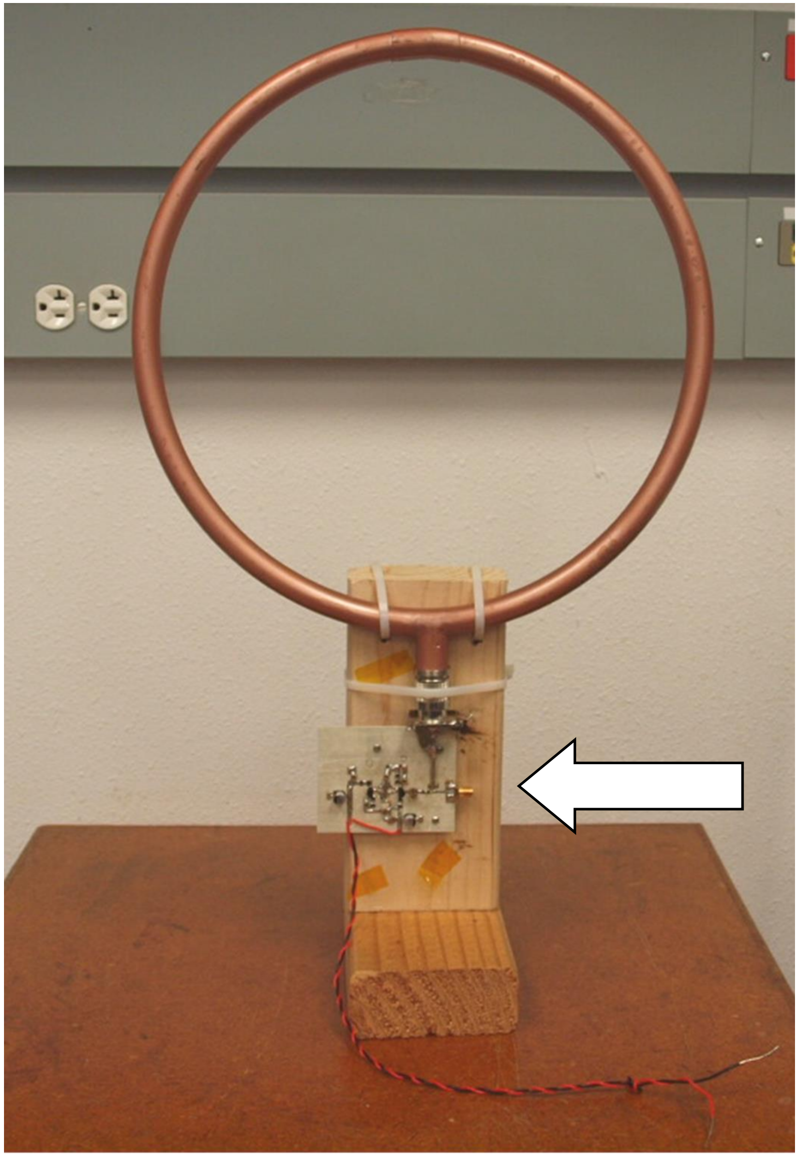


*ATTEN 10dB VAVG 99
RL -60.0dBm 10dB/ 20.100000MHz
CENTER 20.099948MHz SPAN 1.000kHz

Divergent Findings on Noise of Non-Foster Match Circuits

- **Sussman-Fort et al. (2009):**
  - "With receivers, we have experimentally confirmed, on the antenna range, order-of-magnitude improvement in signal to-noise ratio as compared to the best passive match."
- **Stearns (measurements, 2009):**
  - "Non-Foster match gives greater SNR than the R&S preamplifier by a few dB."
- **Best (Allerton AAS, 2012):**
  - "Non-Foster matching offers significant NF improvement. Knowledge of Non-Foster NF is critical in system NF analysis and antenna optimization."
- **Minu et al. (APS-URSI, 2013):**
  - "While the current noise figure is high, we hope to improve this circuit to achieve improved SNR in received signals."
- **Stedler et al. (ISAP, 2014):**
  - "… a NIC circuit as matching element for short active car antennas would degrade the overall SNR performance, compared to a conventional active antenna system... A disadvantage of 6-8 dB in SNR of the NIC-circuit results in comparison with a conventional active antenna."
- **Nagarkoti et al. (EuCAP, 2015):**
  - "The SNR improvement across the frequency band of 100 MHz to 500 MHz is significant except at certain frequencies as shown in Fig. 6… Up to 8 dB SNR improvement has been observed with the non-Foster matching across the frequency band of interest."

Electrically Small Loop Antenna

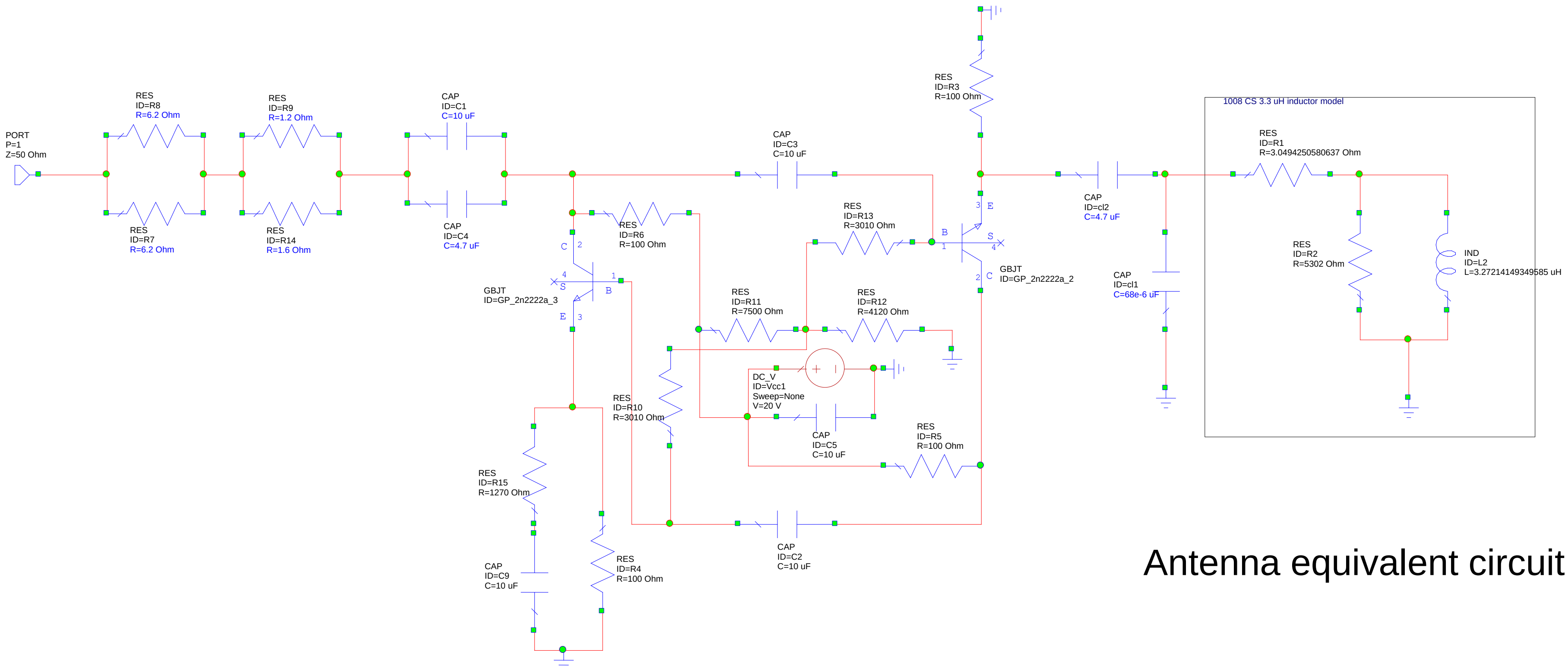


A.H. Systems SAS-563B active shielded loop antenna, 1-ft diameter

Design frequency band: 100 kHz to 200 kHz

Manufacturer’s preamplifier removed and replaced with non-Foster match circuit

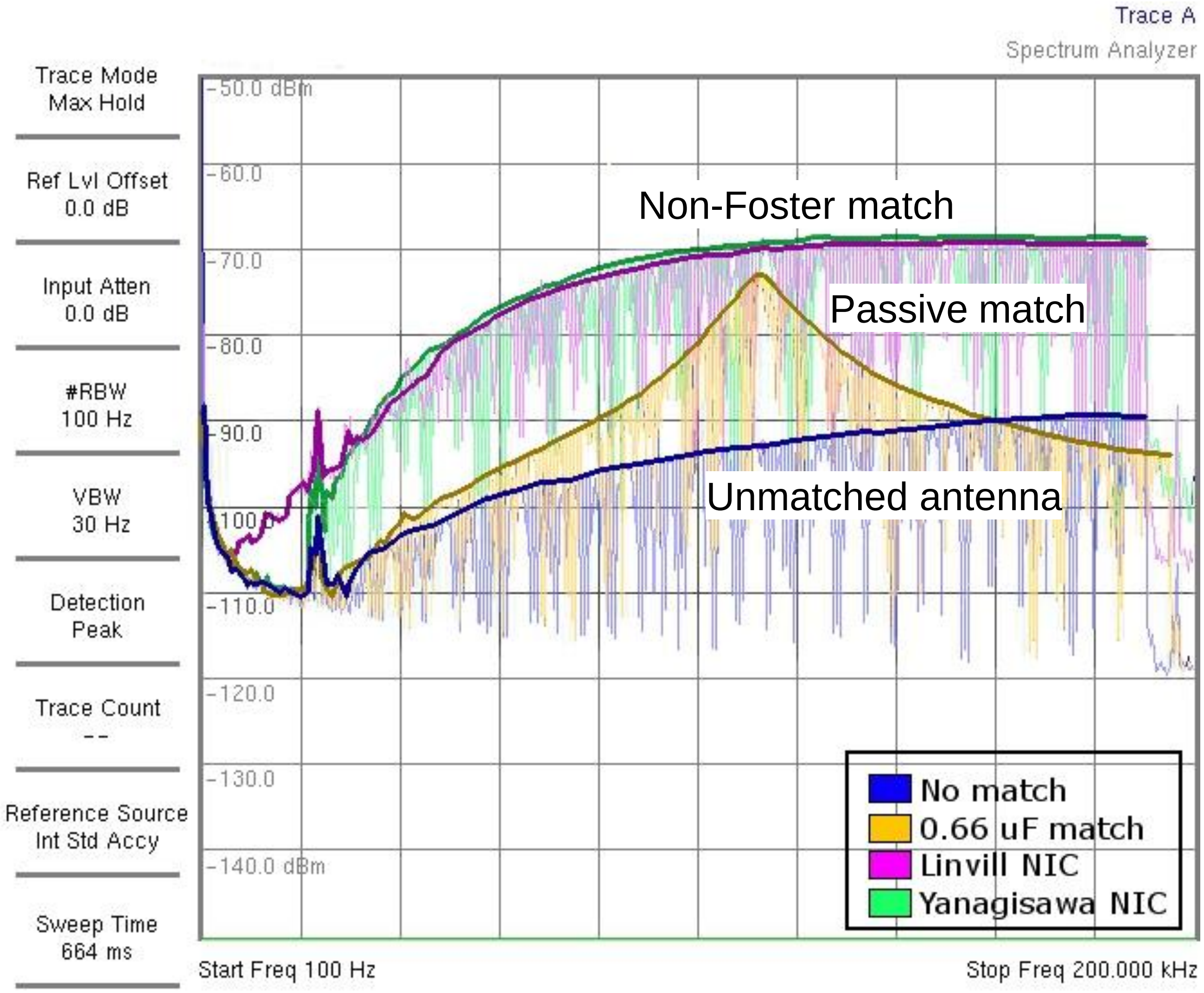
Non-Foster Match Circuit



Antenna equivalent circuit

Negative inductor realized by Yanagisawa NIC

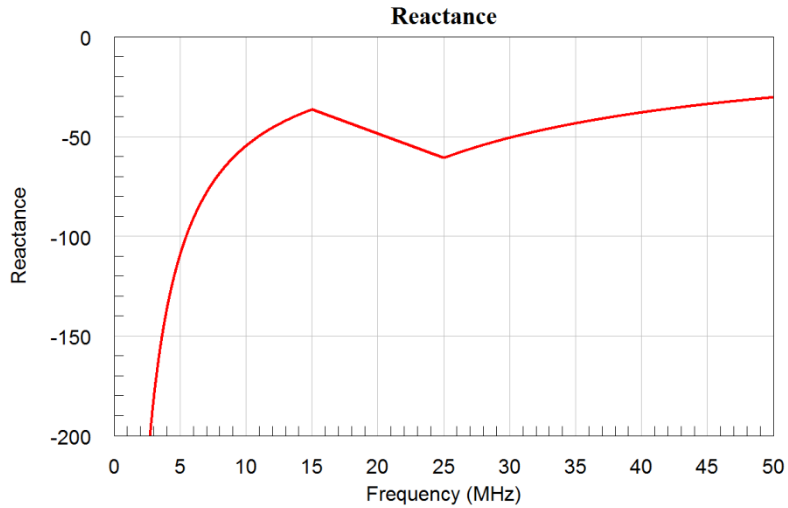
Antenna Response to White Input Noise



Stability of Non-Foster Circuits

- **Stearns (2010) showed among circuits made of just *R*, ±*L*, ±*C*, the unstable circuits form a dense subset – analogous to irrational numbers compared to real numbers**
- **Stearns (2011, 2012) showed counterexample circuits for which unconditional stability 2-port tests always report wrongly on stability and opposite on unconditional stability**
  - Moreover, no improvement to Rollett's criteria or proviso can fix the problem
- **Literature on 2-port unconditional stability is defective**
- **Only stability tests that consider full internal description of a circuit can report stability correctly**
- **Nonlinear stability is also important**
- **Three forms of instability have been observed**
  - Oscillation
  - Lock up
  - Chaos
- **S. Hrabar (2015) showed ideal negative capacitors and inductors are fundamentally flawed concepts and will always be unstable in physical systems**
  - These elements should be avoided in active circuit theory
- **Stearns (2015) showed alternative new elements that solve the theoretical problem and lead to provably stable non-Foster circuits**
  - Ideal negative capacitors → Bandpass negative capacitors
  - Ideal negative inductors → Bandpass negative inductors

Bandpass Negative Inductor



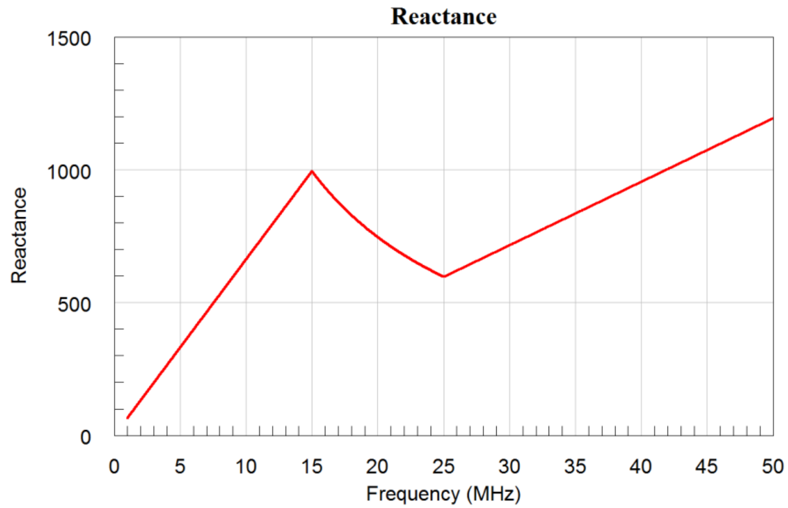
Impedance is continuous function of ω and Δ

$$Z(\omega) = 0 + jX(\omega) \quad \text{with} \quad X(\omega) = \begin{cases} \dfrac{-1}{\omega C_1} & 0 < \omega < \omega_0 - \Delta \\ \omega L & \omega_0 - \Delta < \omega < \omega_0 + \Delta \\ \dfrac{-1}{\omega C_2} & \omega_0 + \Delta < \omega < \infty \end{cases}$$

where

$$C_1 = \frac{1}{(\omega_0 - \Delta)^2 (-L)} \quad \text{and} \quad C_2 = \frac{1}{(\omega_0 + \Delta)^2 (-L)}$$

Bandpass Negative Capacitor



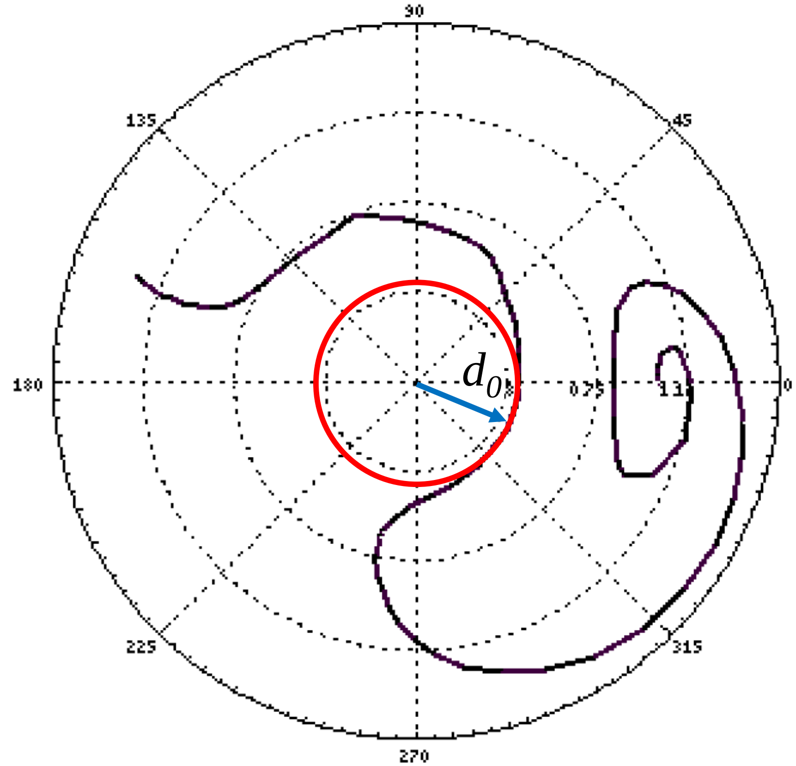
Impedance is continuous function of ω and Δ

$$Z(\omega) = 0 + jX(\omega) \quad \text{with} \quad X(\omega) = \begin{cases} \omega L_1 & 0 < \omega < \omega_0 - \Delta \\ \dfrac{-1}{\omega C} & \omega_0 - \Delta < \omega < \omega_0 + \Delta \\ \omega L_2 & \omega_0 + \Delta < \omega < \infty \end{cases}$$

where

$$L_1 = \frac{1}{(\omega_0 - \Delta)^2 (-C)} \quad \text{and} \quad L_2 = \frac{1}{(\omega_0 + \Delta)^2 (-C)}$$

Stable Non-Foster Bandwidth Exists



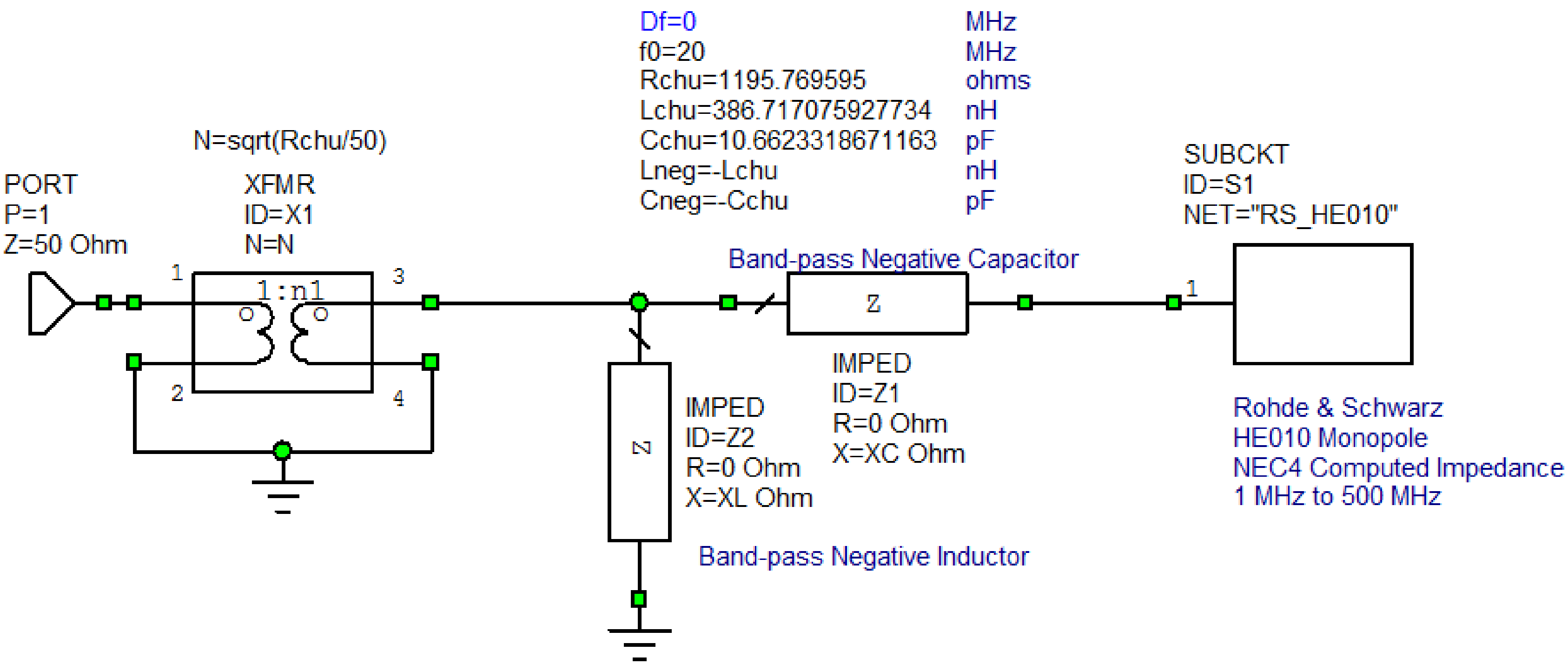
Ends of curve set by positive inductance and capacitance values

- **By continuity, there exists a $\delta > 0$ such that if $0 < \Delta < \delta$, then $0 < |d(\Delta) - d_0| < d_0$**
- **The circuit is stable and exhibits desired non-Foster behavior over bandwidth $2\Delta = 2\delta$**

Pause for demonstration

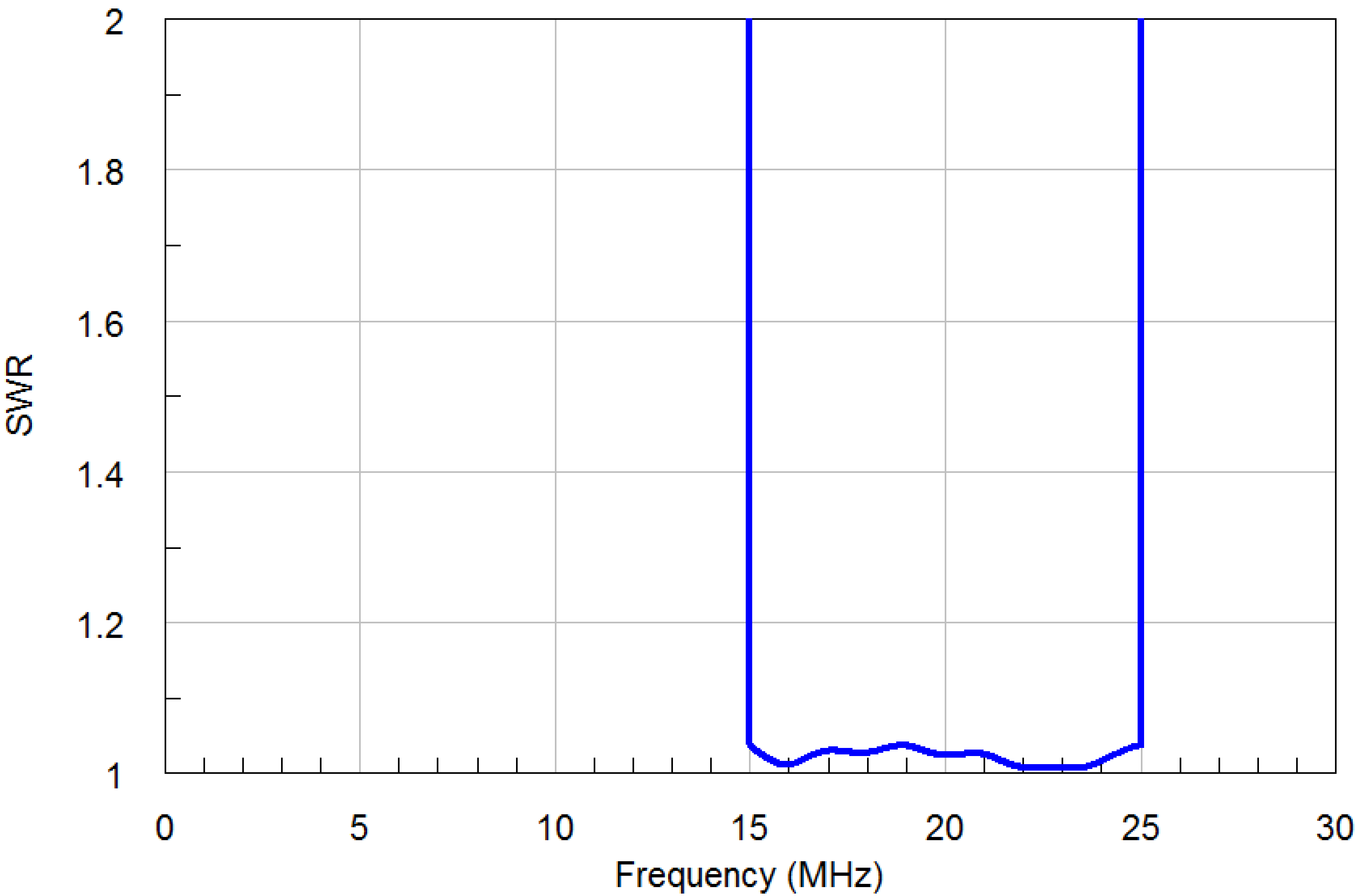
Microwave Office simulation demonstrating a “bandpass non-Foster” match to a dipole antenna and stability according to a Nyquist criterion

Bandpass Non-Foster L Match Network

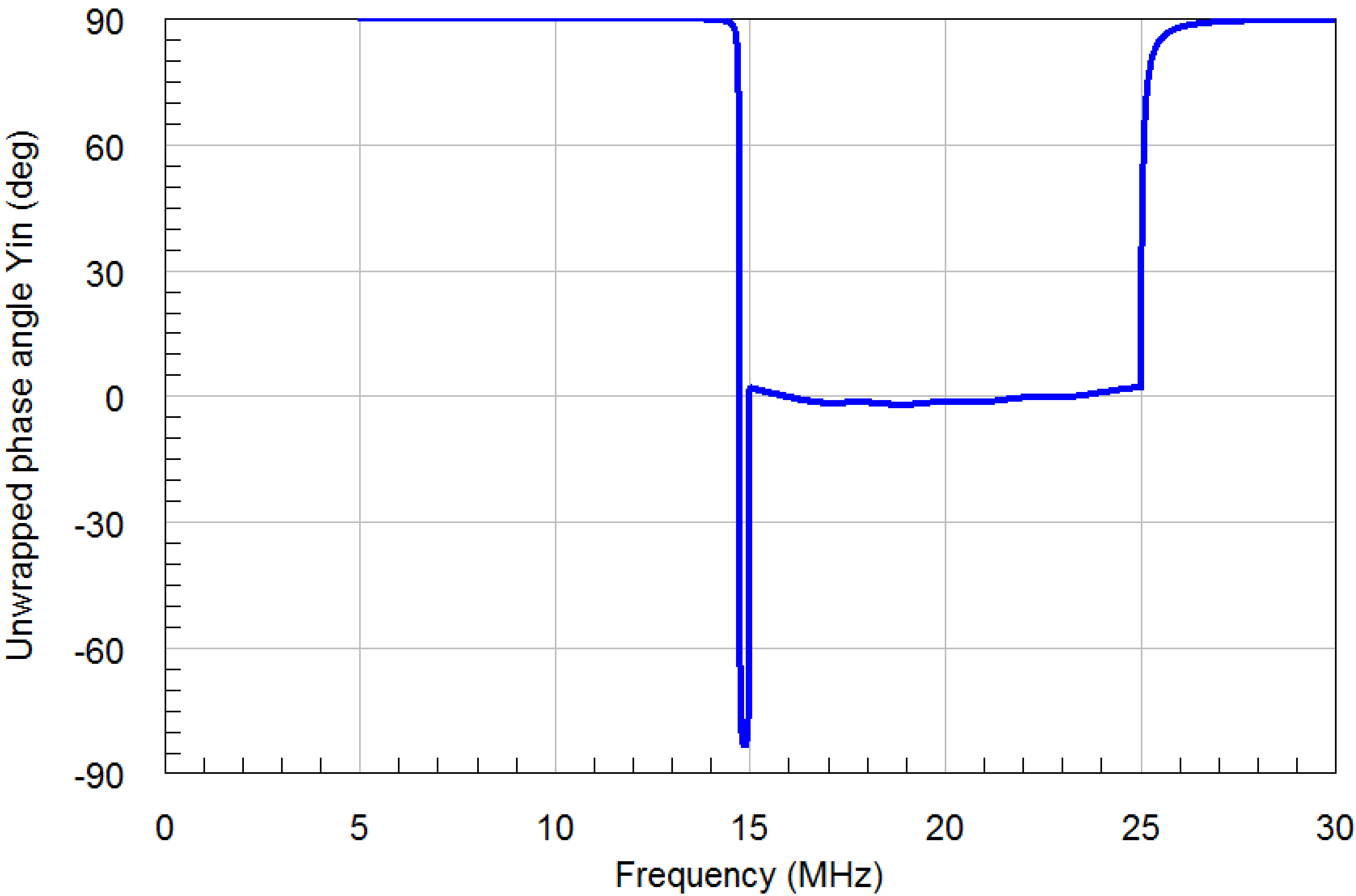


C1=1/((2*_PI*(f0-Df)*1e6)^2*(-Lneg*1e-9))*1e12 pF
C2=1/((2*_PI*(f0+Df)*1e6)^2*(-Lneg*1e-9))*1e12 pF
L1=1/((2*_PI*(f0-Df)*1e6)^2*(-Cneg*1e-12))*1e9 nH
L2=1/((2*_PI*(f0+Df)*1e6)^2*(-Cneg*1e-12))*1e9 nH
XLa=(-1/(2*_PI*_FREQ*C1*1e-12))*(1 - heaviside(_FREQ-(f0-Df)*1e6))
XLb=2*_PI*_FREQ*Lneg*1e-9*(heaviside(_FREQ-(f0-Df)*1e6) - heaviside(_FREQ-(f0+Df)*1e6))
XLc=(-1/(2*_PI*_FREQ*C2*1e-12))*(heaviside(_FREQ-(f0+Df)*1e6))
XL=XLa + XLb + XLc
XCa=2*_PI*_FREQ*L1*1e-9*(1 - heaviside(_FREQ-(f0-Df)*1e6))
XCb=(-1/(2*_PI*_FREQ*Cneg*1e-12))*(heaviside(_FREQ-(f0-Df)*1e6) - heaviside(_FREQ-(f0+Df)*1e6))
XCc=2*_PI*_FREQ*L2*1e-9*(heaviside(_FREQ-(f0+Df)*1e6))
XC=XCa + XCb + XCc

SWR of Bandpass Non-Foster Match



Unwrapped Phase Angle of Admittance Determinant



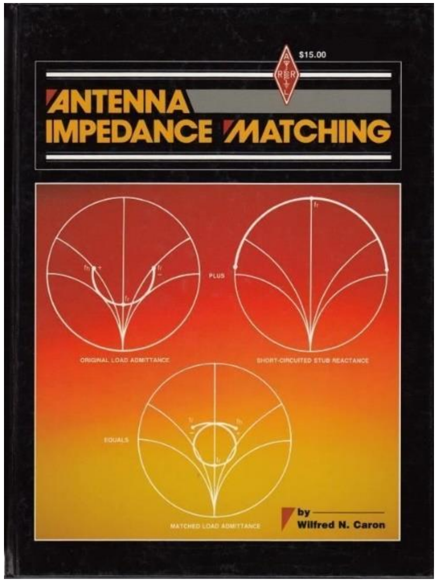
Conclusions

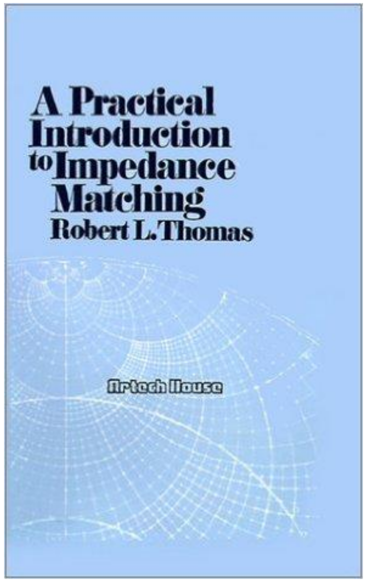
- **The set of networks exhibiting Non-Foster behavior is bigger than the set of networks that have non-Foster elements**
- **Negative capacitors and negative inductors are not required to realize a non-Foster network**
- **Stable non-Foster circuits have been made and reported**
  - S.E. Sussman-Fort
  - S.D. Stearns
  - C. White
  - others
- **Stable non-Foster design requires using accurate device models, correct linear stability tests, and perhaps nonlinear stability analysis**
- **Transistor biasing methods and levels are important**
- **Match bandwidth greater than the Fano limit was observed for match networks for a monopole and a loop**
- **Rules of thumb for ensuring a stable design were found**

Resources for Impedance Match Network Design

A list of useful software tools with links is at
http://www.fars.k6ya.org/others#Software

How to Get The Joy of Matching





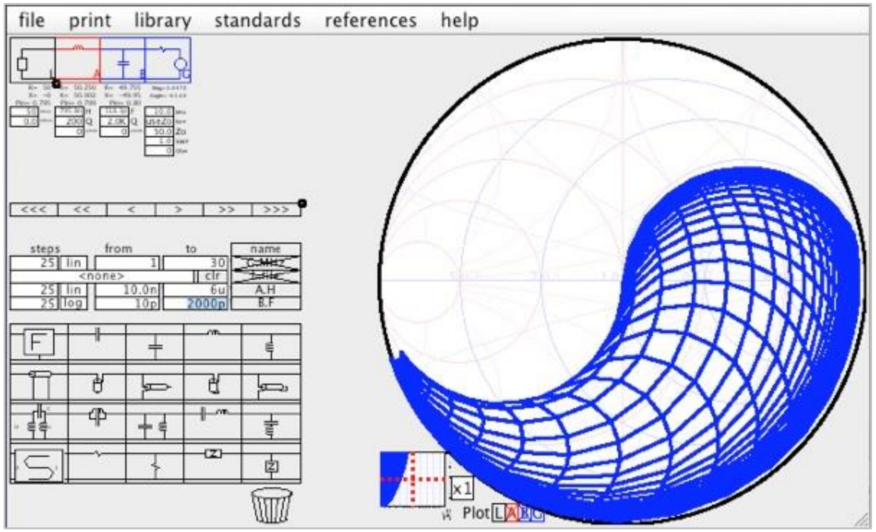
- **Get a copy of Wilfred N. Caron, *Antenna Impedance Matching*, ARRL, 1989, or 1993 edition with errata page**
- **Get a Smith Chart utility program (next slide)**
- **Work through all 11 example matching problems in the book**
  - Confirm Caron's solution
  - Try to find better solutions
- **Study R.L. Thomas, *A Practical Introduction to Impedance Matching*, Artech House, 1976**
- **Study A.R. Lopez, "Impedance Matching Equation: Developed Using Wheeler's Methodology," IEEE Long Island Section APS, Dec. 2013. (free pdf download)**

Software for Smith Charting and Network Design

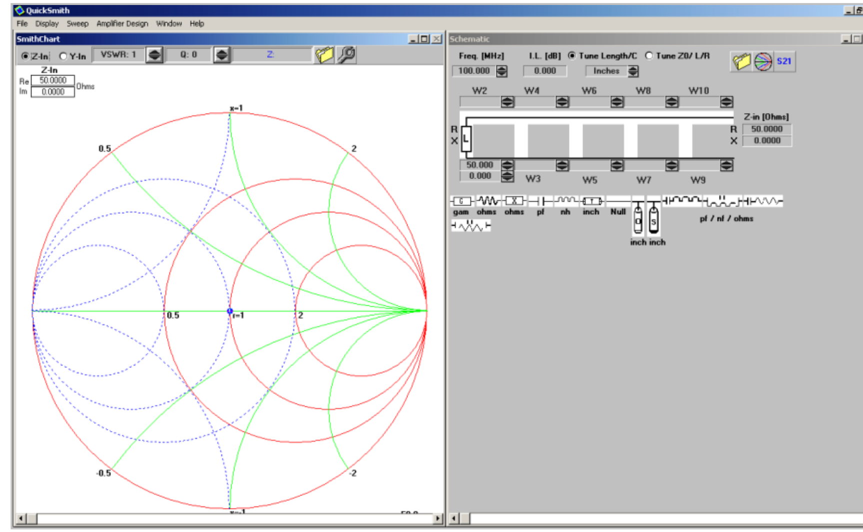
- ***SimSmith* 14.9 by Ward Harriman AE6TY, 2016**
  - Free download from http://ae6ty.com/Smith_Charts.html
- ***Smith Chart Calculator 2.1 by Gorik Stevens, 2015***
  - Free download from http://sourceforge.net/projects/gnssmithchart
- ***JJSmith 2.12, James Bromley K7JEB & James Tonne W4ENE (SK), 2015***
  - Free download from http://tonnesoftware.com/jjsmith.html
- ***QuickSmith 5.0.1 by Nathan Iyer KJ6FOJ, 2014***
  - Free download from https://code.google.com/p/quicksmith
- ***linSmith 0.99.26 by James Coppens ON6JC/LW3HAZ, 2013***
  - Free download from http://jcoppens.com/soft/linsmith/index.en.php
- ***Smith 3.10 by Fritz Dellsperger HB9AJY, 2010***
  - Free download from http://fritz.dellsperger.net
- ***FKSmith 1.15 by Fabian Kung, 2006***
  - Free download from http://pesona.mmu.edu.my/~wlkung
- ***XLZIZL by Dan Maguire AC6LA, 2005. No longer available.***
- ***WinSMITH 2.0, Noble Publishing, 1995. No longer available.***
- ***MicroSmith 2.3, ARRL, 1992. No longer available.***

Smith Chart Programs Assist Ladder Network Match Design

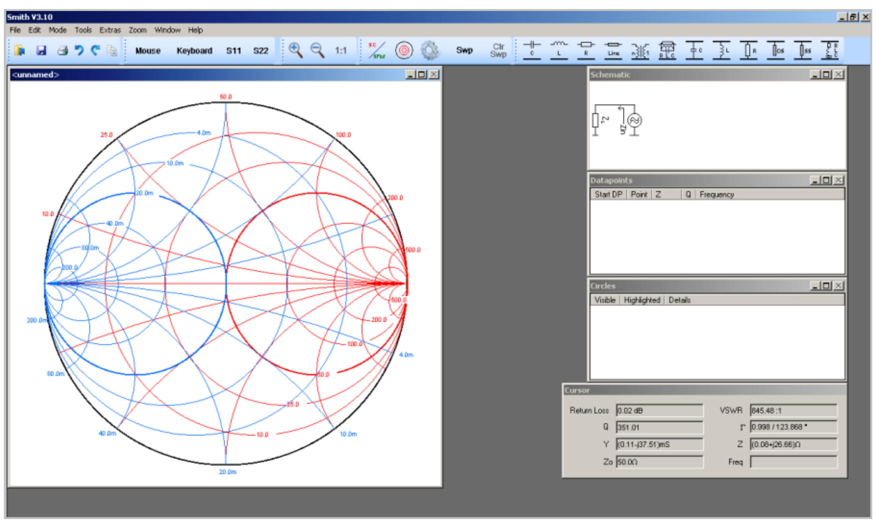
SimSmith 14.9



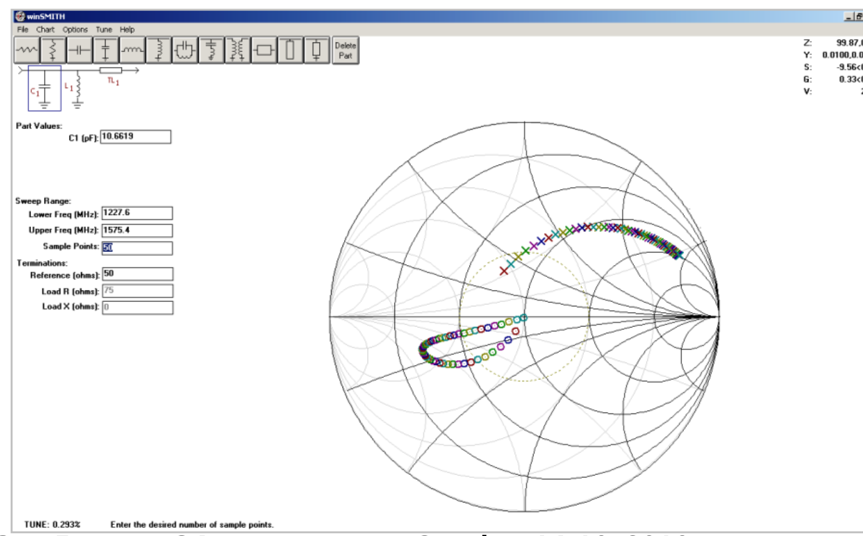
QuickSmith 5.0.1



Smith 3.10



winSMITH 2.0



Smith Chart Program Features Comparison

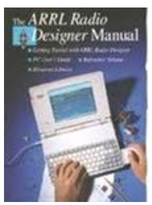
Program	Import files?	Export files?	Tuning sliders?
SimSmith 14.9	Yes, Touchstone	Yes, Touchstone	Yes
Smith Chart Calculator 2.1	No	No	No
JJSmith 2.12	No	No	No
QuickSmith 5.0.1	Yes, .gam	Yes, .gam	Yes
Smith 3.10	Yes, Touchstone	Yes, Touchstone	No
FKSmith 1.15	No	No	No
winSMITH 2.0	No	No	Yes

HP/EEsof Touchstone® (.s1p) File Format

```
!  Impedance of dipole computed 9/11/2004 by EZNEC
!  Len=98.35710564 ft, Dia=0.1071697365 in, Ω′=20
#   MHz   Z   RI   R   1
1.00   1.89876587   -3035.57432668
1.05   2.09705340   -2878.56550812
1.10   2.31069050   -2738.54359431
            •
            •
            •
29.90   1951.92366539   -1360.37582340
29.95   1867.53754109   -1395.10422285
30.00   1786.43800957   -1422.23417869
```

http://www.eda.org/ibis/touchstone_ver2.0/touchstone_ver2_0.pdf
http://www.eda.org/pub/ibis/connector/touchstone_spec11.pdf

General RF Circuit Design, Analysis, and Optimization



- **Software for Radio Amateurs**
  - ***LTspice* IV** 4.23, Linear Technology, 2015
    - Free download from http://www.linear.com/designtools/software
  - ***Quite Universal Circuit Simulator*** (QUCS) 0.0.18, 2014
    - Free download from http://qucs.sourceforge.net
  - ***Serenade SV*** 8.5 (student version), Ansoft, 2000. Still available on web. See article by David Newkirk W9VES in *QST*, Jan. 2001
  - ***ARRL Radio Designer*** 1.5, ARRL, 1995. No longer available.
- **Professional electronic design automation (EDA) software**
  - ***Advanced Design System*** (ADS), Keysight (formerly Agilent)
  - ***Microwave Office*** (MWO), National Instruments, Applied Wave Research
  - ***High Frequency System Simulator*** (HFSS), ANSYS (formerly Ansoft *Designer* and *Serenade*)

Ansoft Serenade SV Described in *QST*, January 2001

By David Newkirk, W9VES

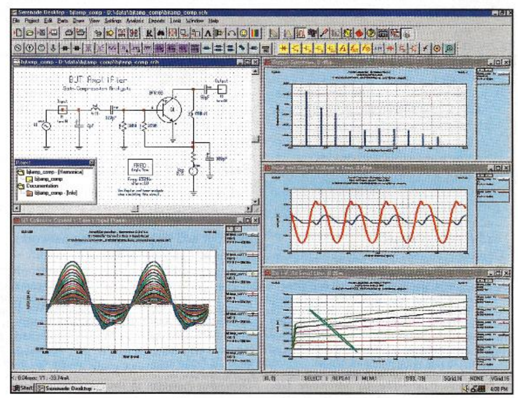
Simulating Circuits and Systems with *Serenade SV*

Teaming linear and harmonic-balance nonlinear circuit simulation of amplifiers, mixers and oscillators with system-level analysis of digital communication systems, *Serenade SV* is like having an RF/microwave communications lab in your computer—and it's free!

As L. B. Cebik's recent series reflects, computer-aided antenna modeling is well-established as a popular Amateur Radio activity, fueled by the free availability of core programs (*NEC* and *MININEC*) capable of realistic antenna modeling.[1] Radio amateurs interested in using their computers for realistic radio and electronic *circuit* simulation have similarly benefited from the free availability of *SPICE*, developed at the University of California at Berkeley. Proprietary alternatives to *NEC*, *MININEC* and *SPICE* are available, of course—at costs that generally put them well beyond the reach of students and hobbyists. *ARRL Radio Designer*, a feature-limited version of one such product (*Super-COMPACT*, a linear circuit simulator) brought realistic, affordable modeling of RF circuits to the market in 1994—if you didn't mind its netlist circuit entry, its lack of even the most basic distributed circuit components (microstrip and stripline structures, essential to useful UHF/microwave circuit modeling), and its inability to simulate any of the nonlinear effects without which a radio can't radio. What radio amateurs and electronics students really need is a free *nonlinear* circuit simulator that, unlike free versions of *SPICE*, incorporates essential *radio*-modeling components and capabilities because it has been designed for RF/microwave/wireless use from its beginning.

This article introduces that simulator,

Notes appear on page 43.



Serenade SV. A feature-limited version of Ansoft Corporation's industry-standard *Serenade Design Environment*, *Serenade SV* runs on *Windows 95, 98, 2000, ME* and *NT 4.0* and includes:[2]

- A harmonic-balance nonlinear circuit simulator (*Harmonica SV*)
- A discrete-time system simulator (*Symphony SV*)
- Graphical circuit entry (schematic capture)
- Linear and nonlinear circuit tuning
- Linear circuit optimization
- Nonlinear oscillator design and analysis
- Two-tone nonlinear analysis, including mixing and intermodulation distortion (IMD)
- Small-signal ac nonlinear analysis
- Stability analysis
- Waveform, modulation, sweep, bit-error rate, and statistical properties analysis
- Interactive, graphical matching-network synthesis
- A transmission-line calculator
- RF-critical circuit components, including black boxes (up to four ports), distributed elements (ideal, coaxial, microstrip and stripline transmission lines, coupled lines and bends), controlled sources, transformers, filters

Professional Impedance Matching Software for Automatic Design

- **Ampsa Impedance-Matching Wizard (IMW)**
  - Developed by Pieter L.D. Abrie
  - http://www.ampsa.com
- **Optenni Lab**
  - Developed by Jussi Rahola
  - https://www.optenni.com
- **Nuhertz Zmatch**
  - http://www.nuhertz.com

Comments on Software

- **All Smith chart programs**
  - Can draw impedance data on a Smith chart with Z and Y grids
  - Allow the user to define ladder networks and see the effect on an impedance locus
- **A good Smith chart program**
  - Has tuning sliders that allow changing parts values and seeing results instantly
  - Allows you to import impedance data as Touchstone .s1p files
  - Can convert impedance data from different file formats to Touchstone .s1p file format
  - Can export impedance data as Touchstone .s1p files
- **A good circuit analysis program**
  - Can draw impedance data on a Smith chart with Z and Y grids
  - Can import and export impedance data in Touchstone .SnP formats (.s1p, .s2p, etc.)
  - Allows analysis and design of arbitrary network topologies, more complicated than ladder networks, e.g. twin-T, bridged-T, and lattice networks
  - Has a robust optimizer such as the Nelder-Mead “amoeba” or “nonlinear simplex” algorithm that can numerically search for optimum parts values that satisfy a user-defined goal criterion

Resources for Antenna Modeling

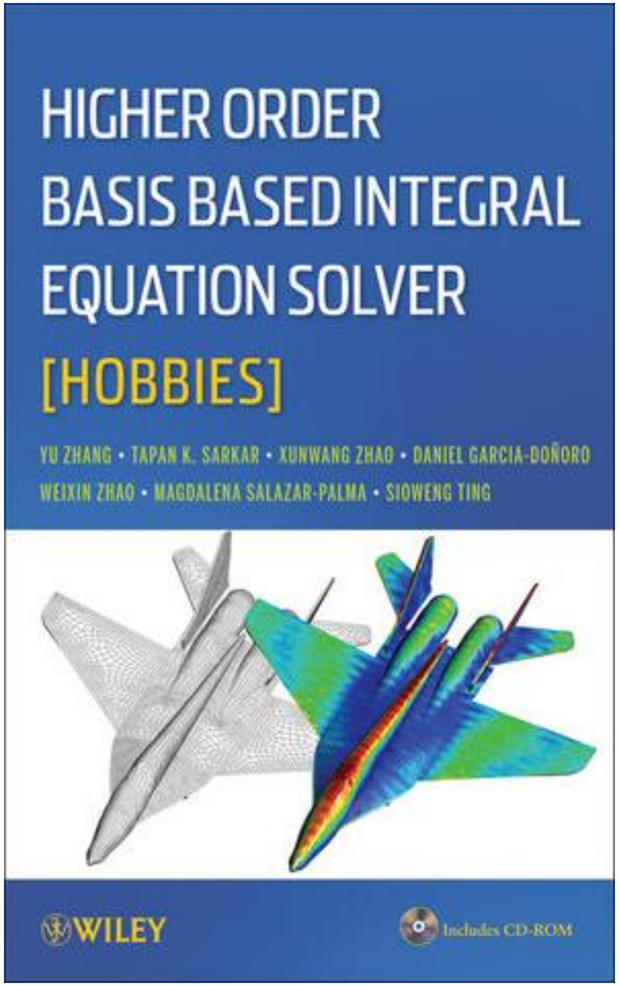
Antenna Modeling Programs for Radio Amateurs

- **EZNEC http://www.eznec.com**
  - EZNEC v.6 Demo program — Free (20 segments, also runs ARRL models)
  - EZNEC v.6 — $100 (500 segments)
  - EZNEC+ v.6 — $150 (2,000 segments)
  - EZNEC Pro/2 v.6 — $525 (45,000 segments)
  - EZNEC Pro/4 v.6 — $675 (sold only to NEC4 licensees)
- **4nec2 http://www.qsl.net/4nec2**
  - Free, 11,000 segments, two optimizers, all NEC commands available
- **MININEC http://www.w8io.com/mininec.htm or http://www.blackcatsystems.com/software**
  - Black Cat Systems offers MiniNEC Pro version 1.4.0, $29
- **MMANA-GAL http://hamsoft.ca/pages/mmana-gal.php**
  - Free Basic version 8,192 segments. Pro version 32,000 segments, $130
- **NEC4 https://ipo.llnl.gov/technologies/nec**
  - Noncommercial user license $300
- **FEKO Student Edition http://www.altairuniversity.com/feko-student-edition**
  - Free to students. Part of HyperWorks 14 Student Edition
- **WiPL-D http://www.wipl-d.com**
  - Free “Microwave Lite” v6.0 (665 unknowns) and free 30-day trial of professional v13.0
- **HOBBIES http://em-hobbies.com**
  - Book includes software registration code, online price varies from $125 to $231 MSRP
- **MEFiSTo http://www.faustcorp.com**
  - Free 2D basic version and free trial of 3D professional version

Accessory Software for EZNEC

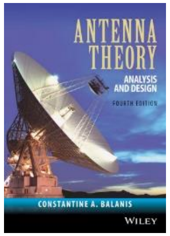
- **AutoEZ 2.0.18 by Dan Maguire, AC6LA, http://www.ac6la.com**
  - Excel/Visual Basic program
    - Demo version, free (30 segment limit)
    - Regular version, $79
  - Requires Excel and EZNEC installed on computer
  - Controls EZNEC to make multiple runs
    - A GUI for a GUI for NEC
  - Has optimizer – Nelder-Mead algorithm
  - Reads NEC, AO, and MMANA-GAL files
  - Doesn’t work with EZNEC-ARRL or EZNEC Demo
  - Replaces MultiNEC, which is no longer available

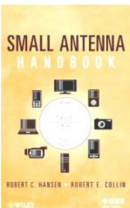
HOBBIES

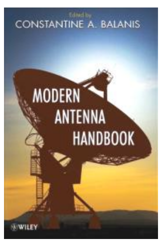


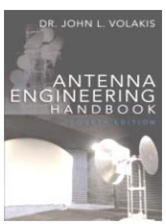
- http://em-hobbies.com
- Y. Zhang, et al., *Higher Order Basis Based Integral Equation Solver* (*HOBBIES*), Wiley, 2012, ISBN 9781118140659
- Make sure to buy a new copy with software license registration code intact and unused

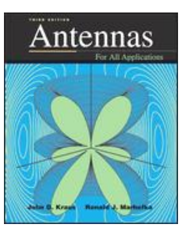
Favorite Antenna Books









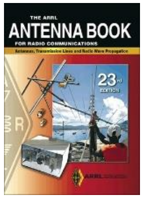


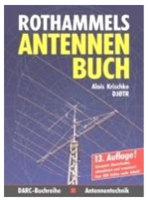
- **Books for antenna engineers and students**
  - C.A. Balanis, *Antenna Theory: Analysis and Design*, 4e, Wiley, 2016
  - R.C. Hansen and R.E. Collin, *Small Antenna Handbook*, Wiley, 2011
  - *Modern Antenna Handbook*, C.A. Balanis, editor, Wiley, 2008
  - *Antenna Engineering Handbook*, 4e, J.L. Volakis, ed., McGraw-Hill, 2007
  - J.D. Kraus and R.J. Marhefka, *Antennas*, 3e, McGraw-Hill, 2001
- **Antenna research papers**
  - IEEE AP-S Digital Archive, 2001-2009 (1 DVD), JD0307
  - IEEE AP-S Digital Archive, 2001-2006 (1 DVD), JD0304
  - IEEE AP-S Digital Archive, 2001-2003 (1 DVD), JD0301
  - IEEE AP-S Digital Archive, 1952-2000 (2 DVDs), JD0351
- **ACES Journal Archives**
  - http://www.aces-society.org/journal.php

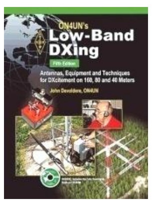
Free Downloadable Books from "Amateur Radio Resources that Save Space"

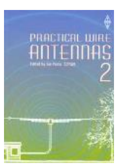
- **P-S. Kildal, *Foundations of Antenna Engineering*, 2015**
  - http://kildal.se/index.php/news-page/28-book
- **S.J. Orfanidis, *Electromagnetic Waves and Antennas*, Rutgers U., 2008**
  - http://www.ece.rutgers.edu/~orfanidi/ewa
- **C.A. Balanis, *Antenna Theory: Analysis and Design*, 3e, Wiley, 2005**
  - https://archive.org/details/Antenna.Theory.Analysis.and.Design3rd.Edition
- **J. Layton, *Directional Broadcast Antennas: A Guide to Adjustment,…* , TAB Books, 1974**
  - http://www.americanradiohistory.com/Archive.../Directional-Broadcast-Antennas-Leyton.pdf
- **A.D. Watt, *VLF Radio Engineering*, Pergamon Press, 1967**
  - http://www.introni.it/pdf/Watt%20-%20VLF%20Radio%20Engineering%2014.pdf
- **G.Z. Ayzenberg, *Shortwave Antennas*, revised edition, translated from Russian, 1962**
  - http://www.dtic.mil/docs/citations/AD0706545
- **S. Seely, *Radio Electronics*, McGraw-Hill, 1956**
  - https://archive.org/details/RadioElectronics
- **S.A. Schelkunoff and H.T. Friis, *Antennas: Theory and Practice*, Wiley, 1952**
  - https://archive.org/details/antennastheorypr00sche
- **E.A. Laport, *Radio Antenna Engineering*, McGraw-Hill, 1952**
  - http://snulbug.mtview.ca.us/books/RadioAntennaEngineering
- **K. Henney, *Radio Engineering Handbook*, McGraw-Hill, 1950**
  - https://archive.org/details/radioengineering00henn
- **F. Langford-Smith, *The Radiotron Designers Handbook*, Wireless Press, 1941**
  - https://archive.org/details/radiotrondesigne00lang
- **K. Henney, *Principles of Radio*, Wiley, 1934**
  - https://archive.org/details/principlesofradi00henn

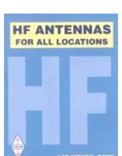
Favorite Antenna Books continued



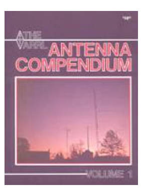


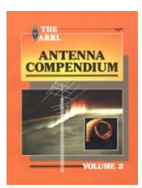


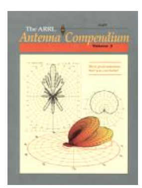


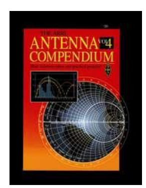


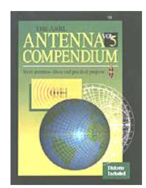
- **Books for Radio Amateurs**
  - *ARRL Antenna Book*, 23e, H.W. Silver, N0AX, editor, American Radio Relay League, 2015
  - *Rothammel's Antennenbuch*, 13e, A. Krischke, DJ0TR, editor, DARC Verlag, 2013
  - J. Devoldere, ON4UN, *ON4UN's Low-Band Dxing*, 5e, American Radio Relay League, 2011
  - *Practical Wire Antennas 2*, I. Poole, G3YWX, editor, Radio Society of Great Britain, 2005
  - J. Sevick, W2FMI, *The Short Vertical Antenna and Ground Radial*, CQ Communications, 2003
  - L. Moxon, G6XN, *HF Antennas for All Locations*, 2e, RSGB, 1983
- **ARRL Antenna Compendium series – eight volumes**

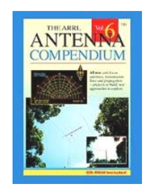


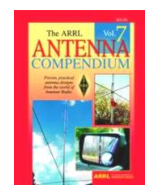


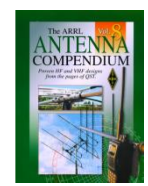




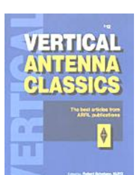


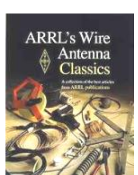


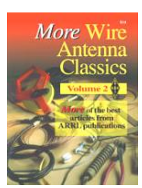


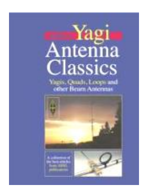


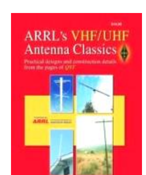
- **ARRL Antenna Classics series – eight titles**

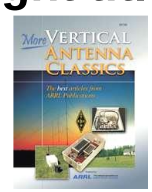


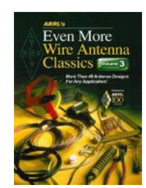


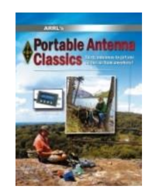




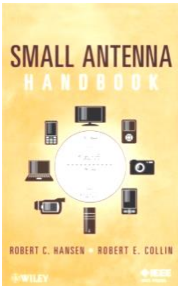








Recent Antenna Books of Interest



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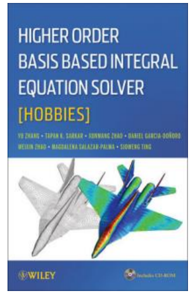

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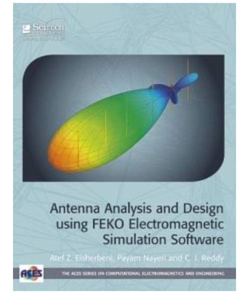

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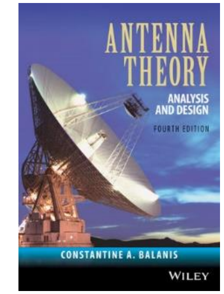

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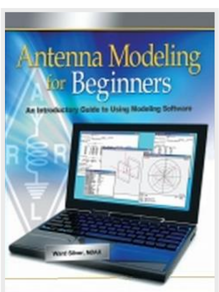

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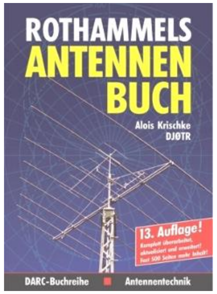

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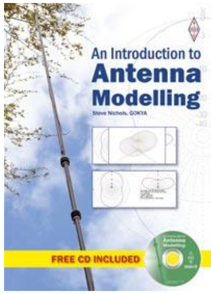

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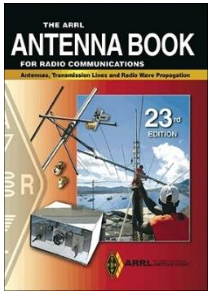

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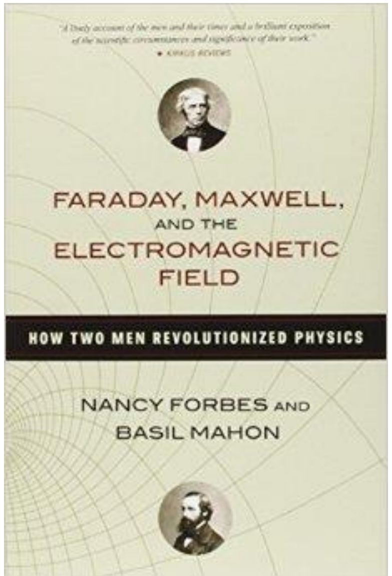

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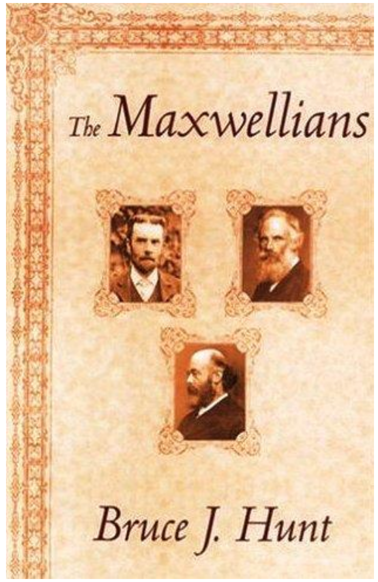

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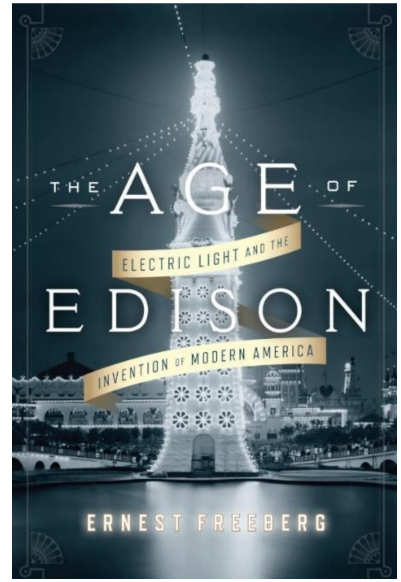

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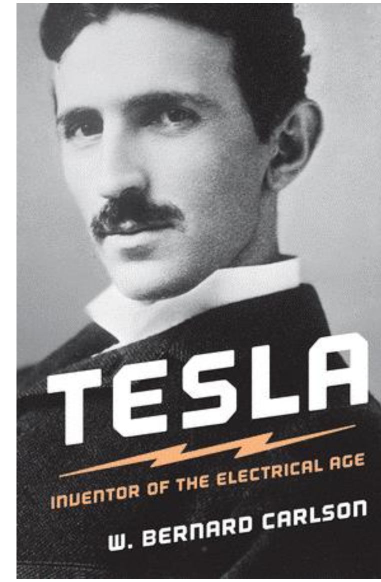

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The End

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