Dipole Impedance, Resonant Length, and Resonant Resistance

Steve Stearns, K6OIK

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k6oik@arrl.net

Abstract

Popular antenna modeling programs do many things well but do some things poorly, even things that are simple and basic. Perhaps most surprising is that many numerical programs do not predict a dipole's resonant frequency or resistance accurately. We explore the simplest antenna, the dipole in free space, from the perspective of antenna theory, and give simple formulas and graphs that are easy to use. This talk expands on "The Mysterious Multiplying Factor *K*," which the author presented at Pacificon 2018, to include the resistance at resonance.

Speaker's Biography



- **Stephen D. Stearns**
- **40 years experience in electronic systems**
  - Northrop Grumman, TRW, GTE Sylvania, Hughes Aircraft
  - Electromagnetic and signal processing systems for communications and radar surveillance, cochannel signal separation, measurement, identification, characterization, polarimetric array signal processing of ionospheric skywave signals for geolocating HF emitters, multi-sensor fusion
  - Recent work: Antenna and scattering theory; Non-Foster circuits for antennas and metamaterials; antennas for radiating OAM Bessel-Vortex beams; reflectionless filters
- **FCC licenses**
  - Amateur Radio Extra Class
  - 1st-Class Radiotelephone
  - General Radio Operator License (GROL)
  - Ship Radar Endorsement
- **Education**
  - PhD Stanford – under Prof. T.M. Cover
  - MSEE USC – under Profs. H.H. Kuehl and C.L. Weber
  - BSEE CSUF – under Profs. J.E. Kemmerly and G.I. Cohn
- **10 patents**
- **More than 100 publications and presentations**
  - Professional https://ieeexplore.ieee.org/Xplore/home.jsp
  - Amateur Radio https://wkwkw.fars.k6ya.org/docs/k6oik

ARRL Pacificon Presentations by K6OIK

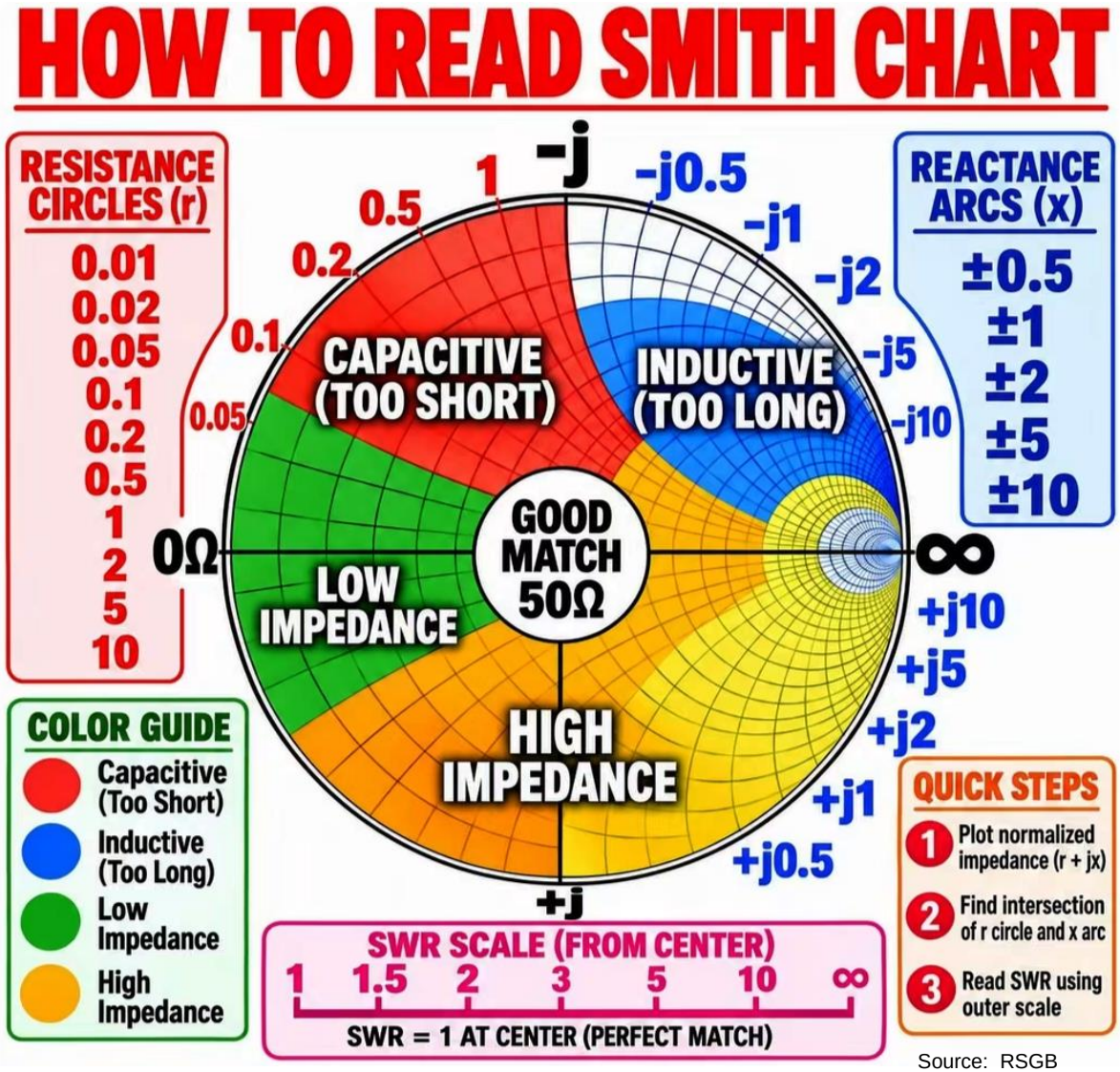
		Archived at http://www.fars.k6ya.org
1999	Mysteries of the Smith Chart	
2000	Jam-Resistant Repeater Technology	
2001	Mysteries of the Smith Chart	✓
2002	How-to-Make Better RFI Filters Using Stubs	
2003	Twin-Lead J-Pole Design	
2004	Antenna Impedance Models – Old and New	✓
2005	Novel and Strange Ideas in Antennas and Impedance Matching	
2006	Novel and Strange Ideas in Antennas and Impedance Matching II	✓
2007	New Results on Antenna Impedance Models and Matching	✓
2008	Antenna Modeling for Radio Amateurs	
2010	Facts About SWR, Reflected Power, and Power Transfer on Real Transmission Lines with Loss	✓
2011	Conjugate Match Myths	✓
2012	Transmission Line Filters Beyond Stubs and Traps	✓
2013	Bode, Chu, Fano, Wheeler – Antenna Q and Match Bandwidth	✓
2014	A Transmission Line Power Paradox and Its Resolution	✓
2015	Weird Waves: Exotic Electromagnetic Phenomena	✓
2015	The Joy of Matching: How to Design Multi-Band Match Networks	✓
2016	The Joy of Matching 2: Multi-Band and Reflectionless Match Networks	
2016-7	Antenna Modeling for Radio Amateurs – Revised and Expanded	✓
2017	VHF-UHF Propagation Planning for Amateur Radio Repeaters	✓
2018	Antennas: The Story from Physics to Computational Electromagnetics	✓
2018	Novel Antennas, The Mysterious Factor *K*, Impromptu Antenna Modeling	
2019	Dipole Basics	✓
2019	Antenna Modeling Half-day Seminar	
2021	Universal Equivalent Circuits for All Antennas	✓
2023	Grow an Antenna … from Seeds	✓
2024	The Best Shape for a Wire Antenna	✓

Recent Work

- **"Insulated Wire Modeling"**
  - Part 1: *QEX*, November/December, 2024
  - Part 2: Dielectric and magnetic losses (to appear)
- **"The Best Shape for a Wire Antenna"**
  - Pacificon Antenna Seminar 2024
  - Using an optimizer to determine wire shape
- **"Grow an Antenna … from Seeds"**
  - Pacificon Antenna Seminar 2023
  - Modeling dielectric objects by using loaded wire grids
  - Antenna performance can be enhanced (or degraded) by the presence of trees. Trees in the right configuration can make good antennas
- **"Universal Equivalent Circuits for All Antennas"**
  - Pacificon Antenna Seminar 2021
  - IEEE Antennas and Propagation Society (APS) and IEEE Microwave Theory and Techniques Society (MTT-S) https://www.youtube.com/watch?v=vQ9BFdmFHCM

Slides: https://www.fars.k6ya.org/docs/k6oik

Question for 2026. What's Wrong Here?



Source: RSGB

Outline

- **Brief history of electricity, magnetism, and electromagnetism**
  - Notable figures in antenna physics and engineering
- **The universe of antennas**
  - A variety of different kinds of antennas
- **Antenna analysis**
  - Antenna analysis via theory and approximations
    - Induced EMF method
  - Computational Electromagnetics (CEM)
    - Method of Moments (MoM)
- **The isolated dipole (in free space)**
  - Dipole impedance theories and models
  - What is resonance?
  - Resonant length and the multiplying factor *K*
  - Resistance at resonance
  - The controversy between IEMF and MoM

19th Century Scientists and Natural Philosophers of Electricity and Magnetism



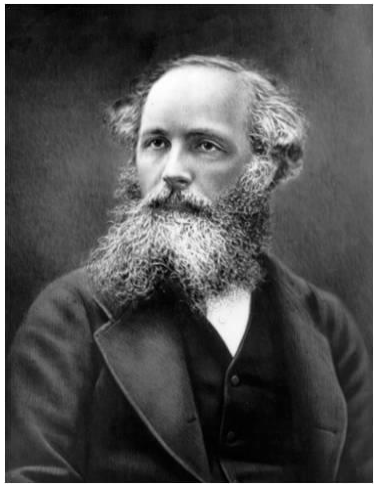
André-Marie Ampère
1775-1836



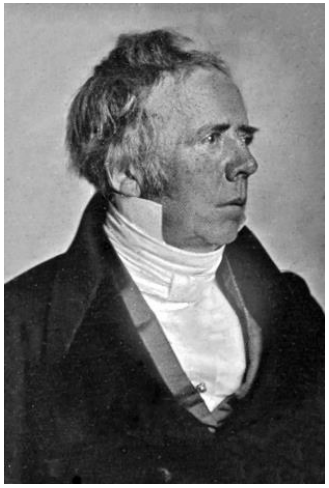
Carl Friedrich Gauss
1777-1855



Michael Faraday
1791-1867



James Clerk Maxwell
1831-1879



Hans Christian Ørsted
1777-1851



Georg Simon Ohm
1789-1854

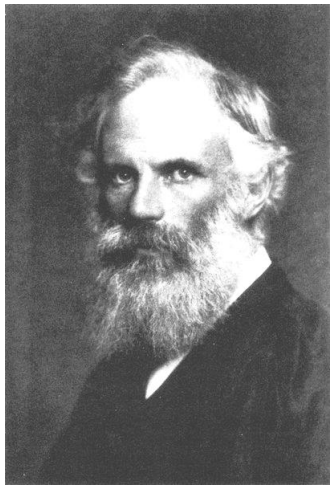


Joseph Henry
1797-1878

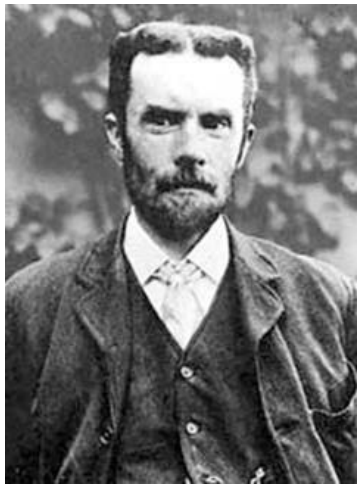


Wilhelm Eduard Weber
1804-1891

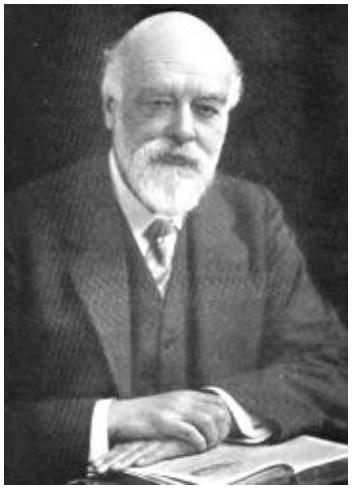
The Maxwellians



George Francis FitzGerald
1851-1901



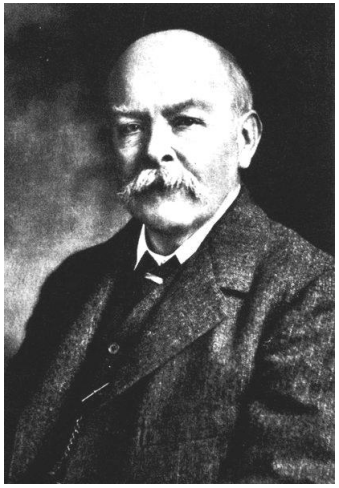
Oliver Heaviside
1850-1925



Oliver Joseph Lodge
1851-1940



Heinrich Rudolph Hertz
1857-1894



John Henry Poynting
1851-1914

Maxwell 1865 and the Equations in His 1873 Treatise

VIII. *A Dynamical Theory of the Electromagnetic Field.* *By* J. Clerk Maxwell, *F.R.S.*

Received October 27,—Read December 8, 1864.

PART I.—INTRODUCTORY.

(1) The most obvious mechanical phenomenon in electrical and magnetical experiments is the mutual action by which bodies in certain states set each other in motion while still at a sensible distance from each other. The first step, therefore, in reducing these phenomena into scientific form, is to ascertain the magnitude and direction of the force acting between the bodies, and when it is found that this force depends in a certain way upon the relative position of the bodies and on their electric or magnetic condition, it seems at first sight natural to explain the facts by assuming the existence of something either at rest or in motion in each body, constituting its electric or magnetic state, and capable of acting at a distance according to mathematical laws.

In this way mathematical theories of statical electricity, of magnetism, of the mechanical action between conductors carrying currents, and of the induction of currents have been formed. In these theories the force acting between the two bodies is treated with reference only to the condition of the bodies and their relative position, and without any express consideration of the surrounding medium.

These theories assume, more or less explicitly, the existence of substances the particles of which have the property of acting on one another at a distance by attraction or repulsion. The most complete development of a theory of this kind is that of M. W. Weber*, who has made the same theory include electrostatic and electromagnetic phenomena.

In doing so, however, he has found it necessary to assume that the force between two electric particles depends on their relative velocity, as well as on their distance.

This theory, as developed by MM. W. Weber and C. Neumann†, is exceedingly ingenious, and wonderfully comprehensive in its application to the phenomena of statical electricity, electromagnetic attractions, induction of currents and diamagnetic phenomena; and it comes to us with the more authority, as it has served to guide the speculations of one who has made so great an advance in the practical part of electric science, both by introducing a consistent system of units in electrical measurement, and by actually determining electrical quantities with an accuracy hitherto unknown.

* Electrodynamische Maassbestimmungen. Leipzic Trans. vol. i. 1849, and Taylor's Scientific Memoirs, vol. v. art. xiv.

† "Explicare tentatur quomodo fiat ut lucis planum polarizationis per vires electricas vel magneticas declinetur."—Halis Saxonum, 1858.

MDCCCLXV. 3 R

$$a = \frac{dH}{dy} - \frac{dG}{dz}$$
$$b = \frac{dF}{dz} - \frac{dH}{dx} \quad \text{(A)} \quad \text{Art. 591}$$
$$c = \frac{dG}{dx} - \frac{dF}{dy}$$

$$P = c\frac{dy}{dt} - b\frac{dz}{dt} - \frac{dF}{dt} - \frac{d\psi}{dx}$$
$$Q = a\frac{dz}{dt} - c\frac{dx}{dt} - \frac{dG}{dt} - \frac{d\psi}{dy} \quad \text{(B)} \quad \text{Art. 598}$$
$$R = b\frac{dx}{dt} - a\frac{dy}{dt} - \frac{dH}{dt} - \frac{d\psi}{dz}$$

$$X = vc - wb$$
$$Y = wa - uc \quad \text{(C)} \quad \text{Art. 603}$$
$$Z = ub - va$$

$$a = \alpha + 4\pi A$$
$$b = \beta + 4\pi B \quad \text{(D)} \quad \text{Art. 605}$$
$$c = \gamma + 4\pi C$$

$$4\pi u = \frac{d\gamma}{dy} - \frac{d\beta}{dz}$$
$$4\pi v = \frac{d\alpha}{dz} - \frac{d\gamma}{dx} \quad \text{(E)} \quad \text{Art. 607}$$
$$4\pi w = \frac{d\beta}{dx} - \frac{d\alpha}{dy}$$

$$\mathfrak{D} = \frac{1}{4\pi} K \mathfrak{E} \quad \text{(F)} \quad \text{Art. 608}$$

$$\mathfrak{K} = C\mathfrak{E} \quad \text{(G)} \quad \text{Art. 609}$$

$$\mathfrak{C} = \mathfrak{K} + \dot{\mathfrak{D}} \quad \text{(H)} \quad \text{Art. 610}$$

$$u = p + \frac{df}{dt}$$
$$v = q + \frac{dg}{dt} \quad \text{(H*)}$$
$$w = r + \frac{dh}{dt}$$

$$\mathfrak{C} = (C + \frac{1}{4\pi} K \frac{d}{dt})\mathfrak{E} \quad \text{(I)} \quad \text{Art. 611}$$

$$u = CP + \frac{1}{4\pi} K \frac{dP}{dt}$$
$$v = CQ + \frac{1}{4\pi} K \frac{dQ}{dt} \quad \text{(I*)}$$
$$w = CR + \frac{1}{4\pi} K \frac{dR}{dt}$$

$$\rho = \frac{df}{dx} + \frac{dg}{dy} + \frac{dh}{dz} \quad \text{(J)} \quad \text{Art. 612}$$

$$\sigma = lf + mg + nh + l'f' + m'g' + n'h' \quad \text{(K)} \quad \text{Art. 613}$$

$$\mathfrak{B} = \mu \mathfrak{H} \quad \text{(L)} \quad \text{Art. 614}$$

Heaviside's "Duplex" Equations for Maxwell's Theory

$$\nabla \times \mathbf{E} = -\mathbf{M} - \frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla \cdot \mathbf{D} = \rho_e$$

$$\nabla \cdot \mathbf{B} = \rho_m$$

$$\mathbf{D} = \varepsilon \mathbf{E} \qquad \mathbf{J} = \sigma_e \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H} \qquad \mathbf{M} = \sigma_m \mathbf{H}$$

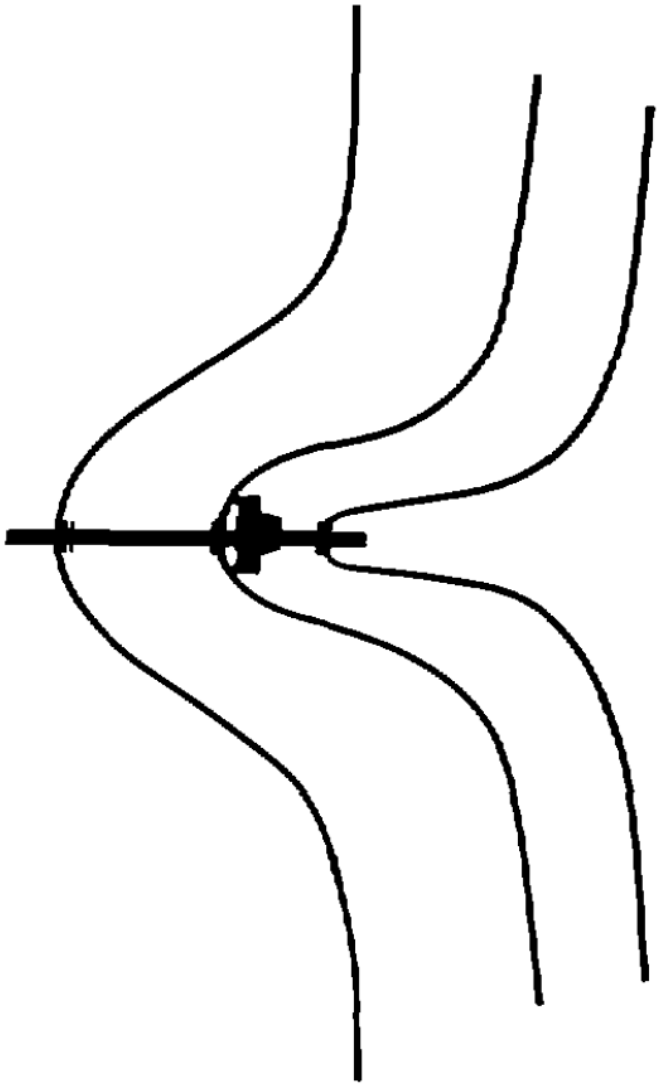
"And God said, Let there be light; and there was light." *Genesis* 1:3

The Universe of Antennas

Amateur Antenna Paradigms

- Antennas made of straight elements (wires, rods, and tubes)
- Antennas made of conductors (metals)
- Resonant antennas
- Narrowband antennas
- But … many interesting antennas break these rules!

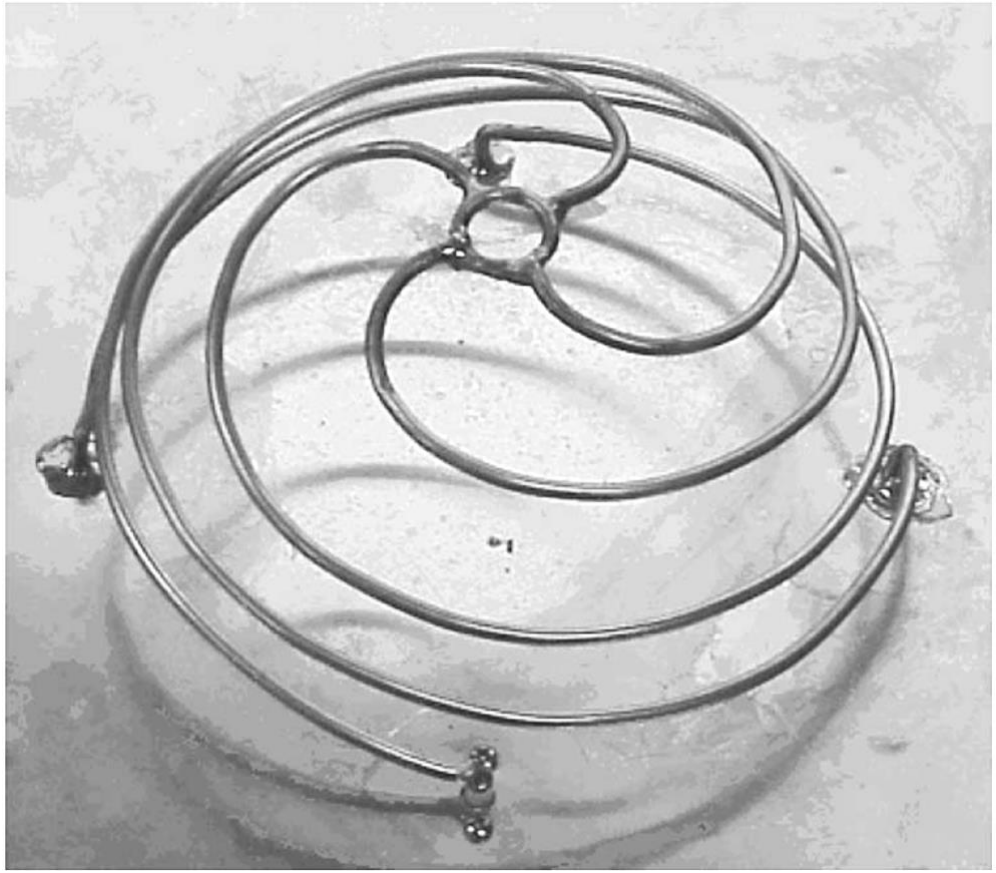
Landstorfer Antenna (1976)



- **Elements: 3**
- **Element shape: Optimized (approximately Gaussian)**
- **Gain: 11.5 dBi**
- **Sidelobes: < -20 dB**
- **F/B ratio: 26 dB**
- **Performance similar to 10-element Yagi – except…**
- **Bandwidth: > 3% (W4RNL)**

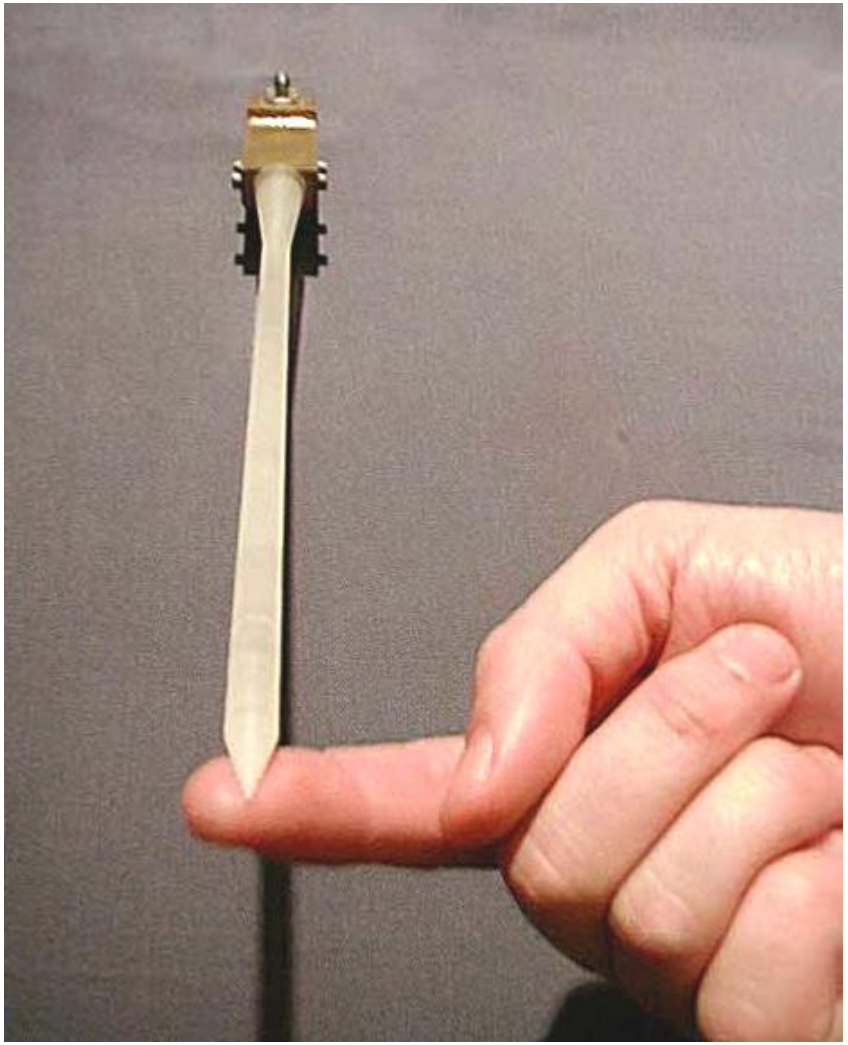
Stearns, K6OIK, "The Best Shape for a Wire Antenna," ARRL Pacificon Antenna Seminar, 2024.

Folded Hemi-Spherical Helix Over Ground Plane (2004)



- Helix: 4 arms, 1 turn
- Height (radius): λ/16.5
- Frequency: 300 MHz
- Polarization: vertical
- Z: 50 ohms real
- VSWR: < 1.16
- Efficiency: > 94%
- Bandwidth: 22.8 MHz (7.6%)
- Q_A: 32 (Q_{Chu} = 22.8)

Polyrod Antenna (1947)

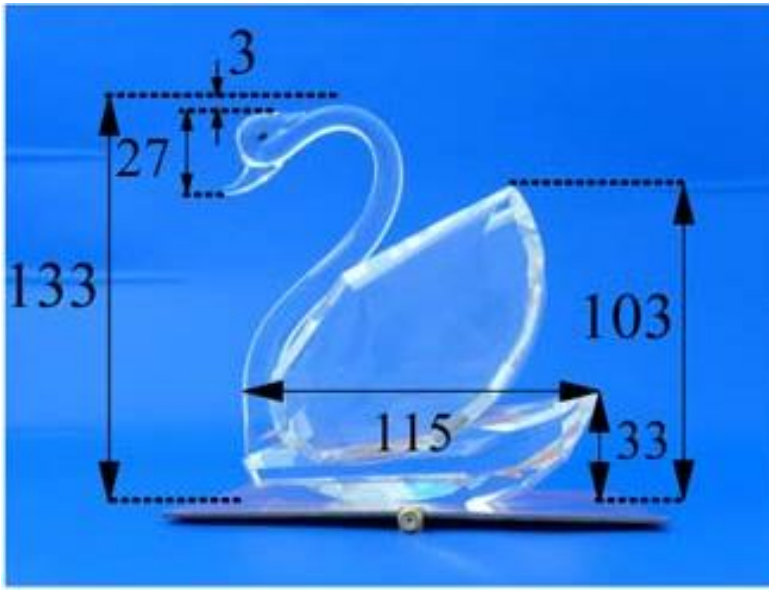


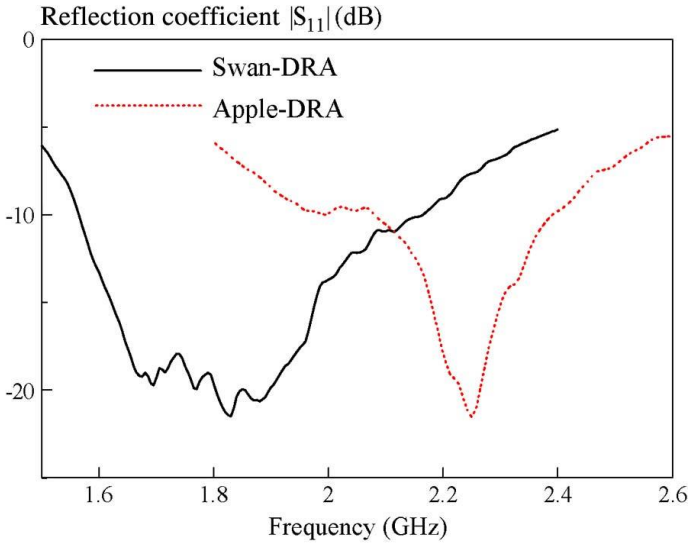
- **Material: Polystyrene**
- **Frequency: 11.6 GHz**
- **Gain: 20 dBi**
- **Bandwidth: 40%**

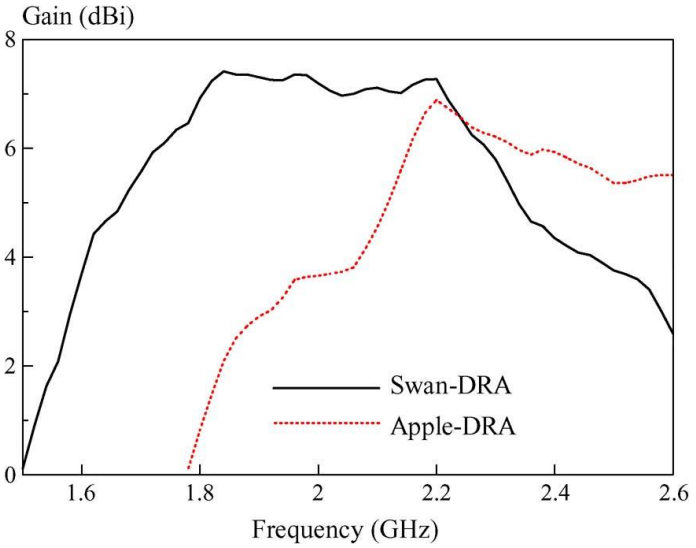
Dielectric rods made of ceramic or fused quartz can handle high power

Dielectric Resonator Antennas – Apple and Swan





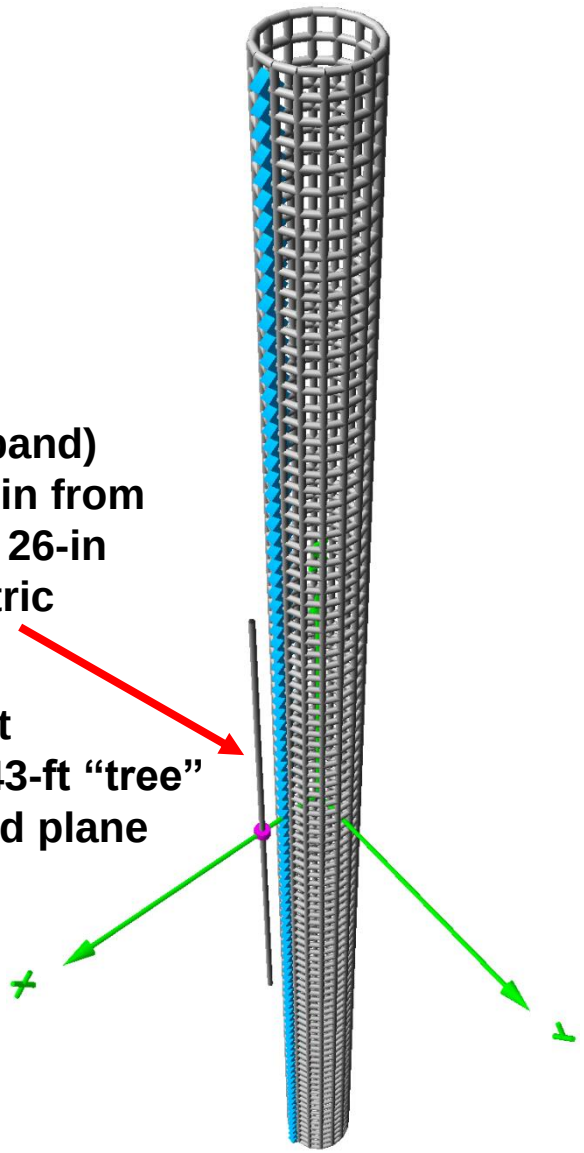




Dipole or Monople Using a Tree as Reflector

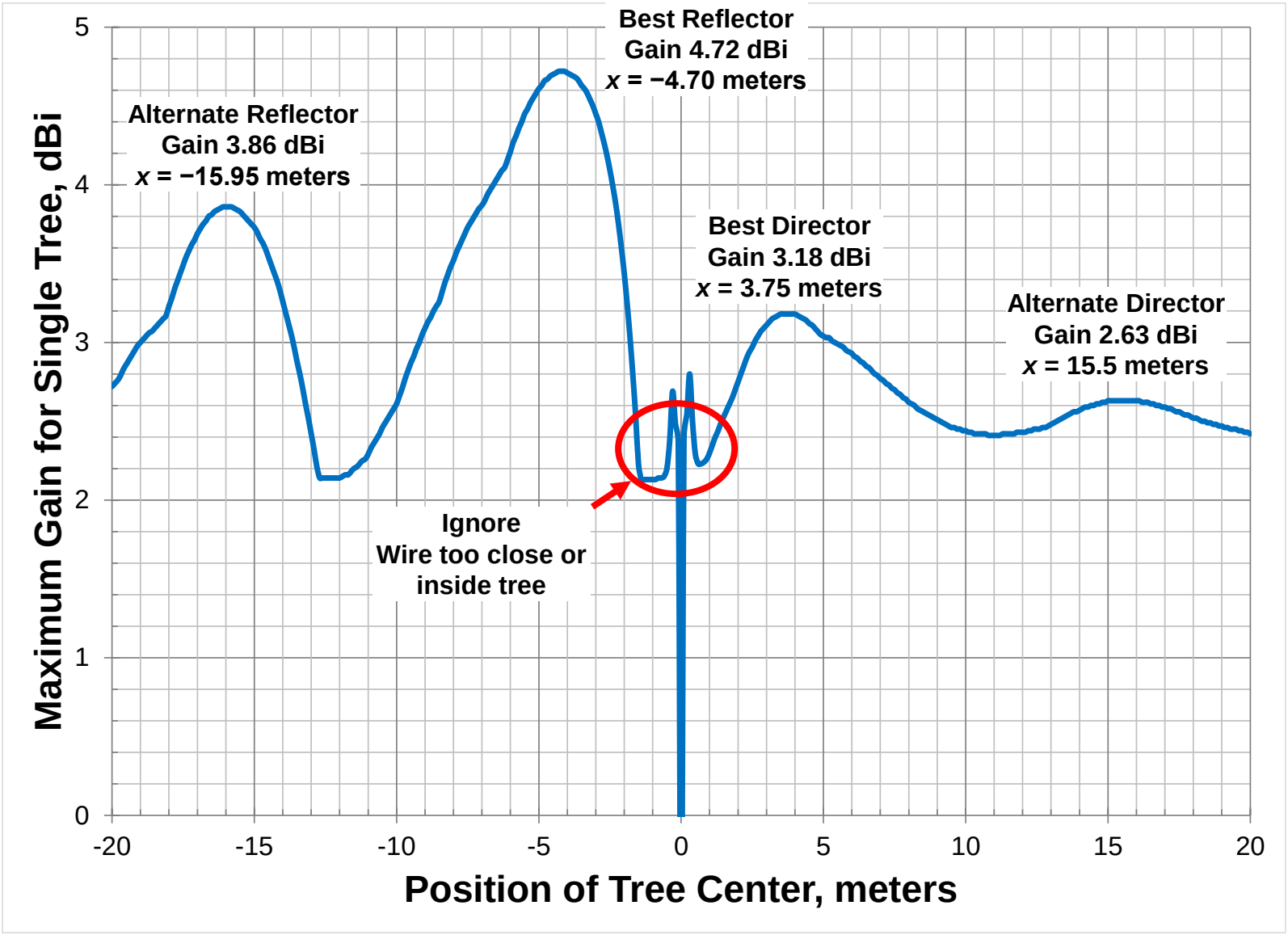
34-ft (20 meter band) metal dipole 20-in from surface of 86-ft, 26-in diameter dielectric cylinder

Represents 17-ft monopole and 43-ft "tree" over PEC ground plane

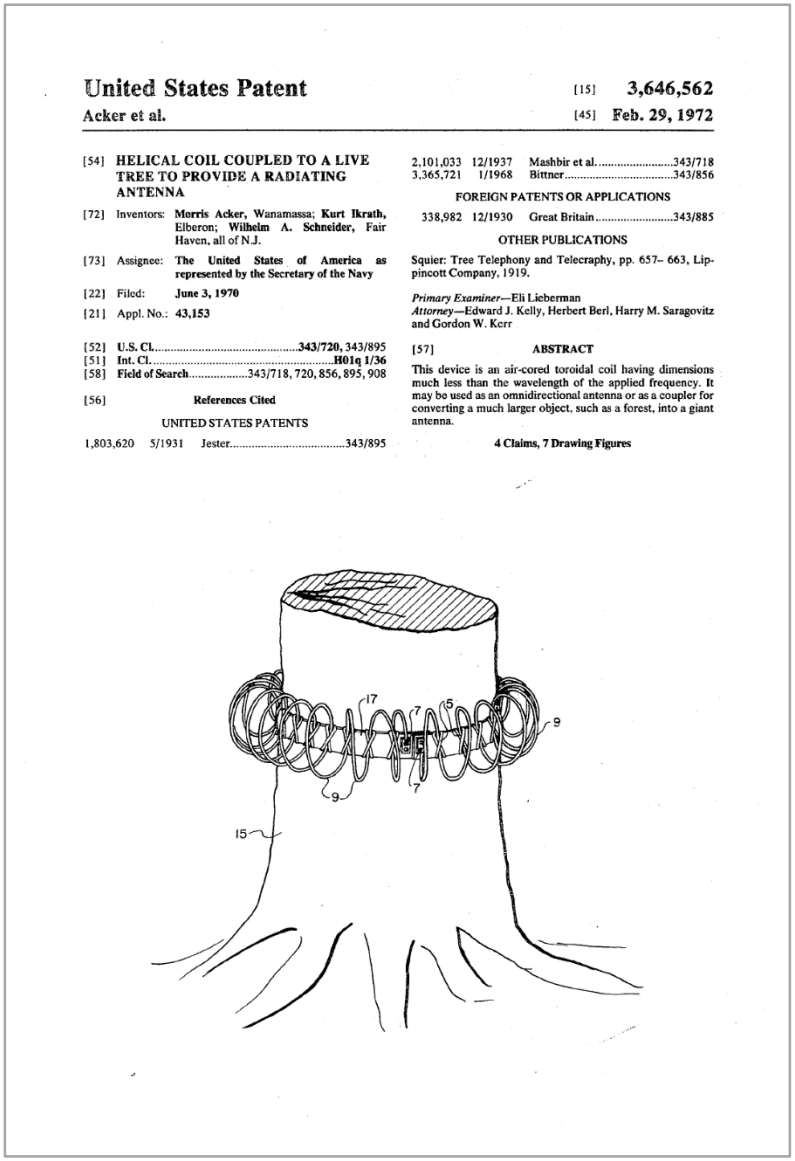


- 16-sided tree model
- 16 vertical wires on tree perimeter
- Grid facets are square
- Segment lengths equal $R \times 2 \sin(11.25°)$
- Segment len ÷ diameter = π
- Satisfies Equal Area Rule
- Wires are loaded to match tree's surface impedance
- Dipole and tree segments equal and aligned
- Model runs in 4nec2 in minutes, not hours

Forward Gain vs Tree Position on *X*-Axis



Hybrid Electromagnetic Antenna Couplers (HEMAC)



United States Patent

Acker et al.

[15] 3,646,562

[45] Feb. 29, 1972

[54] HELICAL COIL COUPLED TO A LIVE TREE TO PROVIDE A RADIATING ANTENNA

[72] Inventors: Morris Acker, Wanamassa; Kurt Ikrath, Elberon; Wilhelm A. Schneider, Fair Haven, all of N.J.

[73] Assignee: The United States of America as represented by the Secretary of the Navy

[22] Filed: June 3, 1970

[21] Appl. No.: 43,153

[52] U.S. Cl. 343/720, 343/895

[51] Int. Cl. H01q 1/36

[58] Field of Search 343/718, 720, 856, 895, 908

[56] References Cited

UNITED STATES PATENTS

1,803,620	5/1931	Jester	343/895
2,101,033	12/1937	Mashbir et al.	343/718
3,365,721	1/1968	Bittner	343/856

FOREIGN PATENTS OR APPLICATIONS

338,982	12/1930	Great Britain	343/885

OTHER PUBLICATIONS

Squier: Tree Telephony and Telecraphy, pp. 657– 663, Lippincott Company, 1919.

Primary Examiner—Eli Lieberman

Attorney—Edward J. Kelly, Herbert Berl, Harry M. Saragovitz and Gordon W. Kerr

[57] ABSTRACT

This device is an air-cored toroidal coil having dimensions much less than the wavelength of the applied frequency. It may be used as an omnidirectional antenna or as a coupler for converting a much larger object, such as a forest, into a giant antenna.

4 Claims, 7 Drawing Figures



Saguaro Cactus (Carnegia giantea)?





Evergreen Trees?







Deciduous Tree?





A Drive-Up Discone Antenna in Green Valley, Arizona



- HF Discone antenna at the Titan Missile Museum
- Built by Collins Radio Company in 1963
- Height: 80 feet
- Elevation: 2,962 feet
- Frequency: Full HF band
- Amateur Radio operators may use the antenna for free
- At the Visitor Center, request and sign the antenna log book
- Drive to the antenna and park by the picnic table
- Open the metal box and roll out the coax cable
- Connect your radio using the PL-259 connector on the end of the cable
- When finished, please roll up and return the cable to the box

Why Little Transmitters Get Heard

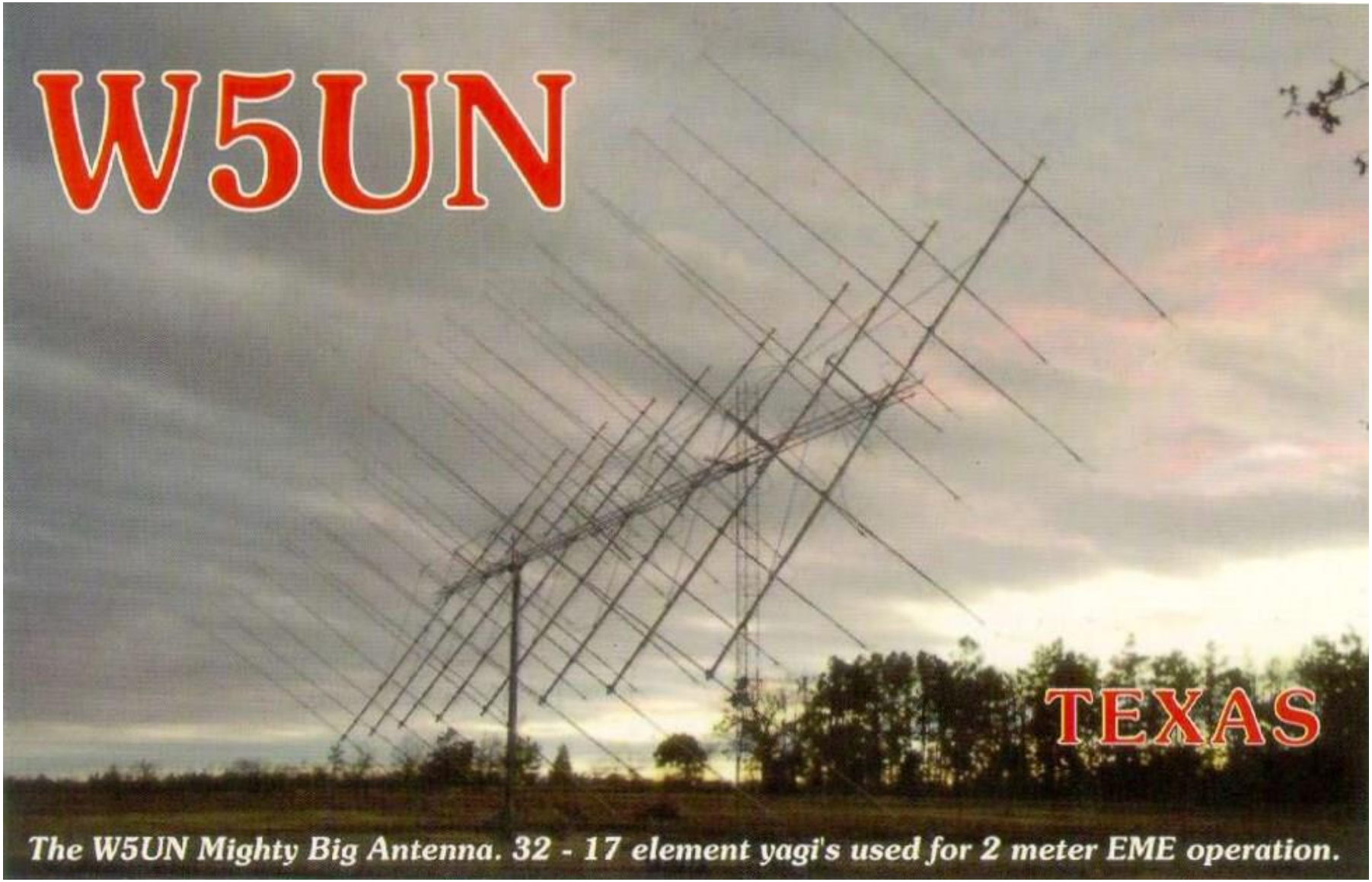


K0DK in Boulder, Colorado

K4JA at Callao, Virginia



W5UN at Mount Pleasant, Texas

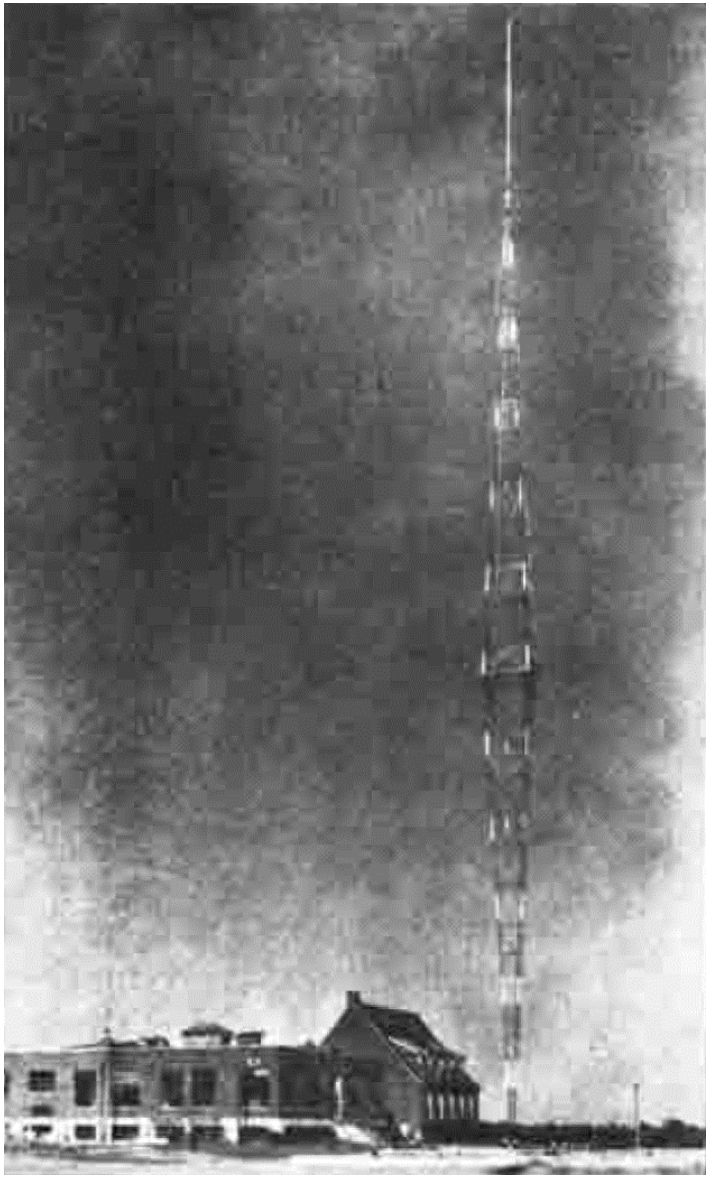


W6AM's Antennas As Seen from Space



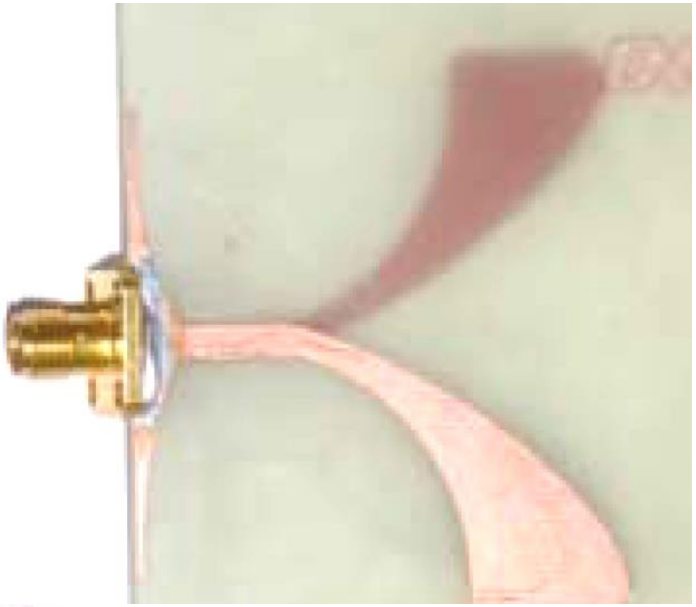
W6AM at Rancho Palos Verdes, California

WLW AM Broadcast Vertical in Mason, Ohio

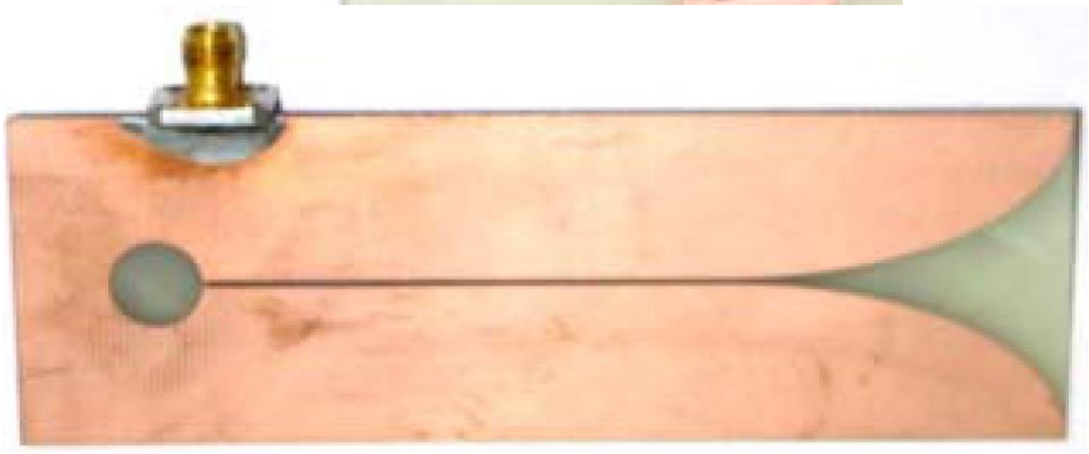


- **Station Owner: Powel Crosley, Jr.**
- **Builder: Blaw-Knox**
- **Erected 1934**
  - No. 2 built
- **Original height 831 feet**
- **Tx power 500 kW**
  - Applied for increase to 1 MW
- **Service areas**
  - Prime: Entire U.S.
  - Secondary: Europe during WWII
- **Quit the Mutual Broadcasting System because station's coverage exceeded the network's**
- **Local residents saw their lights flicker on modulation peaks**
- **People picked up WLW radio on the metal coils of mattress and bedsprings**
- **Arcing often occurred near the transmitter site**

Vivaldi Antenna (1974)

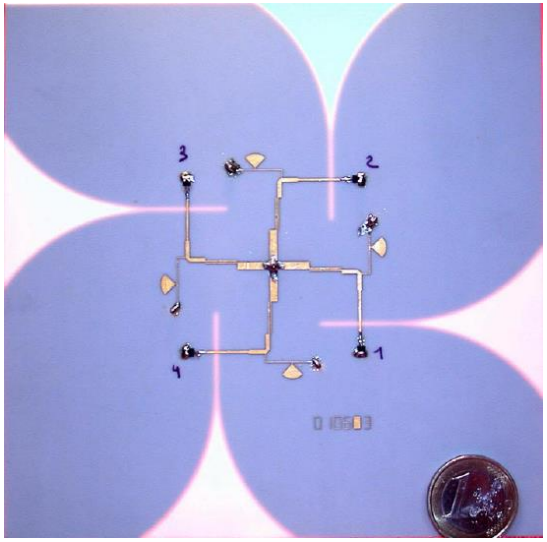


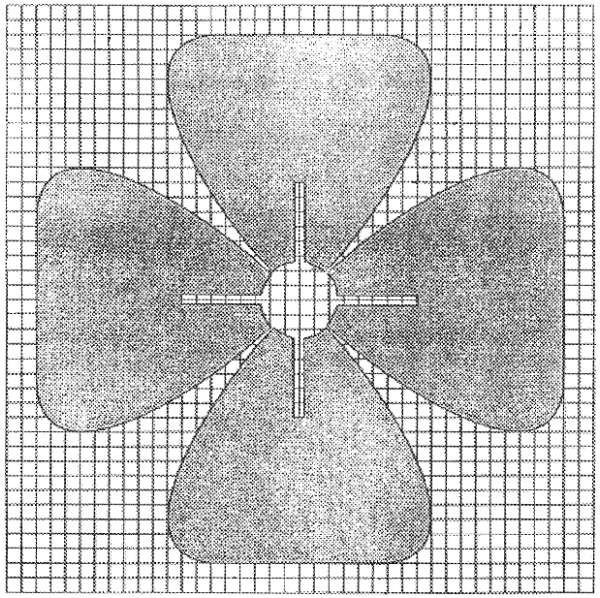
- **Exponentially tapered slot antenna**
- **Gain: 8 to 9 dBi**
- **Bandwidth: no fundamental limit**
- **Arbitrary polarizations obtained by feeding two crossed antennas**
- **Construction: PC board**



Bandwidths of one octave to one decade can be achieved

Four-Sector Vivaldi Antennas

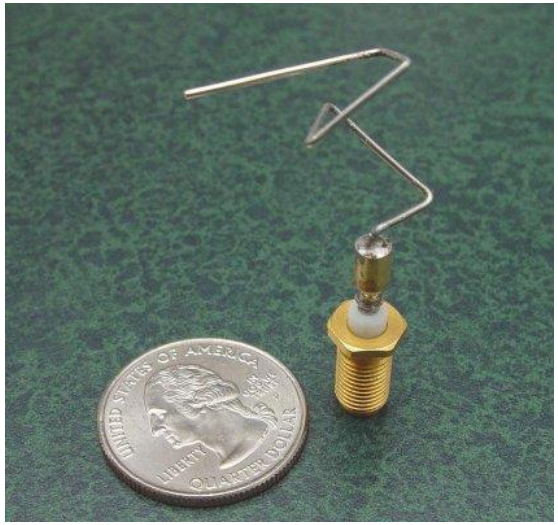


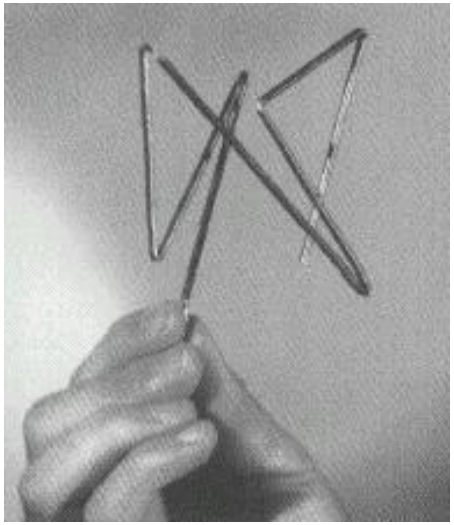


- **Antenna mounts horizontally**
- **Polarization: horizontal**
- **Four directional beams**
- **Beam selected by PIN diodes**
- **Gain: 8 to 9 dBi per sector**

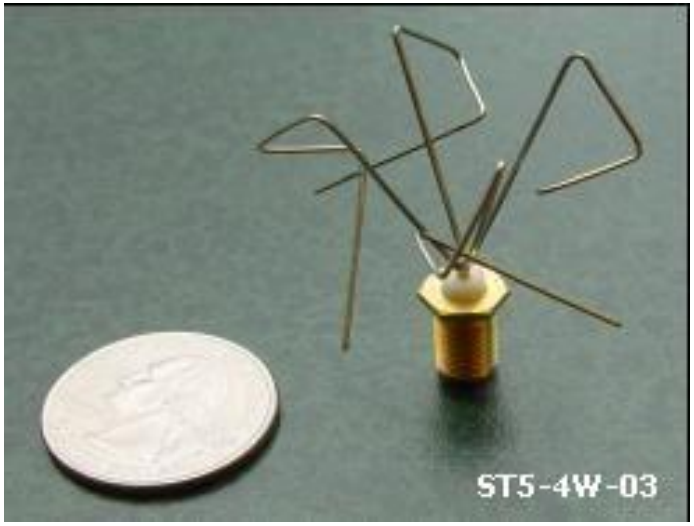
- **Dual use!**
  - Mount as a horizontal capacity hat on a short HF monopole to cover all UHF bands
  - Mount two vertically and crossed for vertical polarization in four sectors

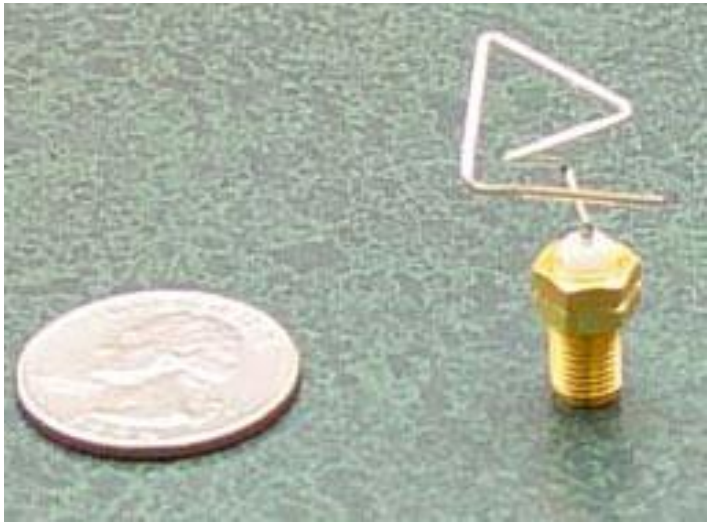
Evolved Antennas (AKA Crooked Wire Antennas)











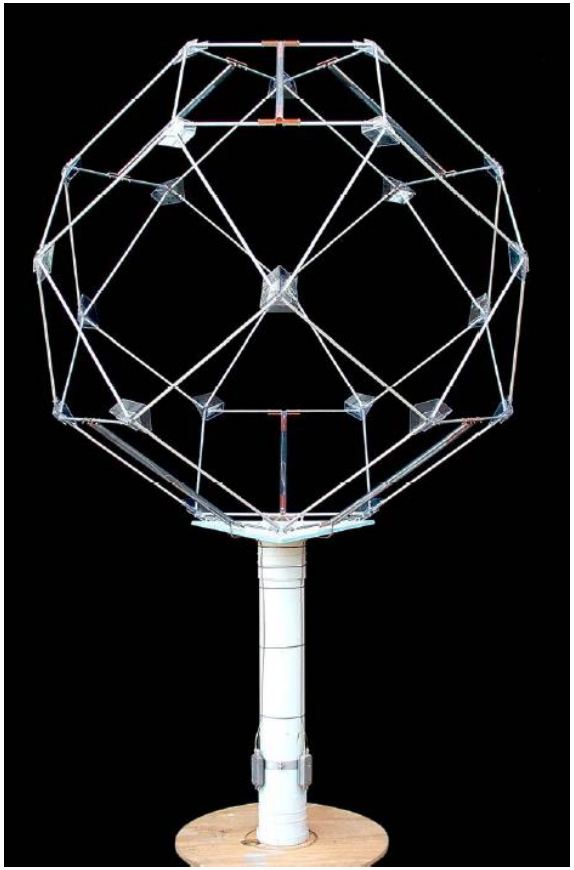
Vector Sensor Polarimeter Arrays



U.S. Navy 6-Axis Array



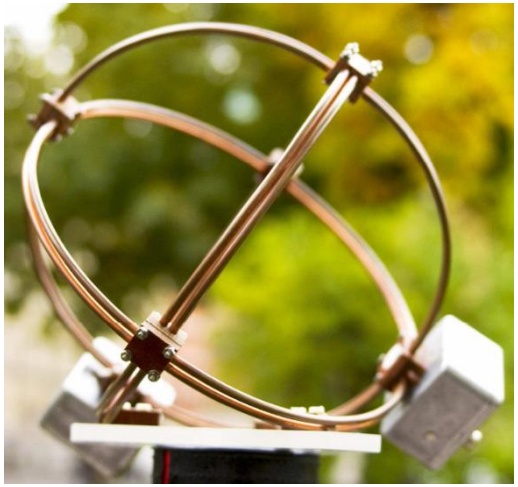
LOIS 3-Axis E Array



Martinsen's Giselle Rhombicuboctahedron Array

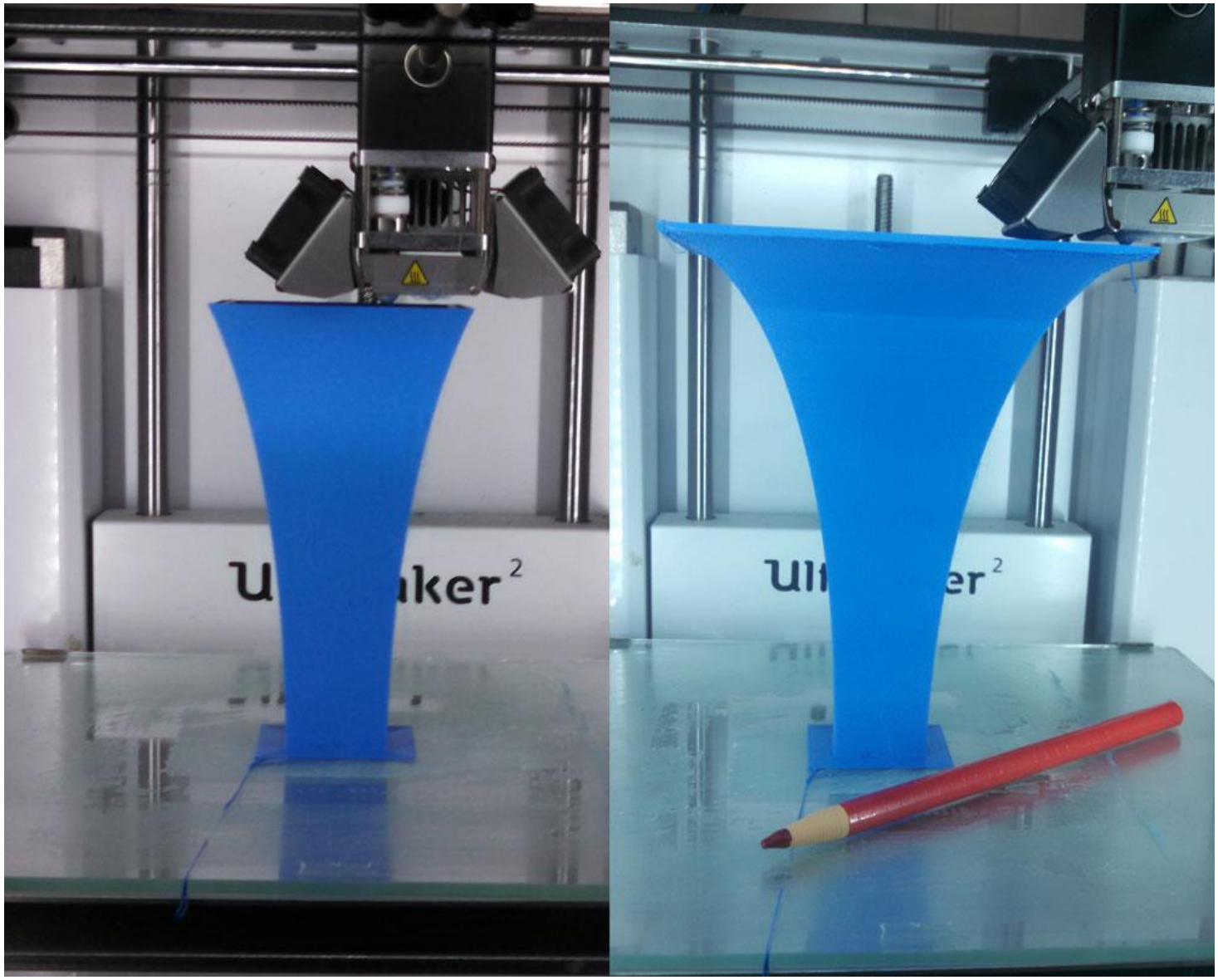


Flam & Russell CART Array



LOIS 3-Axis H Array

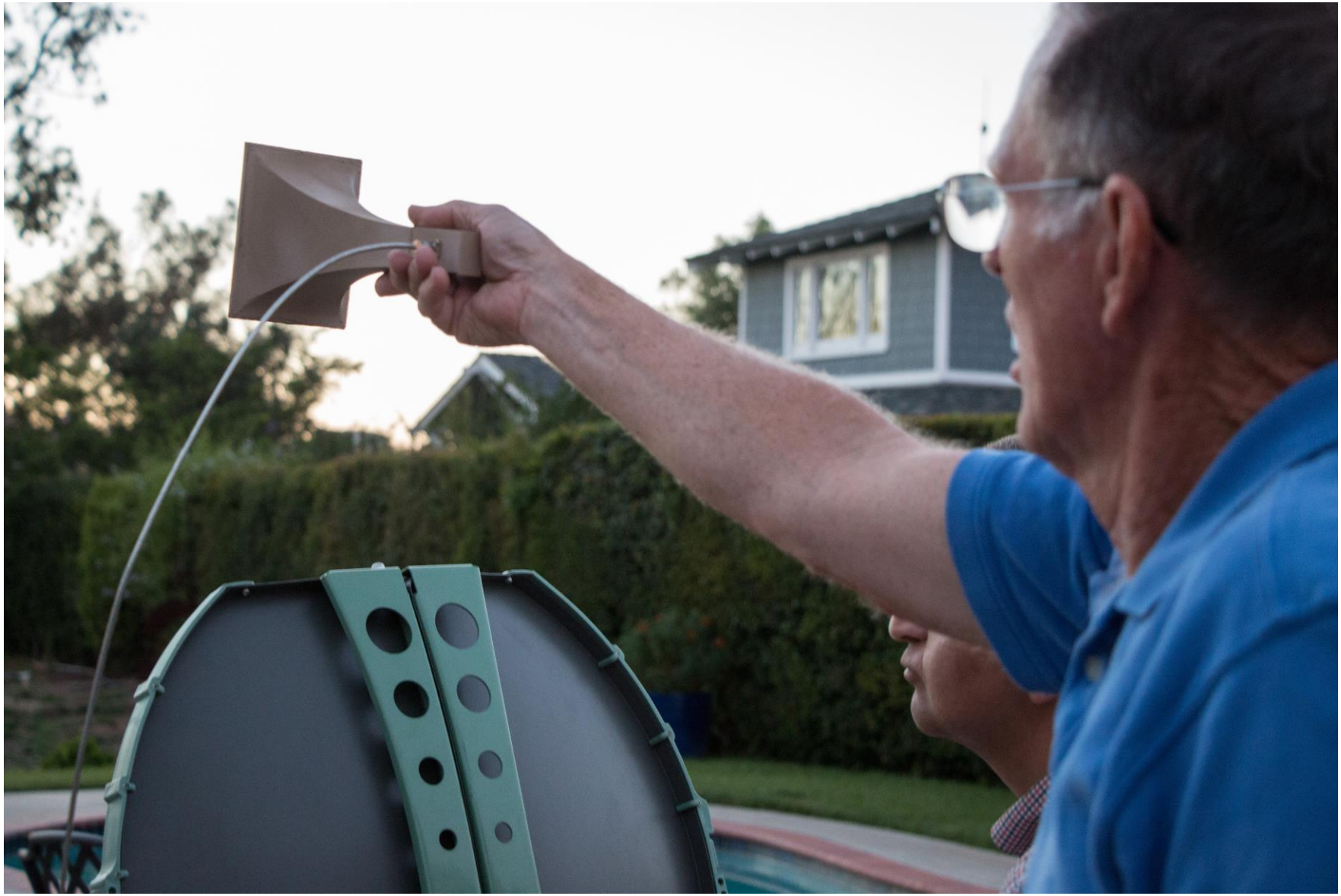
3D-Printed Antennas
W5NYV and N6IZW 10-GHz Elliptical Horn



Spray-On Metalization



Testing 10-GHz Elliptical Horn, *QEX*, Sep/Oct 2016



What is It ?
Open-Boundary Quad-Ridged Horn or Crossed Vivaldis



- **Frequency range 100 kHz to 1.5 GHz**
- **Weight 152 lbs**

V. Rodriquez, “Super-Open-Boundary Quadridge Horn Antennas,” *IEEE Int. Symp. Antennas and Propagation*, July 2011

Metamaterial Antennas

DNG, NRI, NIM Radomes

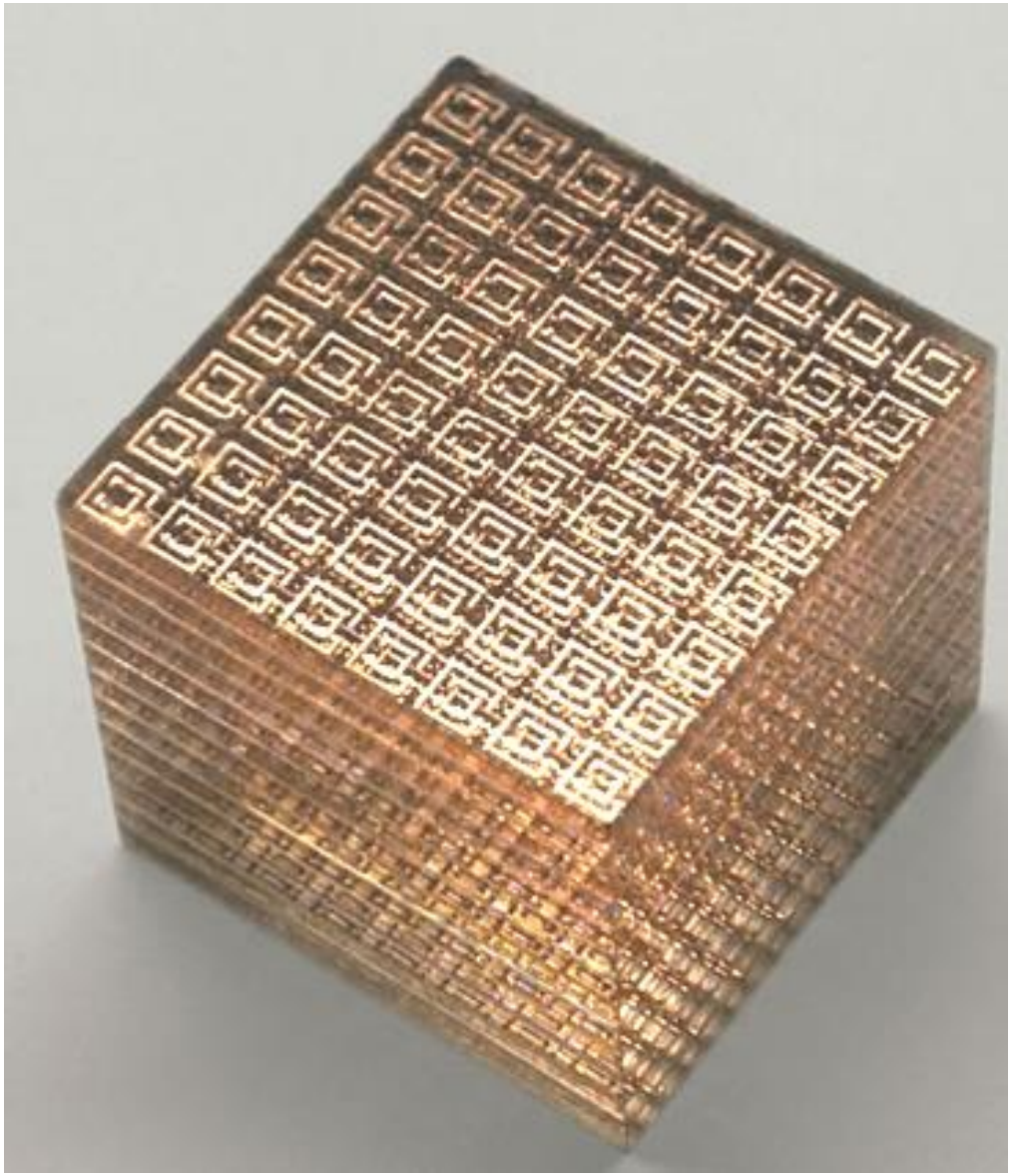
Material Classifications

Classification	Constitutive Parameters	Wave Behaviors
Artificial dielectrics	$\varepsilon_r > 1$ and $\mu_r = 1$	Ordinary waves
Sub-unity materials	$\varepsilon_r < 1$ or $\mu_r < 1$	Superluminal phase velocity
Single-negative materials (SNG) Epsilon negative (ENG) Mu negative (MNG)	 $\varepsilon_r < 0$ and $\mu_r \geq 0$ $\varepsilon_r \geq 0$ and $\mu_r < 0$	Evanescent fields Traveling waves do not exist
Double-negative materials (DNG) Negative refractive index (NRI) Negative index materials (NIM)	$\varepsilon_r < 0$ and $\mu_r < 0$	Backward waves: Energy velocity is opposite to phase velocity

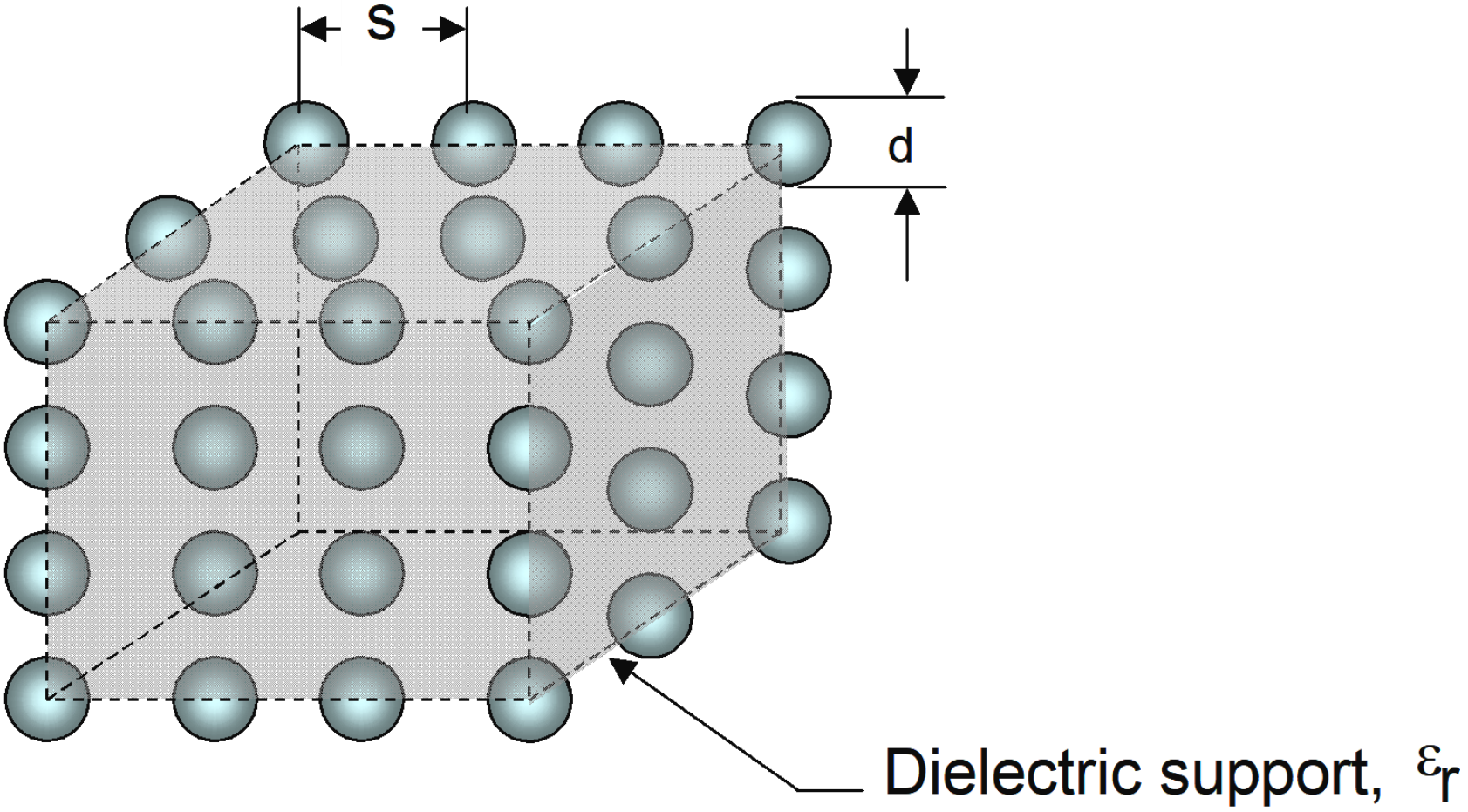
ε_r is relative permittivity (dielectric constant)

μ_r is relative magnetic permeability

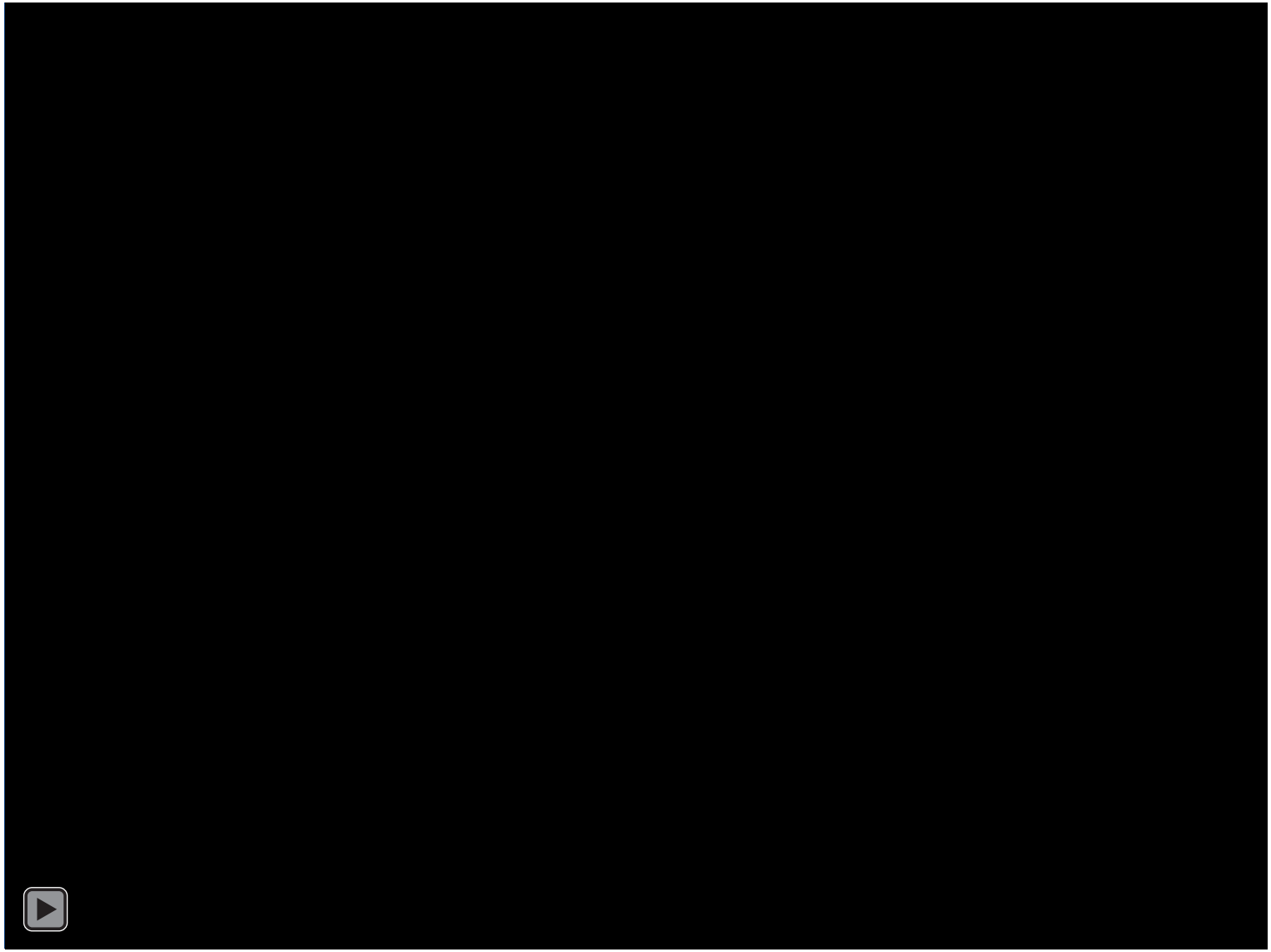
Professional Metamaterial - The Boeing Cube



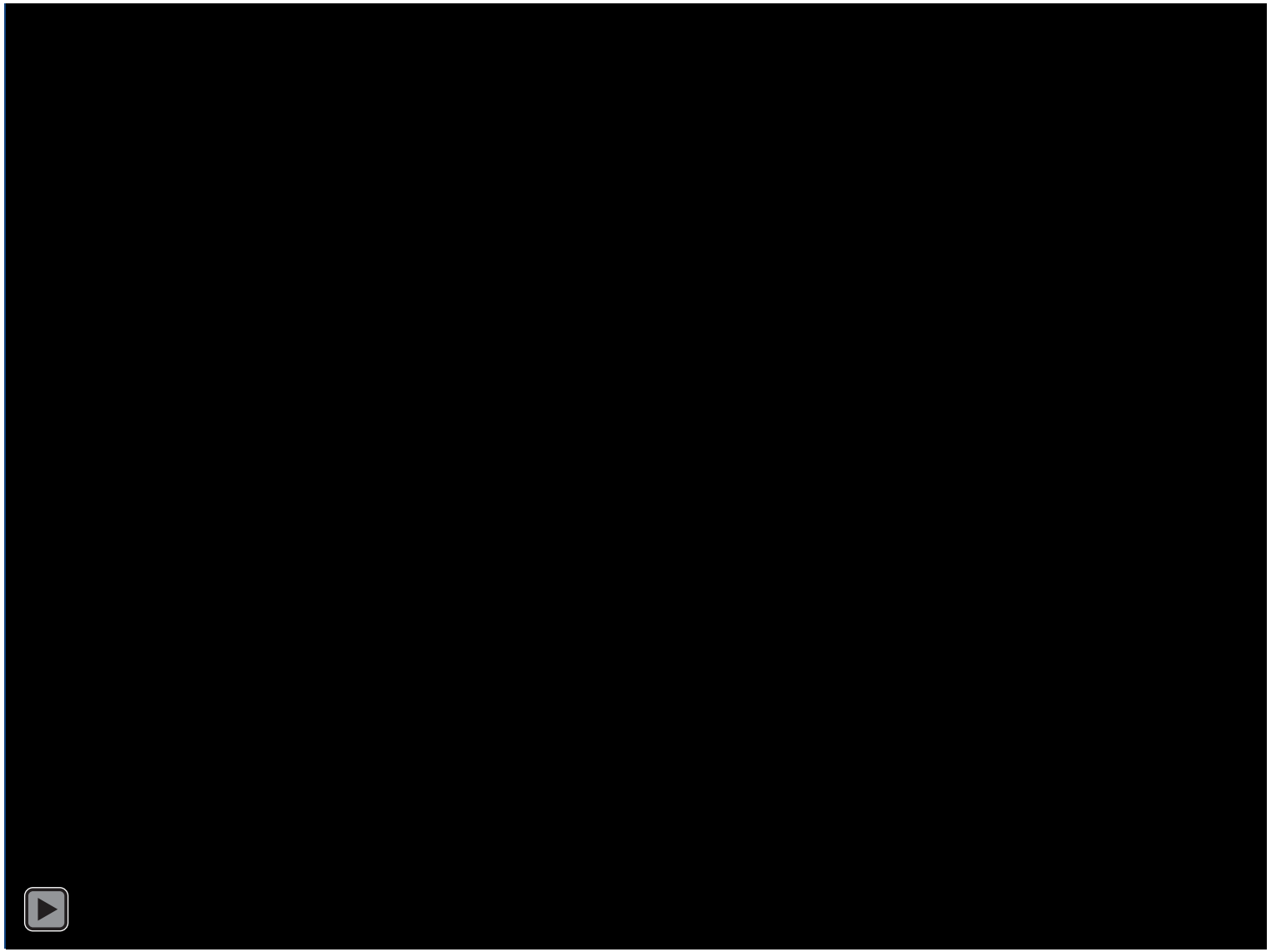
Amateur Metamaterial



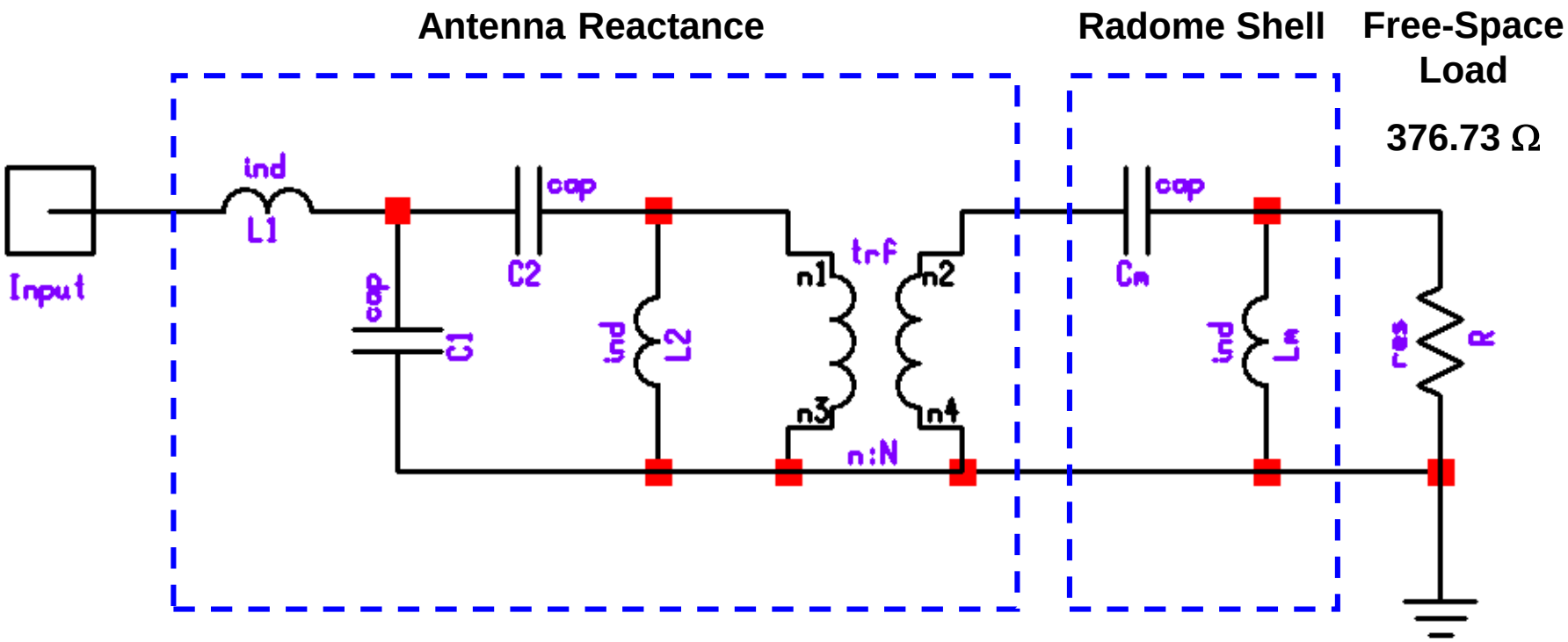
Pendry's Perfect Lens



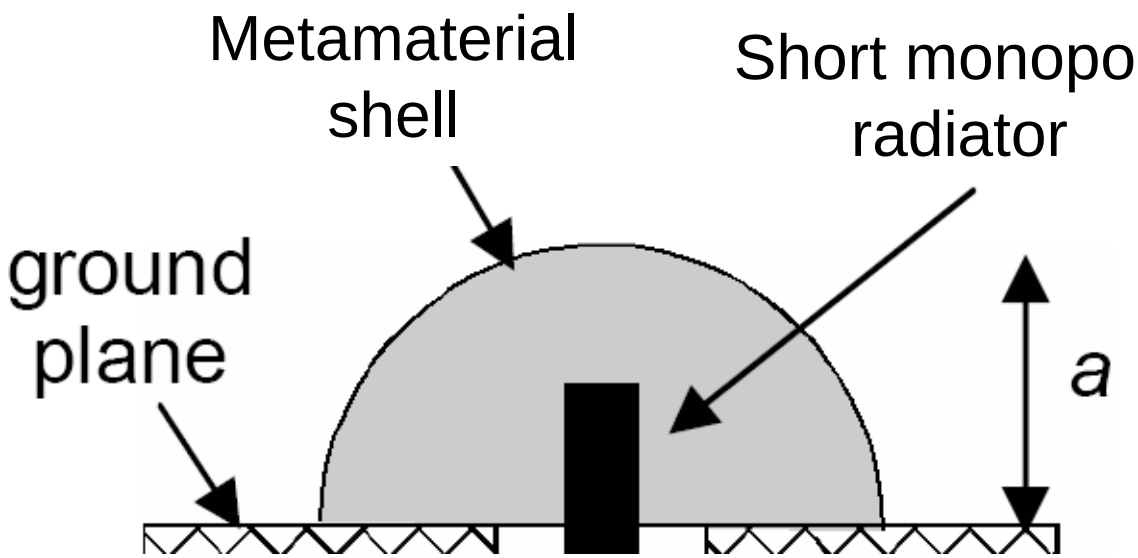
Backward Waves in a Metamaterial Slab Lens



Antenna Matching by Using a Radome Shell

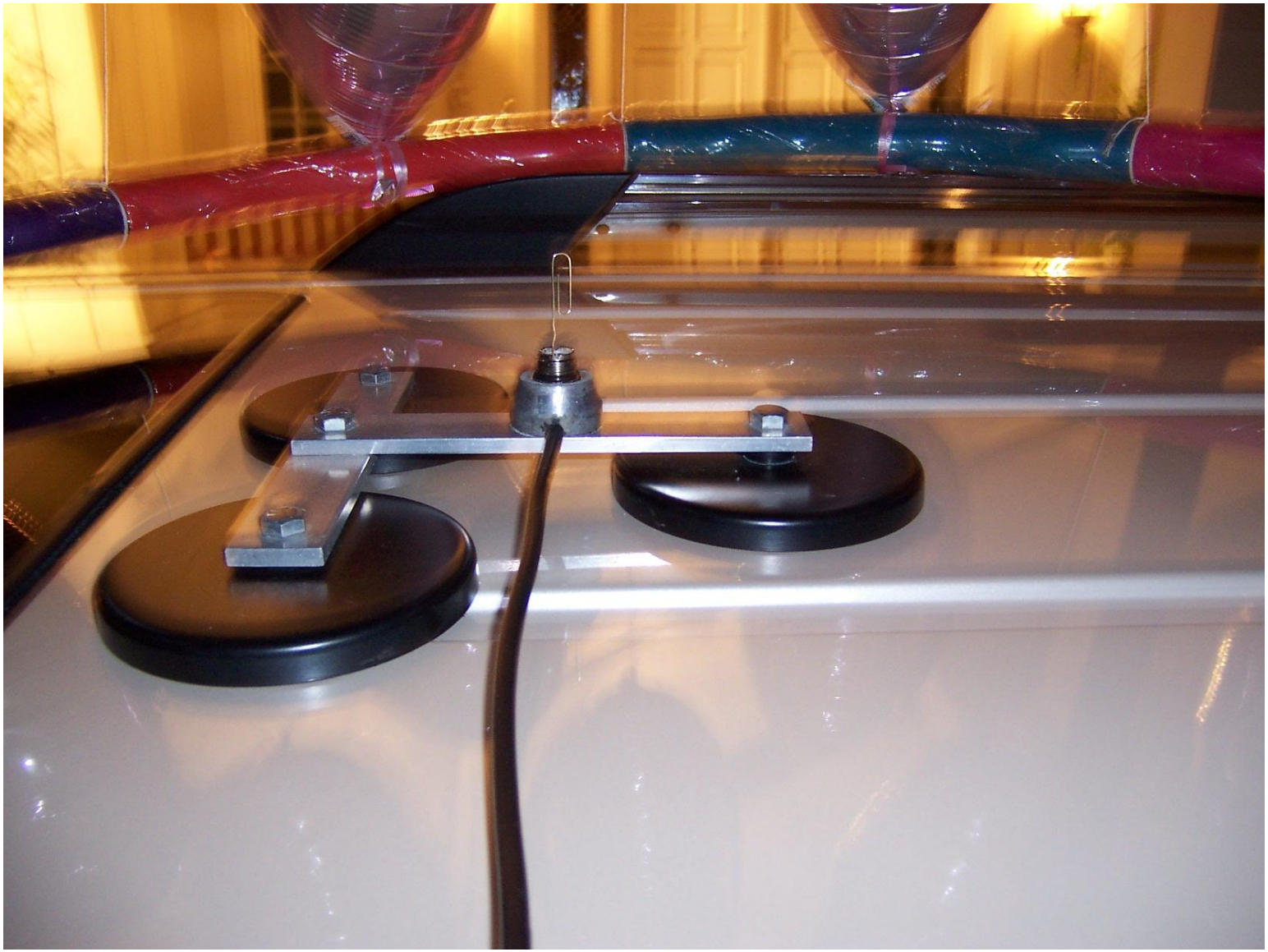


Bell Laboratories Monopole in Metamaterial Shell

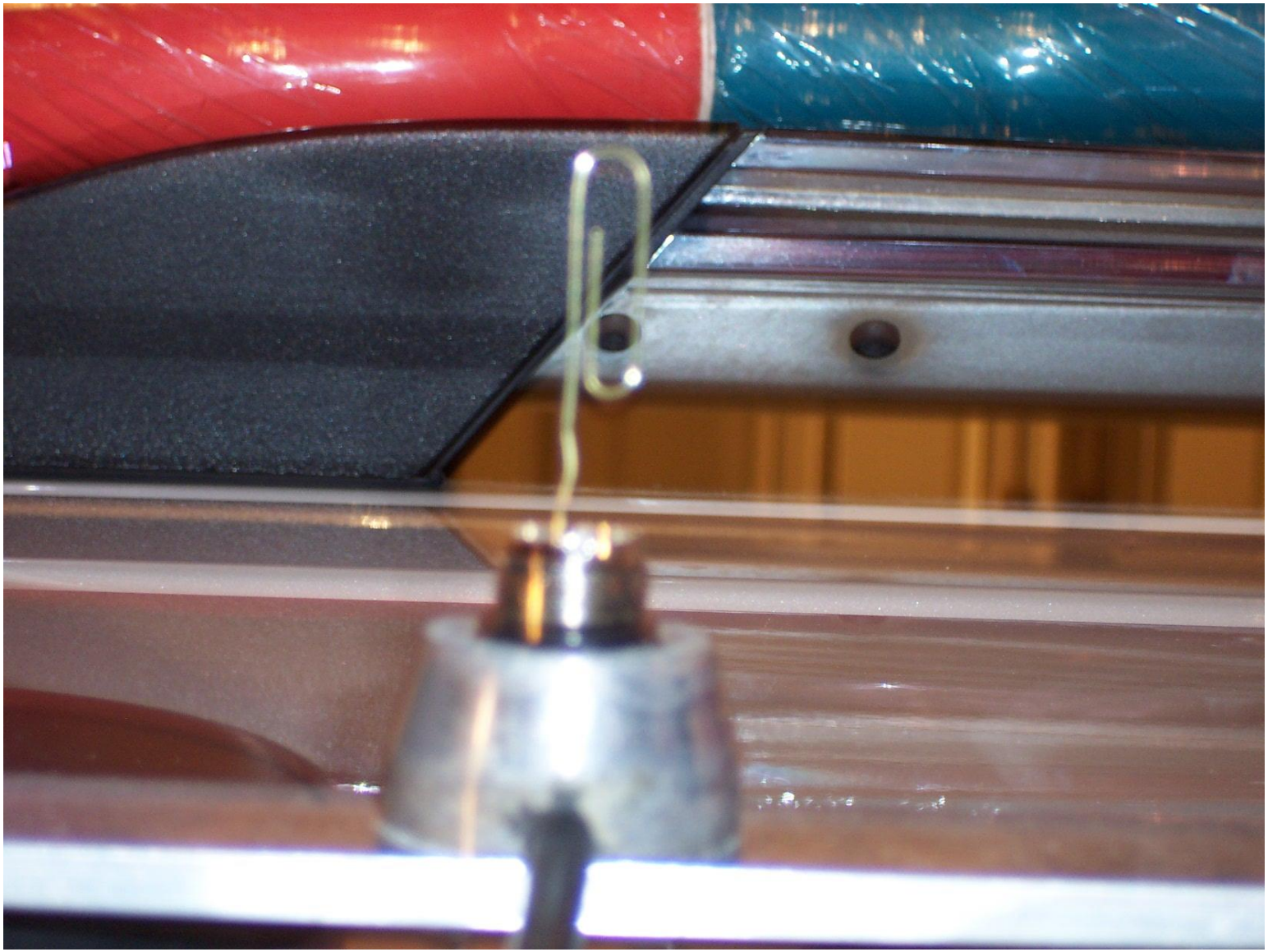


- Shell type: ENG
- Stub length: $\lambda/50$
- Shell radius: $\lambda/18.5$
- Frequency: 2.025 GHz
- *Z*: 50 ohms real
- VSWR: < 1.02
- Bandwidth: 4.76%
- Q_A: 42 (Q_{Chu} = 28.9)
- Polarization: vertical
- Efficiency: > 61%

K6OIK's Electrically-Small 2-Meter Radiator



Close Up View of the Non-Evolved 2-Meter Radiator



A Double-Negative Metamaterial Radome



2-Meter Double-Negative Radome



Linear Cylindrical Antennas

Definitions

- **A linear cylindrical antenna is a wire, rod, or tube driven at an arbitrary point (a gap) along its length**
  - Length is arbitrary
  - Feedpoint location is arbitrary
  - Arms are colinear
  - Cross-section need not be circular or convex or topologically connected
  - But current should be longitudinal
- **A simple dipole is a linear cylindrical antenna that is symmetric about its feedpoint**
  - Geometrically symmetric about feedpoint (equal arm lengths)
  - Current distribution is symmetric about feedpoint
  - Feedpoint is balanced
  - The half-wave dipole is called a “Hertz” dipole or “doublet”
- **Examples**
  - A cage dipole is a dipole
  - An off-center fed (OCF) dipole is misnamed; it is a linear cylindrical antenna but not a dipole
  - A fan dipole is not a dipole nor a linear cylindrical antenna but is made of several dipoles

Facts

- **A simple dipole is symmetric and center-fed**



A.C. source

- **For lossless antennas, directivity and gain are the same**
- **An antenna's radiation resistance is not unique. It depends on a reference current or location**
- **The resonant length of a dipole depends on its diameter**
- **Dipoles are resonant at lengths slightly shorter than an odd number of half-wavelengths**
  - The resonant length of a "half-wave" dipole or doublet is $L = \frac{K\lambda}{2}$
  - $K < 1$ depends on resonance number and dipole fatness
- **Dipoles are anti-resonant at lengths slightly shorter than an even number of half-wavelengths**
- **If linear antenna is resonant, then its feedpoint impedance is real everywhere along its length**
- **If a dipole is a half-wavelength, then its current phase is about 30° everywhere along its length (phase reference is feed voltage)**

Fictions

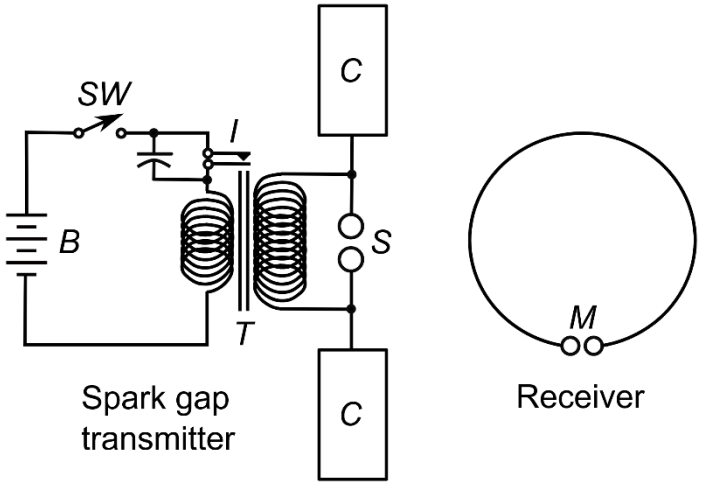
- A dipole should be one-half wavelength long
- Half-wavelength dipoles are resonant
- Dipoles are 75 ohms
- In free space, a half-wavelength dipole has a real (resistive) feedpoint impedance
- A half-wavelength dipole is 50 ohms
- The feedpoint resistance of a half-wavelength dipole depends on its diameter
- The feedpoint reactance of a half-wavelength dipole depends on its diameter
- Dipoles are anti-resonant at lengths slightly longer than an even number of half-wavelengths

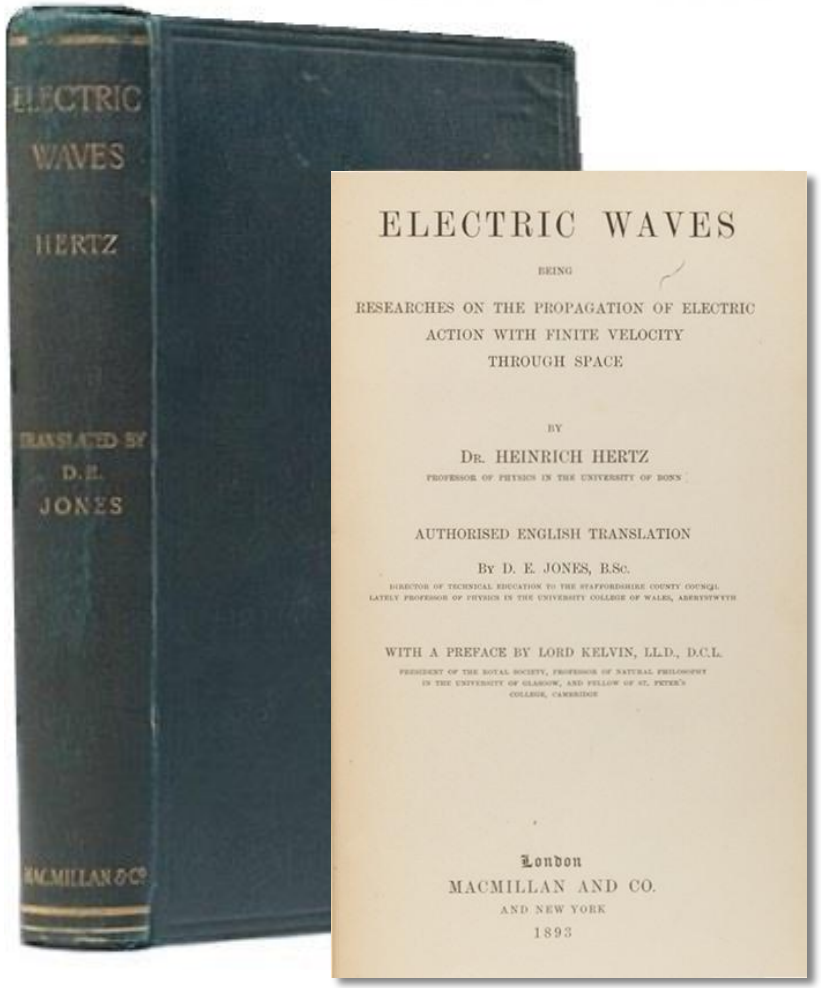
Dipole Current, Impedance, and Resonant Length

The Dipole's Origin – Heinrich Hertz, 1887



Heinrich Rudolph Hertz, 1857-1894





The first antenna book, 1893

Milestones in Cylindrical Antennas

- **Heinrich Hertz's experiments (1887)**
- **Pocklington's integro-differential equation (1897)**
  - Solved numerically in NEC
- **Induced EMF (IEMF) method (1922)**
  - L. Brillouin, *Radioélectricité*, April 1922
  - A.A. Pistolkors, *Proc. IRE*, March 1929
  - P.S. Carter, *Proc. IRE*, June 1932
  - S.A. Schelkunoff, papers and books 1941-1954
  - C-T. Tai, *J. Applied Physics*, July 1949
- **Hallén's integral equation (1938)**
  - E. Hallén at Uppsala University, Sweden
  - C.J. Bouwkamp at Philips Labs, Holland
  - R.W.P. King and students at Harvard University
    - F.G. Blake
    - C.W. Harrison
    - D. Middleton
  - S.A. Schelkunoff and M.C. Gray at Bell Labs
- **Numerical methods for antenna analysis (1960's)**

Sergei Alexander Schelkunoff, 1897-1992



Sergei Schelkunoff was a pioneer in antenna theory and electromagnetics at Bell Telephone Laboratories.

Marion Cameron Gray, 1902-1979



Marion Gray was a prolific mathematician from Scotland who worked at Bell Telephone Laboratories. She devised a better way to solve Hallén's equation.

Linear Cylindrical Antennas



Ronald Wyeth Percival King, 1905-2006

- Speaking at his 100th birthday party, October 2005
- Cruft Laboratories, Harvard University
- Authority on linear cylindrical antennas
- Spent his career on solving Hallén's equation, starting in 1938
- Over 100 doctoral students, many of whom worked on Hallén's equation – Tai, Blake, Harrison, Middleton, Long, Wu, Papas, Simpson, …

Integral Equations for Dipole Current

- **Pocklington's equation (1897)**

$$\int_{-l}^{l} I_z(z')\left[\left(\frac{\partial^2}{\partial z^2}+k^2\right)G(z,z')\right]dz' = -j\omega\varepsilon E_z^i(\rho=a)$$

- **Hallen's equation (1938)**

$$\int_{-l}^{l} I_z(z')\frac{e^{-jkR}}{4\pi R}d\,z' = -j\sqrt{\frac{\varepsilon}{\mu}}\left[B_1\cos(kz)+C_1\sin(k\,|\,z\,|)\right]$$

- **General form**

$$L(f) = g$$

Linear operator → $L(f)$ ← **Driving function**

Unknown function → f

David Middleton, 1920-2008



David Middleton was a brilliant theoretician in statistical communication theory. Early in his career, he developed better solutions to Hallén's equation.

Chen-To Tai, 1915-2004



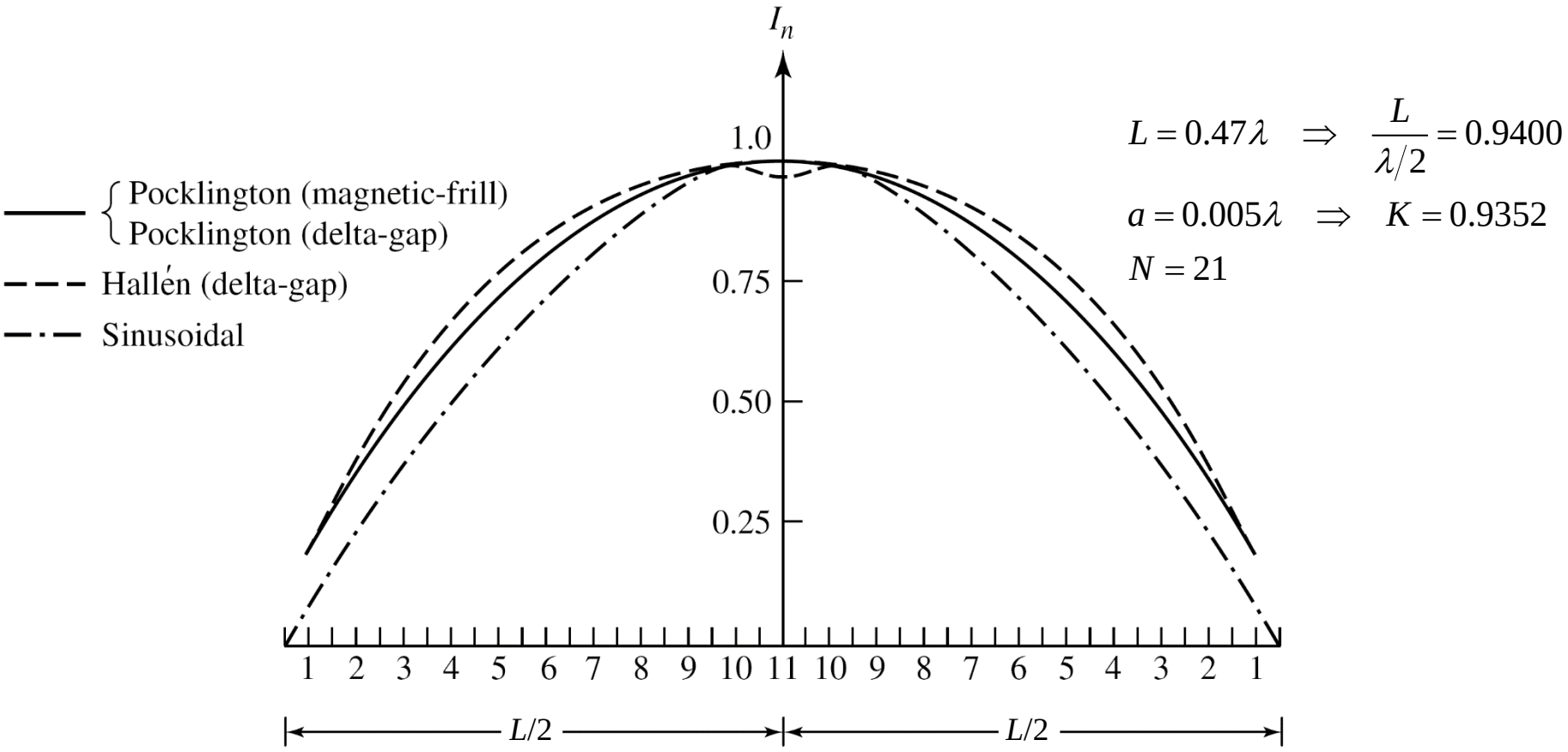
Professor Chen-To Tai of the University of Michigan was a famous researcher in electromagnetics and antenna theory.

John Daniel Kraus, 1910-2004



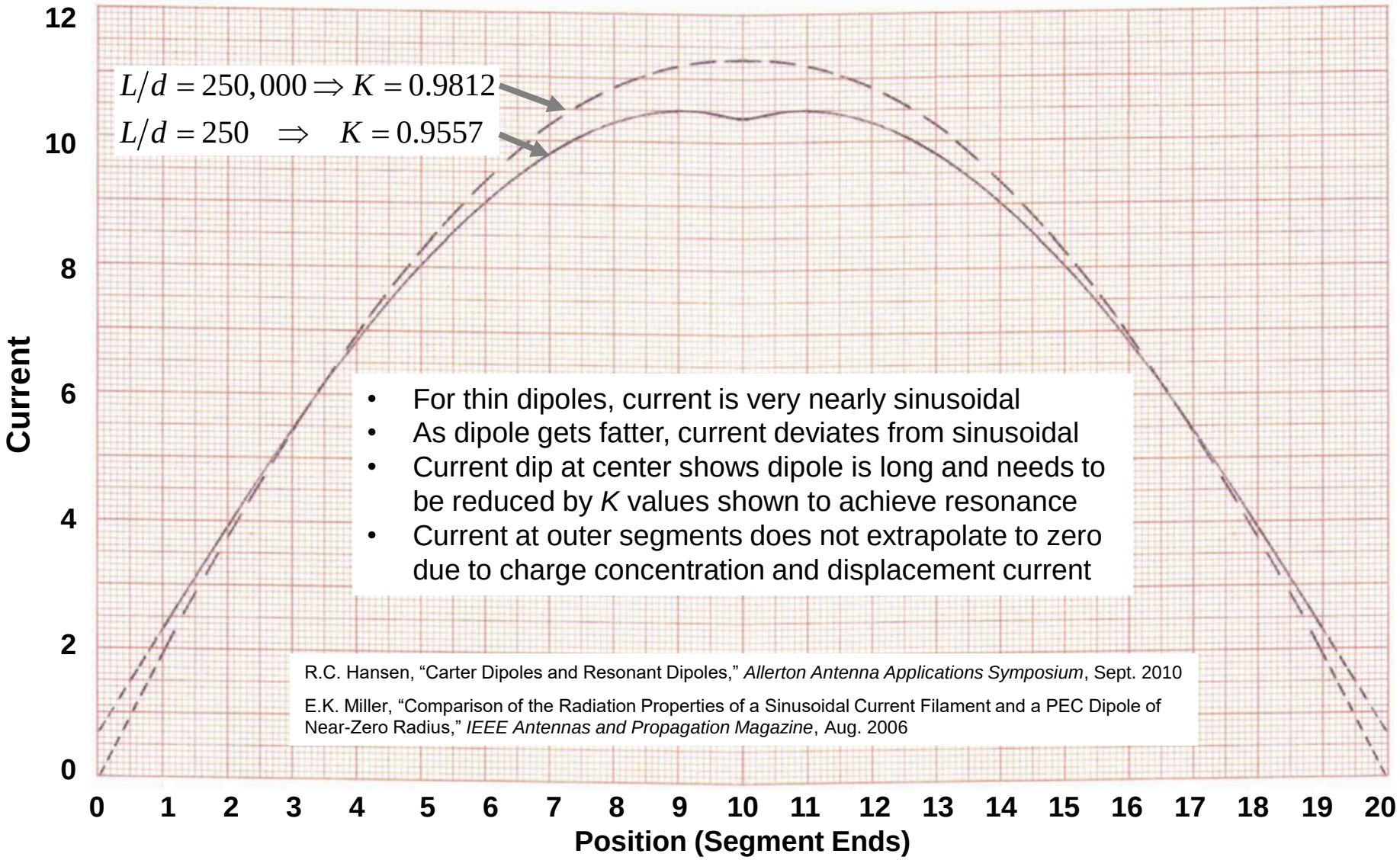
Professor John Kraus of the Ohio State University authored the famous book *Antennas* and taught a generation of antenna engineers.

Current Distribution of an Almost Resonant Dipole



C.A. Balanis, *Antenna Theory: Analysis and Design*, 4th ed., p. 457, Wiley, 2016

Current Distribution on Two Halfwave Dipoles



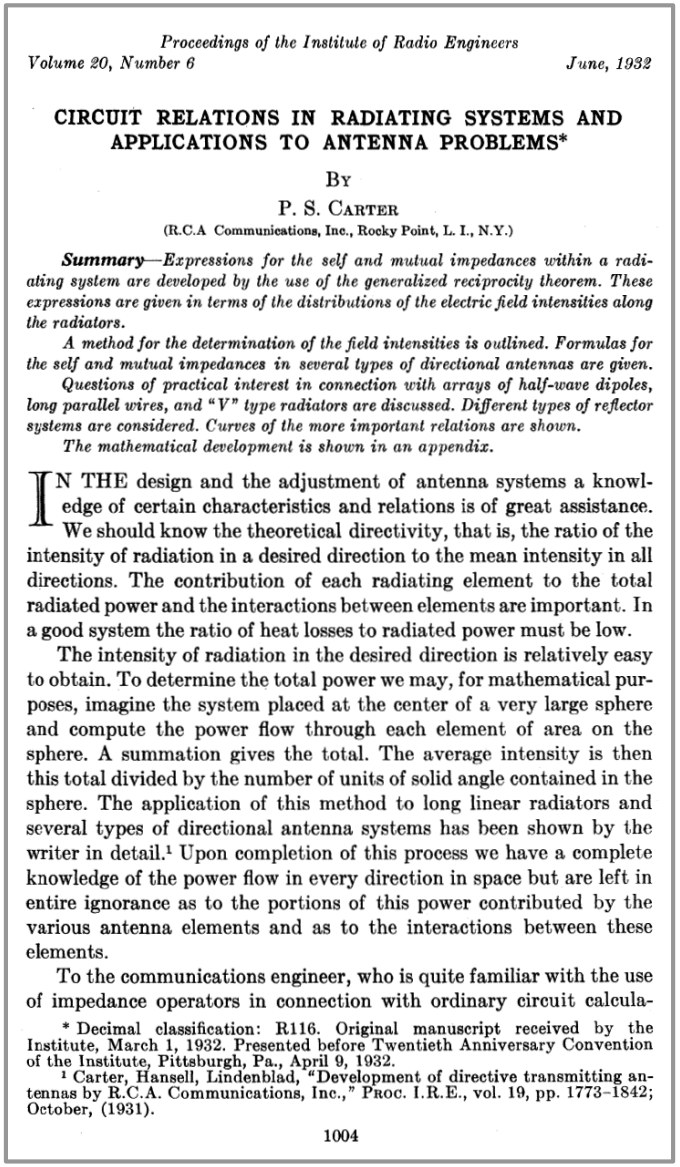
- For thin dipoles, current is very nearly sinusoidal
- As dipole gets fatter, current deviates from sinusoidal
- Current dip at center shows dipole is long and needs to be reduced by *K* values shown to achieve resonance
- Current at outer segments does not extrapolate to zero due to charge concentration and displacement current

R.C. Hansen, "Carter Dipoles and Resonant Dipoles," *Allerton Antenna Applications Symposium*, Sept. 2010

E.K. Miller, "Comparison of the Radiation Properties of a Sinusoidal Current Filament and a PEC Dipole of Near-Zero Radius," *IEEE Antennas and Propagation Magazine*, Aug. 2006

Induced EMF Method

- L. Brillouin, *Radioélectricité*, April 1922
- A.A. Pistolkors, *Proc. IRE*, March 1929
- P.S. Carter, *Proc. IRE*, June 1932
- C-T. Tai, *J. Applied Physics*, July 1949
- Assume sinusoidal current distribution
- Obtain pattern, radiation resistance and reactance
- Accurate for pattern and impedance of dipoles up to half-wavelength and verticals up to quarter-wavelength
- Inaccurate for impedance of dipoles longer than half-wavelength and verticals longer than quarter-wavelength
- Widely used for the design of AM broadcast towers
- Obsoleted by numerical methods



Proceedings of the Institute of Radio Engineers
Volume 20, Number 6 *June, 1932*

CIRCUIT RELATIONS IN RADIATING SYSTEMS AND APPLICATIONS TO ANTENNA PROBLEMS*

By

P. S. Carter

(R.C.A Communications, Inc., Rocky Point, L. I., N.Y.)

Summary—*Expressions for the self and mutual impedances within a radiating system are developed by the use of the generalized reciprocity theorem. These expressions are given in terms of the distributions of the electric field intensities along the radiators.*

A method for the determination of the field intensities is outlined. Formulas for the self and mutual impedances in several types of directional antennas are given.

Questions of practical interest in connection with arrays of half-wave dipoles, long parallel wires, and "V" type radiators are discussed. Different types of reflector systems are considered. Curves of the more important relations are shown.

The mathematical development is shown in an appendix.

IN THE design and the adjustment of antenna systems a knowledge of certain characteristics and relations is of great assistance. We should know the theoretical directivity, that is, the ratio of the intensity of radiation in a desired direction to the mean intensity in all directions. The contribution of each radiating element to the total radiated power and the interactions between elements are important. In a good system the ratio of heat losses to radiated power must be low.

The intensity of radiation in the desired direction is relatively easy to obtain. To determine the total power we may, for mathematical purposes, imagine the system placed at the center of a very large sphere and compute the power flow through each element of area on the sphere. A summation gives the total. The average intensity is then this total divided by the number of units of solid angle contained in the sphere. The application of this method to long linear radiators and several types of directional antenna systems has been shown by the writer in detail.[1] Upon completion of this process we have a complete knowledge of the power flow in every direction in space but are left in entire ignorance as to the portions of this power contributed by the various antenna elements and as to the interactions between these elements.

To the communications engineer, who is quite familiar with the use of impedance operators in connection with ordinary circuit calcula-

* Decimal classification: R116. Original manuscript received by the Institute, March 1, 1932. Presented before Twentieth Anniversary Convention of the Institute, Pittsburgh, Pa., April 9, 1932.

[1] Carter, Hansell, Lindenblad, "Development of directive transmitting antennas by R.C.A. Communications, Inc.," Proc. I.R.E., vol. 19, pp. 1773–1842; October, (1931).

1004

Dipole Impedance via the Induced EMF Method

- **Resistance**

$$R_{in} = \frac{\eta}{4\pi \sin^2(kl)} \left\{ 2\gamma + 2\ln(2kl) - 2\mathrm{Ci}(2kl) + \sin(2kl)\left[\mathrm{Si}(4kl) - 2\mathrm{Si}(2kl) \right] + \cos(2kl)\left[\mathrm{Ci}(4kl) - 2\mathrm{Ci}(2kl) + \gamma + \ln(kl) \right] \right\}$$

Terms vanish when l / λ is a half integer; impedance is independent of wire diameter for such lengths

- **Reactance**

$$X_{in} = \frac{\eta}{4\pi \sin^2(kl)} \left\{ 2\mathrm{Si}(2kl) - \cos(2kl)\left[\mathrm{Si}(4kl) - 2\mathrm{Si}(2kl) \right] + \sin(2kl)\left[\mathrm{Ci}(4kl) - 2\mathrm{Ci}(2kl) + \gamma + 2\ln(ka) - \ln(kl) \right] \right\}$$

Wire radius affects only reactance

- **Assumptions**
  - Circular cylindrical dipole
  - In free space
  - Made of PEC → no losses
  - Length $L = 2l \leq \lambda/2$
  - Radius a

Halfwave Dipole – Impedance to 10 Digits

- **Resistance**

$$R_{in}\left(\frac{\pi}{2}, ka\right) = \frac{\eta}{4\pi}\left[\gamma + \ln(2\pi) - \mathrm{Ci}(2\pi)\right] = \frac{\eta}{4\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(2\pi)^{2n}}{2n(2n)!} = 29.9792458 \times 2.437653393$$

$$R_{in} = 73.07901025 \text{ ohms}$$

- **Reactance**

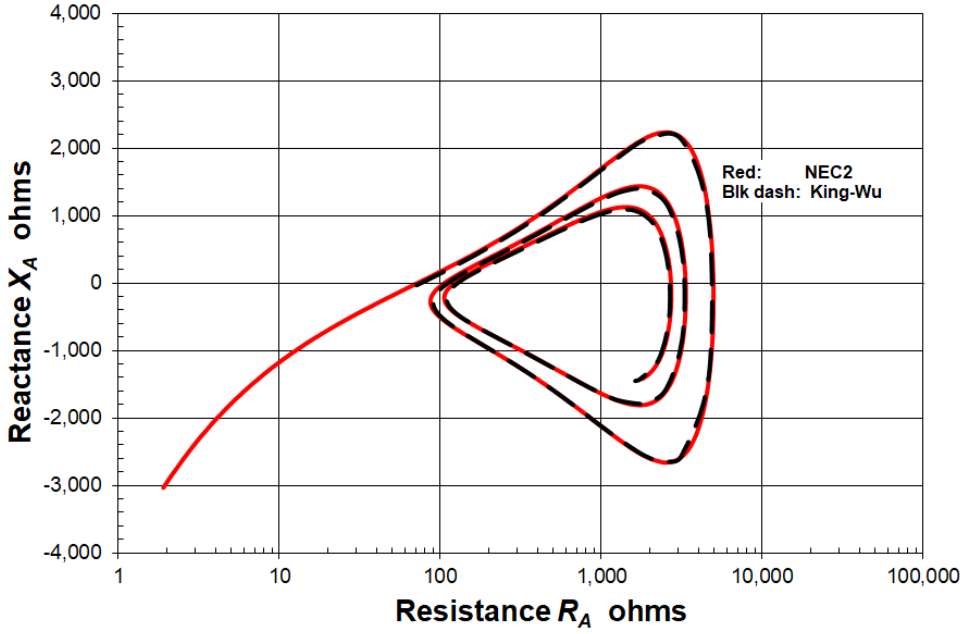
$$X_{in}\left(\frac{\pi}{2}, ka\right) = \frac{\eta\,\mathrm{Si}(2\pi)}{4\pi} = \frac{\eta}{4\pi}\sum_{n=0}^{\infty}\frac{(-1)^{n}(2\pi)^{2n+1}}{(2n+1)(2n+1)!} = 29.9792458 \times 1.418151576$$

$$X_{in} = 42.51511468 \text{ ohms}$$

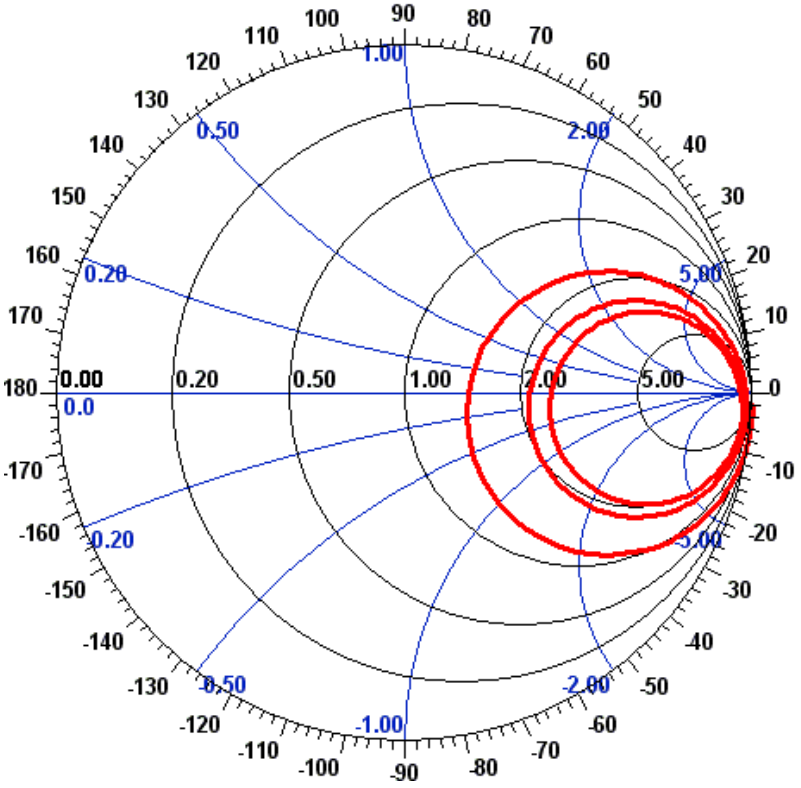
- **Is there practical value to such precise numbers?**
- **Yes, exact theoretical values are needed to validate the accuracy of numerical codes like NEC, FEKO, WIPL-D, and HOBBIES**

Complex Impedance as Frequency is Swept

ARRL Semilog Impedance Plane

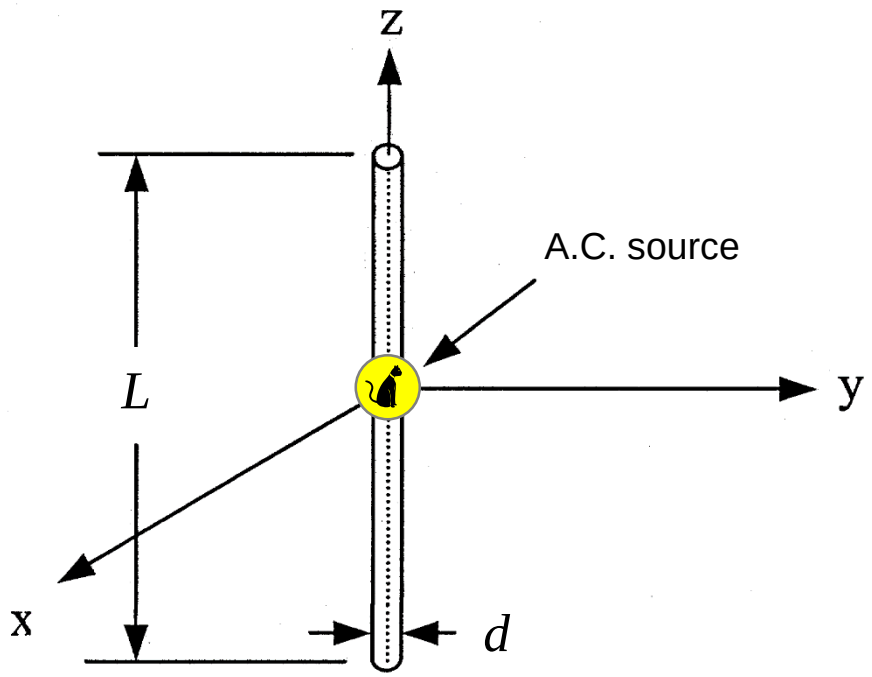


Smith Chart



- Antenna: 98.4-foot dipole in free space
- Wire: #10 AWG
- *L/d* = 11,000
- Frequency: 1 MHz to 30 MHz

Q of Small Dipole from Electromagnetic Field Analysis

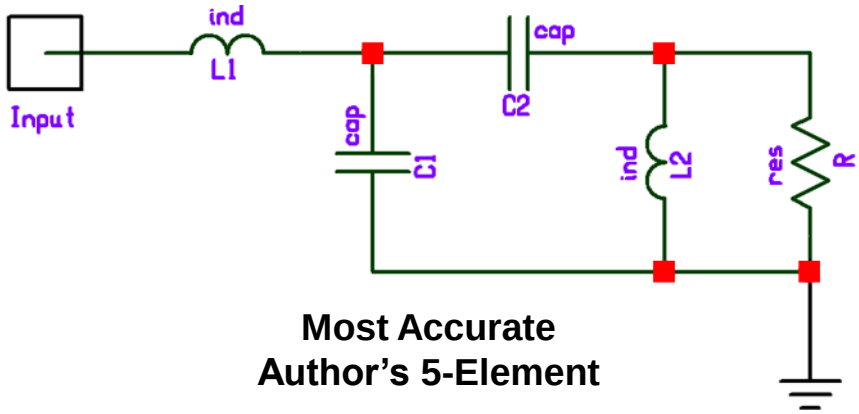


$$Q_{dipole} \approx \frac{6\left[\ln\left(\frac{L}{d}\right)-1\right]}{\pi^3\left(\frac{fL}{c}\right)^3}$$

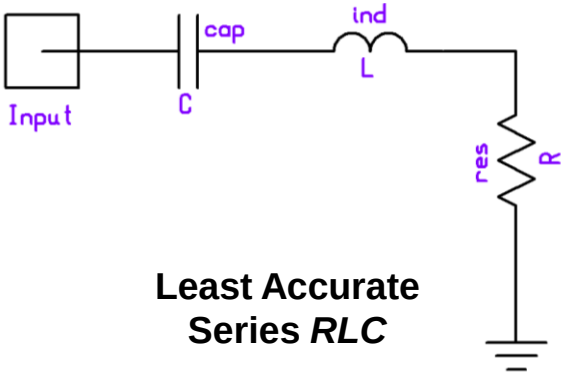
Equivalent Circuits for Dipole Impedance

Narrowband and Very Broadband

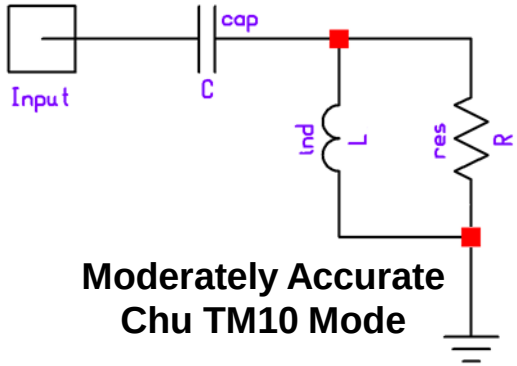
Narrowband Equivalent Circuits Near 1st Resonance



Most Accurate
Author's 5-Element



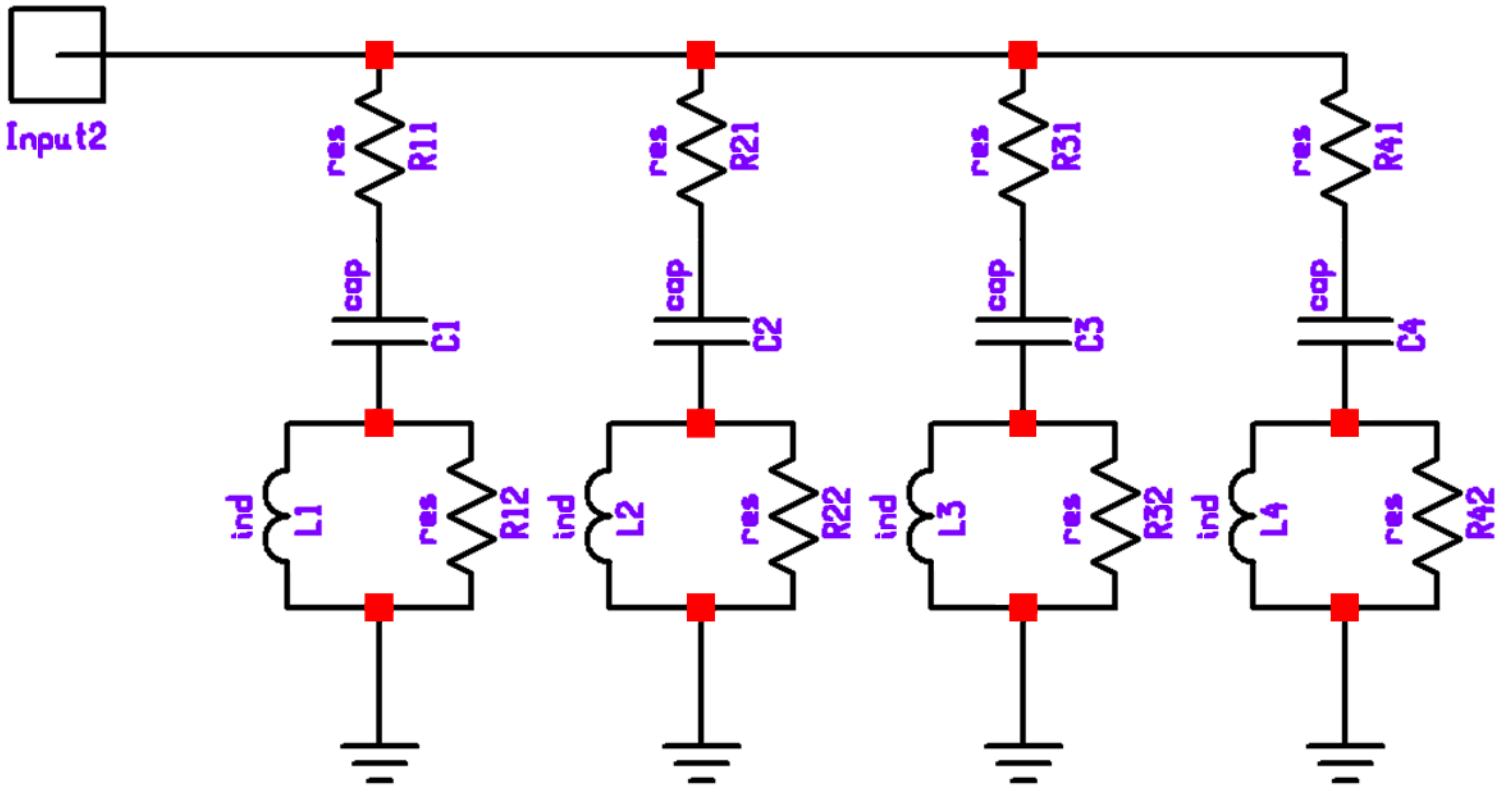
Least Accurate
Series *RLC*



Moderately Accurate
Chu TM10 Mode

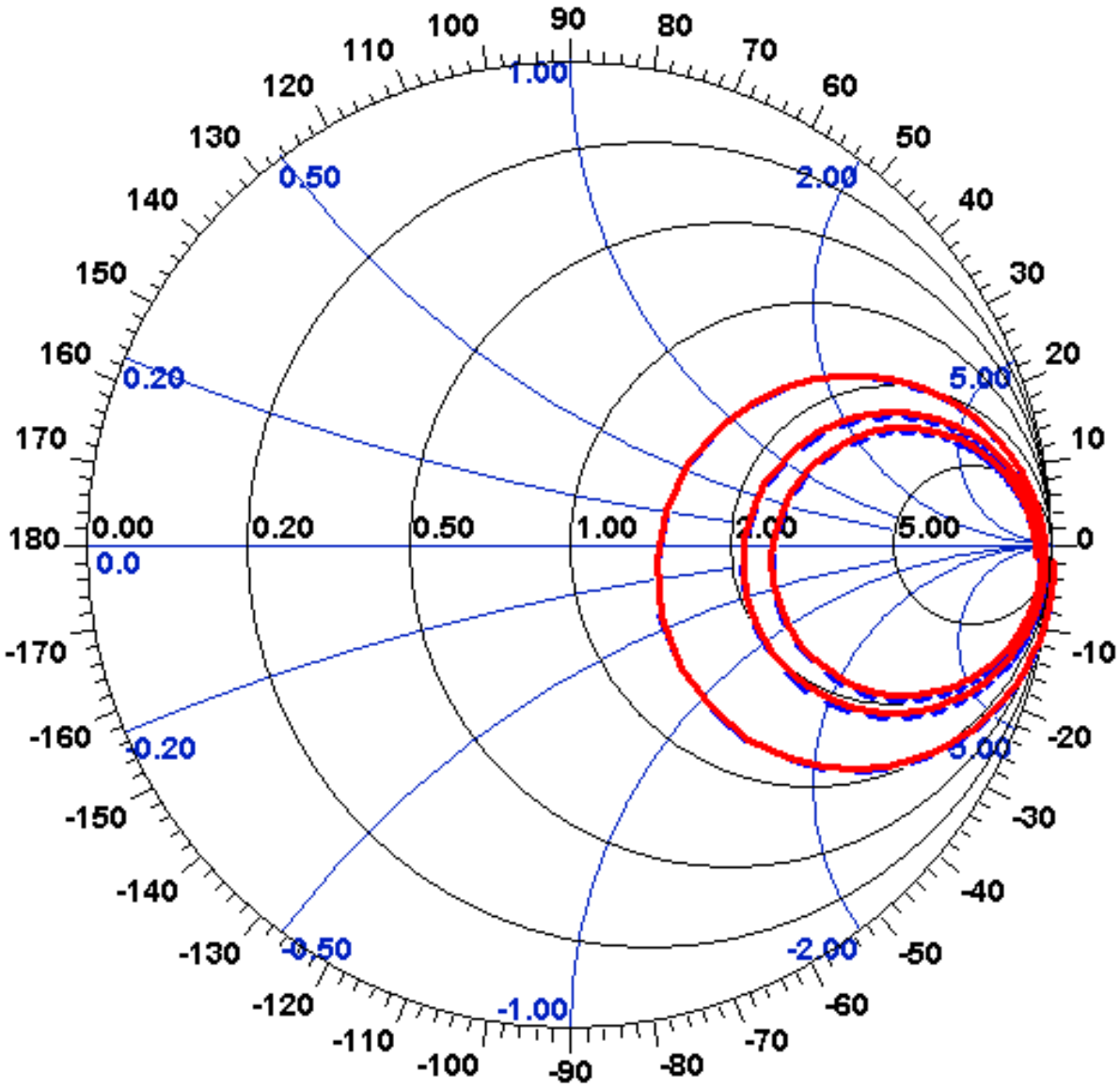
- **All are lumped element circuits in Darlington form**
  - Cauer reactance 2-port with resistor termination)
- **Resistor represents radiation plus loss resistances**
- **All are determined by fitting to dipole impedance data, computed or measured, or by continued fraction synthesis**

Broadband Equivalent Circuit for 98.4-foot Dipole (*L*/*d* = 11,000) from Zero to 30 MHz

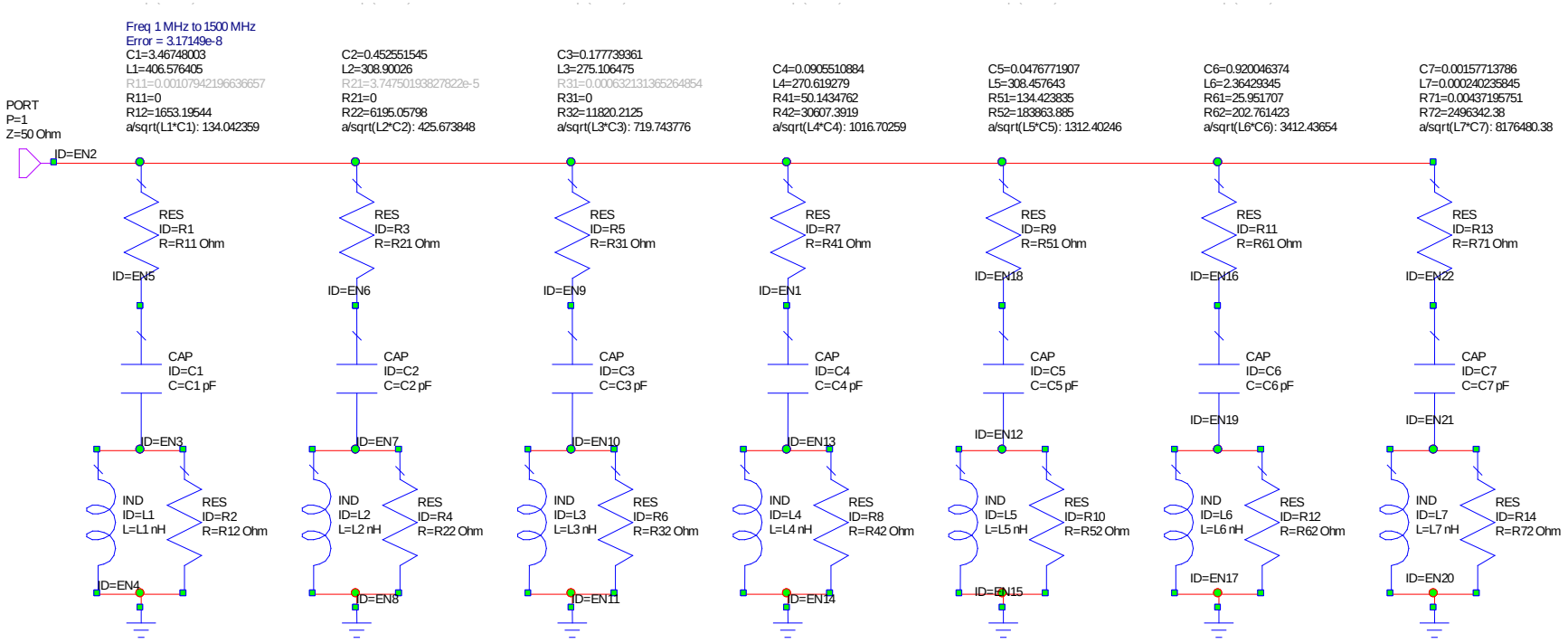


*R*11 = 5.06 Ω	*R*21 = 0 Ω	*R*31 = 25.5 Ω	*R*41 = 0 Ω
*C*1 = 39.9 pF	*C*2 = 4.64 pF	*C*3 = 4.69 pF	*C*4 = 1.68 pF
*L*1 = 27.1 μH	*L*2 = 24.9 μH	*L*3 = 2.26 μH	*L*4 = 24.5 μH
*R*12 = 10.1 kΩ	*R*22 = 50.1 kΩ	*R*32 = 2.68 kΩ	*R*42 = 116 kΩ

Accuracy of Broadband Equivalent Circuit

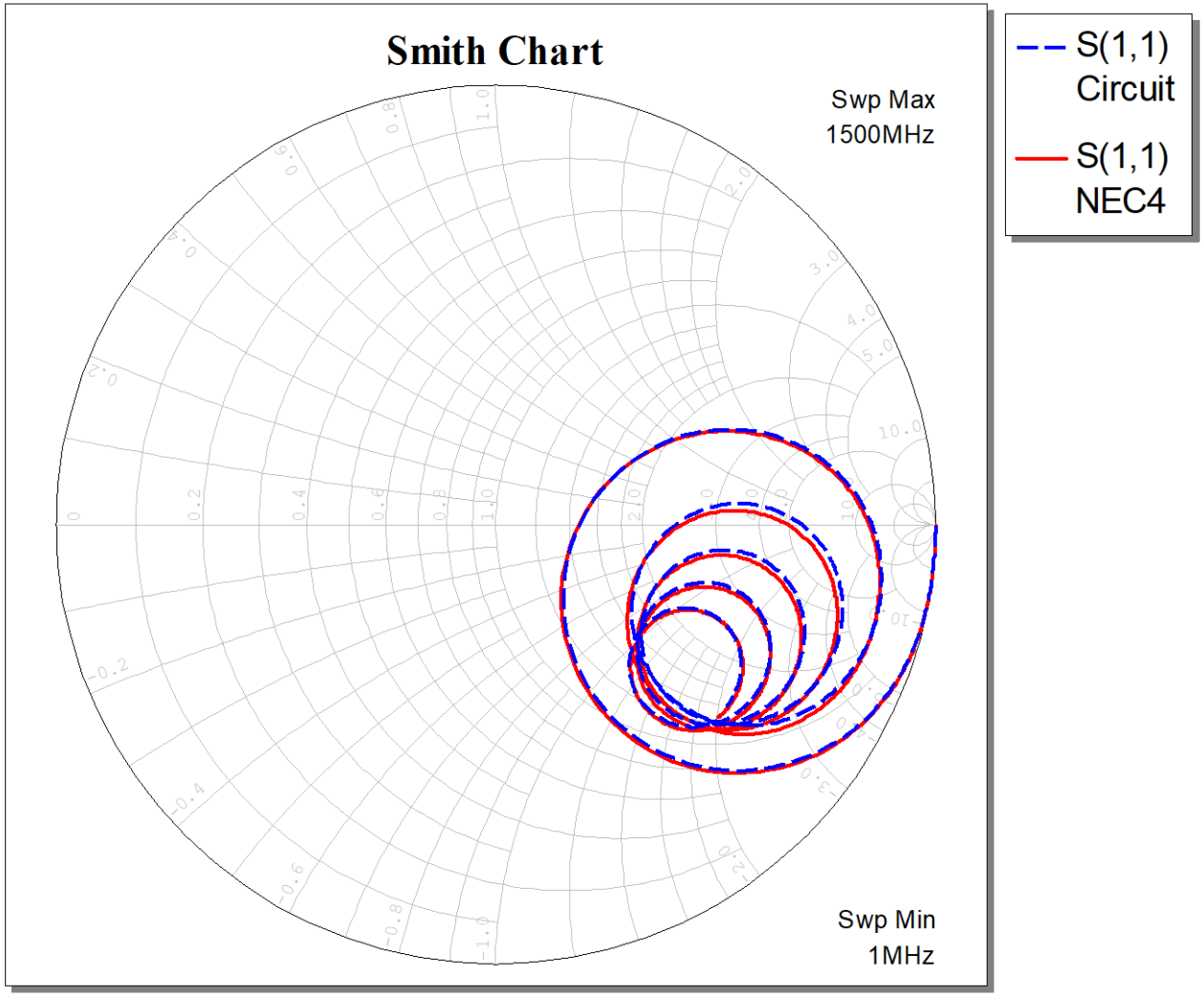


Broadband Equivalent Circuit for 2-meter Dipole (L/d = 50) from Zero to 1.5 GHz

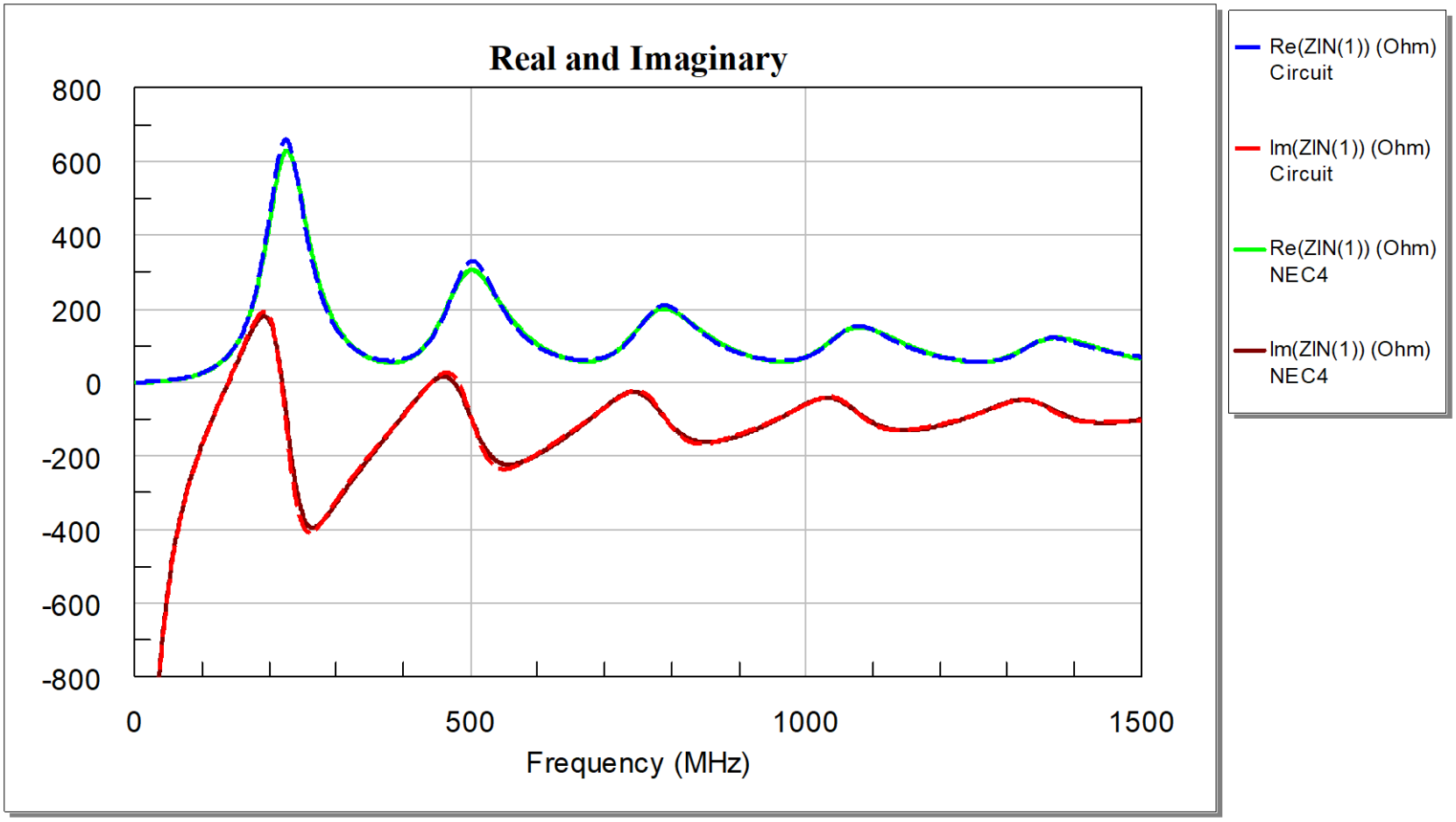


- Introduced by the author (2007)
- Partial fraction expansion of dipole admittance
- A modification of Foster's 2nd canonical form
- More accurate than other broadband equivalent circuits for dipoles, viz. Hamid-Hamid (1997), Rambabu-Ramesh-Kalghatgi (1999), and Streable-Pearson (1981)
- Six stages sufficient to cover *d-c* to 1.5 GHz

Accuracy of Broadband Equivalent Circuit



Impedance Accuracy of Broadband Equivalent Circuit



Resonant Dipole Length

The *K* Factor

What is Resonance?

- **Electric field energy equals magnetic field energy**

$$\iiint (\varepsilon_0 |E|^2 - \mu_0 |H|^2) dV = 0$$

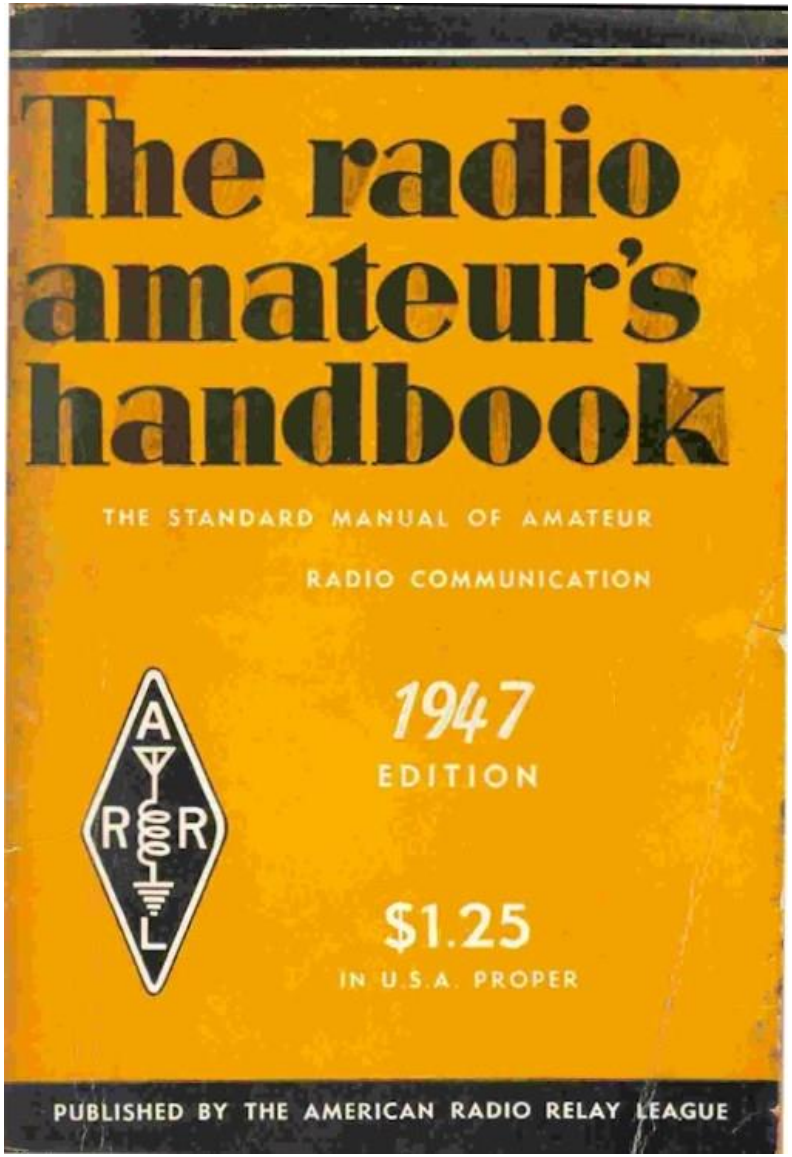
  - Some authors exclude radiation energy and consider only stored energy that is not associated with radiation, i.e. traveling outward
- **Feedpoint reactance is zero**

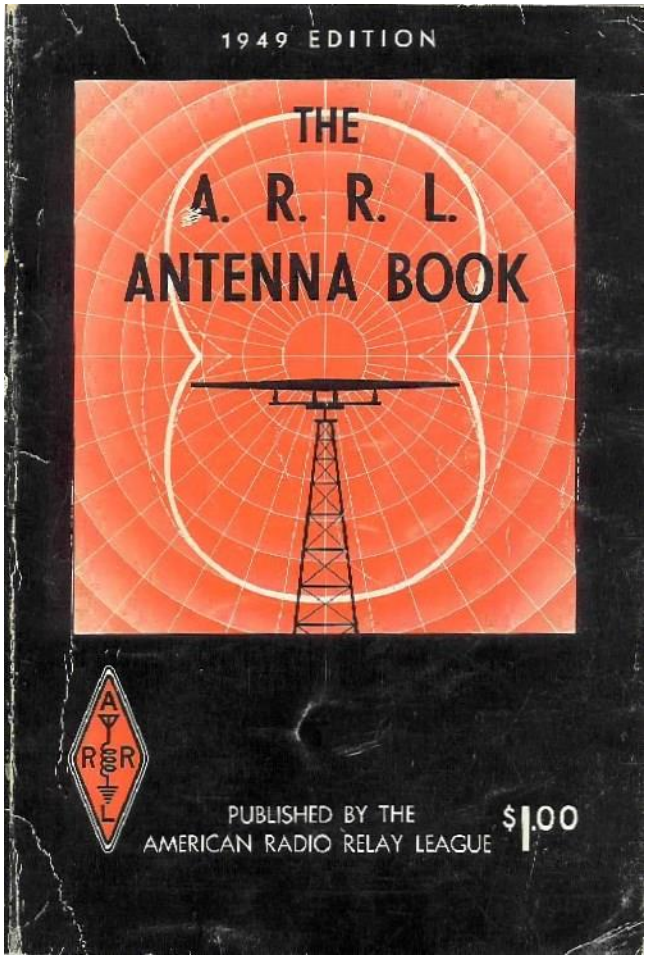
$$X(f) = 0$$

  - This definition is less fundamental and can be fooled, i.e. wrong, as for example, if the antenna has transmission lines built in
  - A nonresonant antenna can be made to appear resonant by adding a transmission line of appropriate length
  - If an antenna's impedance curve lies entirely in the upper or lower half of the Smith chart and does not cross the horizontal *X* = 0 midline, then it has no resonances

Circuit resonance at a feedpoint is different from field resonance in free space.

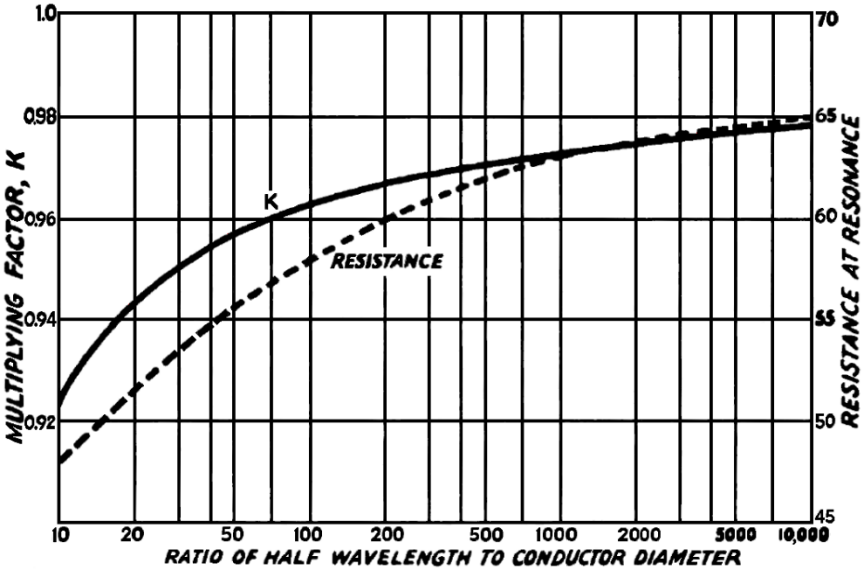
Dipole Resonant Length



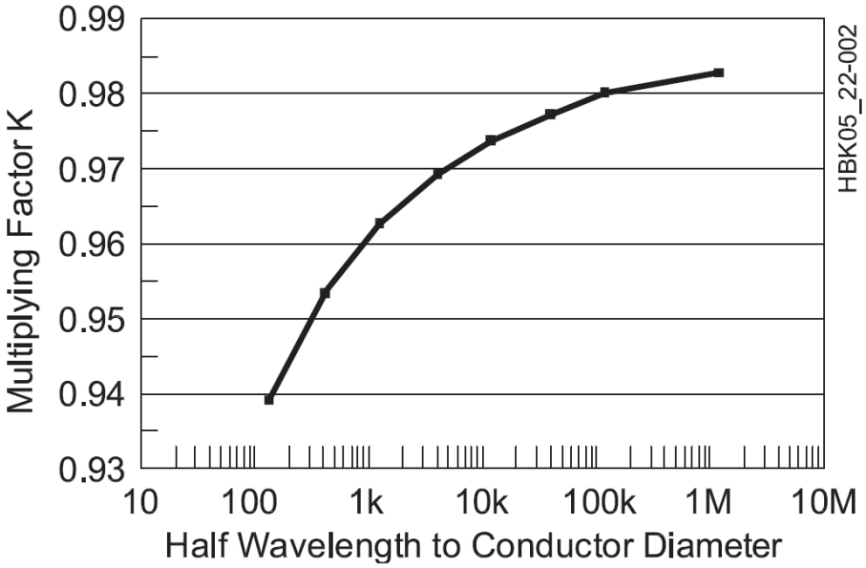


The Multiplying Factor *K*

ARRL 1947-1997



ARRL 1998-2018



$$L_{res} = K \times \frac{\lambda}{2} = K \times \frac{491.786}{f_{MHz}} \quad \text{feet}$$

***K* is not a velocity factor !**

The Source of ARRL's Original Curve is Lost
QEX, March 1996

Notes

[1]*The Radio Amateur's Handbook*, 24th Ed (West Hartford: ARRL, 1947), pp 194-195. I am indebted to Michael Tracy, KC1SX, of the ARRL Technical Information Service, who added his efforts to my own in the search for the source of the *Handbook* graph. Nothing in *QST* or other League publications has shown itself. If anyone has knowledge of an authoritative source for the graph, I would appreciate correspondence or email.

[2]Schelkunoff, S. A., *Electromagnetic Waves* (New York: Van Nostrand, 1943), p 465. For example, see the treatment of end "caps." At best, one can draw some inferences from sources such as this, but they do not show the definitive development of the Handbook graph. See Note 1. The designation of the shortening factor as "K" is also problematic in connection with transmission-line analogy calculations, which often use *K* to designate the characteristic impedance of a biconical antenna and sometimes of its cylindrical counterparts.

The Source of ARRL's 1998 Revised Curve
QEX, March 1996

Calibrating K to NEC

The Handbook *rule-of-thumb for determining electrical antenna length may not be adequate for computer modeling of antennas.*

By L. B. Cebik, W4RNL

K is the factor by which you shorten the physical length of an antenna to achieve the correct electrical length. A half-wave dipole in free space or a vertical quarter-wave monopole over perfect ground will, at resonance, be physically shorter than a half or a quarter of a wavelength, respectively, as shown in Fig 1. The amount of shortening is regular; that is, it is a function of the antenna and not of the surroundings (which will also have an effect upon the length of a resonant antenna). Nor does the shortening depend upon deformations, usually loops, at the ends of wire antennas, which simply add to the end effect.

For most practical purposes, hams cut half-wavelength wire antennas to a length in feet approximating the constant 468 divided by the frequency in MHz. This standard formulation has been in *ARRL Handbooks* since at least 1930. In 1947, the *Handbook* introduced a graph relating the ratio of the antenna element diameter and a half wavelength in free space to a multiplier, *K*. *K* is always less than 1 and approximates the amount by which the antenna is shortened relative to a full half wavelength by virtue of its diameter. *The Antenna Book* has carried the graph continuously since 1949.

Unfortunately, the 1947 discussion refers to end effect only in terms of the additional system capacitance added by antenna end deformation.[1] In point of fact, deformation is an additional end effect. The value of *K* dependent upon antenna diameter is also an end effect to the degree that it is applicable only to the end quarter-wavelength sections of a dipole, however many wavelengths long it may be. Likewise, it is applicable to the final quarter-wavelength section of a vertical antenna over perfect ground, no matter how many wavelengths long the monopole might be. Other sections of such antennas (each a half wavelength for dipoles or a quarter wavelength for monopoles) will be roughly (but not exactly) a full half or quarter wavelength, respectively.

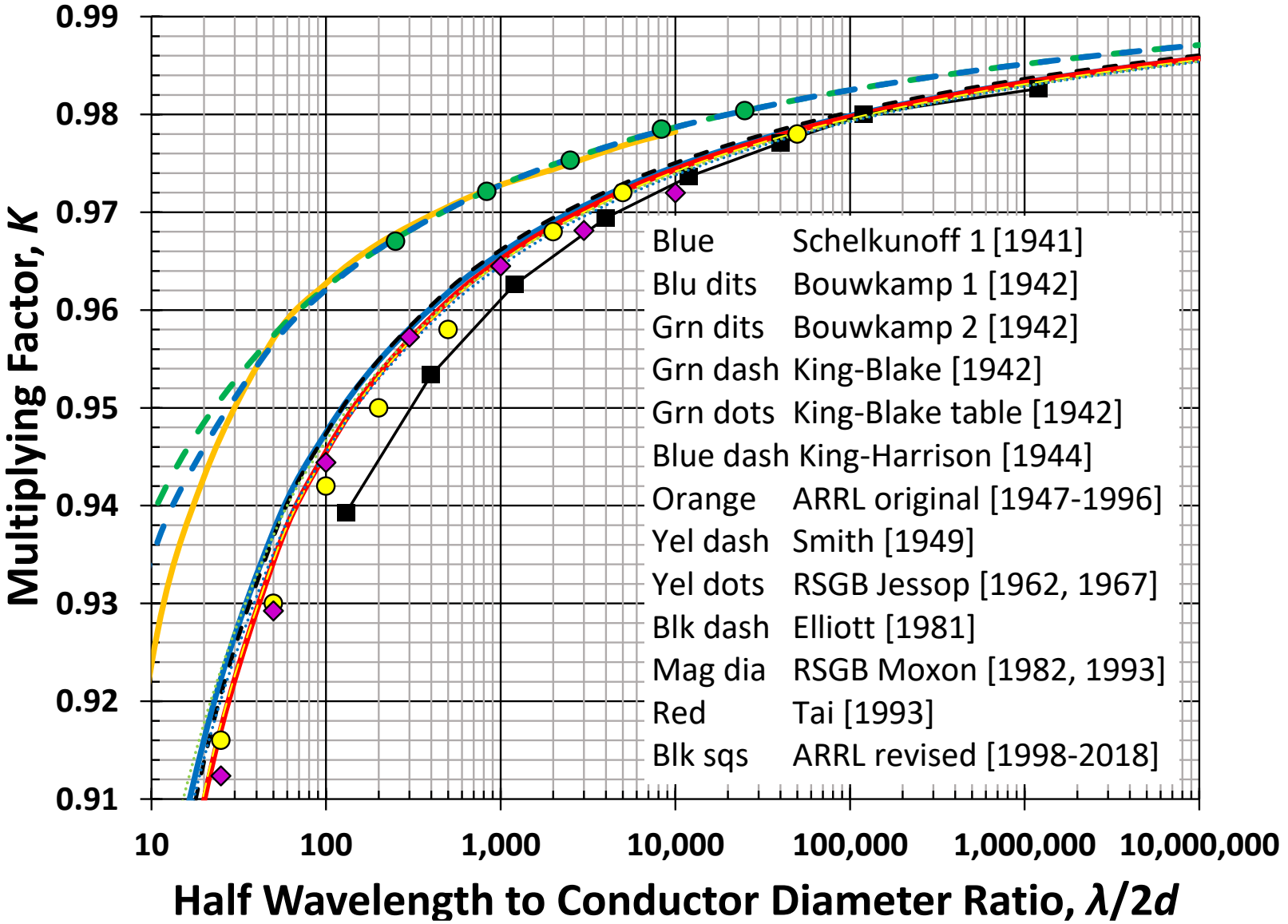
Explanations of the shortening effect are ordinarily traceable to accounts of the end capacitance of an antenna element, as if the end consisted of a spherical cap with the wire's radius.[2] Within the antenna-transmission-line analogy upon which classical mathematical treatments of antennas rest, the shortening effect is reducible for practical work to a constant, often called *K*, corresponding to the diameter-to-half-wavelength ratio at the frequency of interest.[3]

Working values of *K*, the decimal multiplier used to account for regular

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Knoxville, TN 37938-4443
email: cebik@utk.edu

[1]Notes appear on page 8.

The "*K*" Universe – Who is Right?

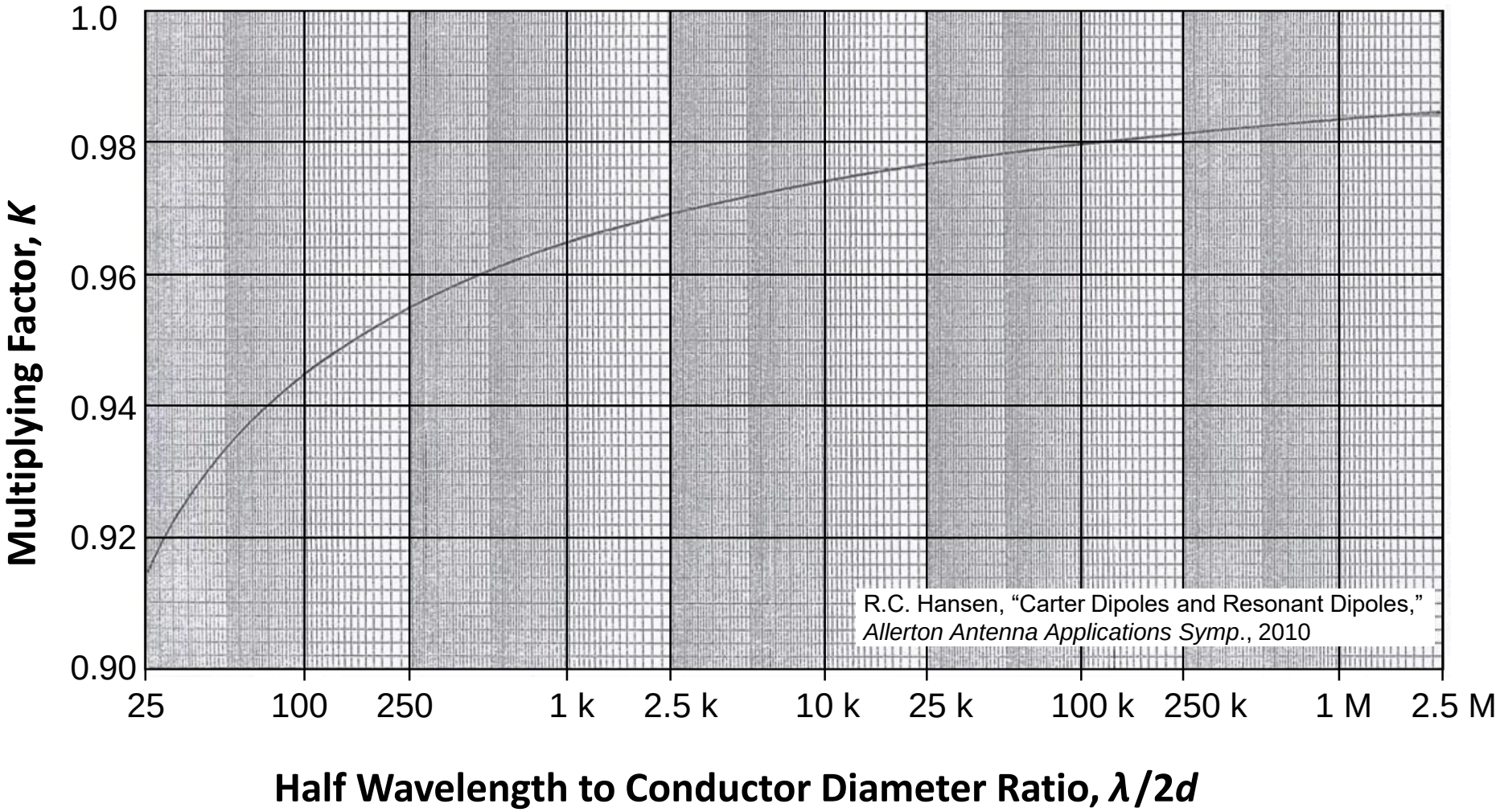


Robert Clinton Hansen, 1926-2018



Dr. R.C. Hansen fostered antenna science as an engineering discipline of the IEEE and was the U.S. government's consulting authority on antennas

Hansen's 2010 Graph of *K*



Three Half-Baked Theories of the Multiplying Factor *K*

- **Theory 1: *K* is a velocity factor**
  - Claim: Dipole is a transmission line and *K* is a velocity factor
  - No physical basis exists for non-unity velocity factor
    - Only materials are PEC metal and vacuum, no dielectric or losses
  - Schelkunoff's wave theory of antennas has velocity factor = unity
- **Theory 2: *K* is an "end" effect**
  - Claim: Dipole is a quarter-wave transmission line, velocity factor = 1, but fringing capacitance at ends transforms to inductance at feedpoint
  - Forward and reverse traveling waves would give a sinusoidal current standing wave, but this is not the case
- **Theory 3: *K* is due to current distribution not being sinusoidal**
  - True, the magnitude of the current distribution deviates from sinusoidal as a dipole gets fatter, but this does not explain *K*
  - *K* is predicted accurately by the induced e.m.f. method which assumes sinusoidal current distribution; so non-sinusoidal current is not the explanation

Five Formulas for Dipole Reactance

- **Induced EMF method (1929)**

$$X(kl,ka)=\frac{\eta}{4\pi\sin^2(kl)}\{2\mathrm{Si}(2kl)-\cos(2kl)[\mathrm{Si}(4kl)-2\mathrm{Si}(2kl)]+\sin(2kl)\,[\mathrm{Ci}(4kl)-2\mathrm{Ci}(2kl)+\gamma+2\ln(ka)-\ln(kl)]\}$$

- **Hallén theory (1938)**

$$X(kl,ka)=\mathrm{Im}\left\{\frac{-j\Omega\eta}{2\pi}\left[\frac{\cos(kl)+\frac{\alpha_1}{\Omega}+\frac{\alpha_2}{\Omega^2}+\cdots}{\sin(kl)+\frac{\beta_1}{\Omega}+\frac{\beta_2}{\Omega^2}+\cdots}\right]\right\}$$

- **Schelkunoff theory (1941)**

$$X(kl,ka)=\frac{K_a\left[\frac{1}{2}(G^2+F^2+M^2-N^2-K_a^2)\sin(2kl)+(MN-K_aF)\cos(2kl)+MF-K_aN\right]}{G^2\cos^2(kl)+[(K_a+M)\sin(kl)+(F+N)\cos(kl)]^2}$$

- **Tai-Elliott cubic polynomial (1993)**

$$X(kl,ka)=9.6031(kl)^3-8.6793(kl)^2+17.0082(kl)-0.4456-\frac{\eta}{\pi}\left(\ln\left(\frac{2l}{a}\right)-1\right)\cot kl$$

- **K6OIK's quartic polynomial (2018)**

$$X(kl,ka)=6.76275(kl)^4-11.6669(kl)^3+12.4080(kl)^2+10.1423(kl)-\frac{\eta}{\pi}\left(\ln\left(\frac{2l}{a}\right)-1\right)\cot kl$$

Summary of Formulas for the Multiplying Factor *K*

K6OIK's IEMF formula (2018)

Best

$$K = 1 - \frac{0.225706}{\ln\left(\frac{\lambda}{2d}\right) - 0.429451}$$

K6OIK's quartic polyfit (2018)

Very good

$$K = 1 - \frac{0.225630}{\ln\left(\frac{\lambda}{2d}\right) - 0.436223}$$

Tai-Elliott cubic polyfit (1993)

$$K = 1 - \frac{0.223370}{\ln\left(\frac{\lambda}{2d}\right) - 0.492770}$$

Very good

Schelkunoff's 1st formula (1941)

Good

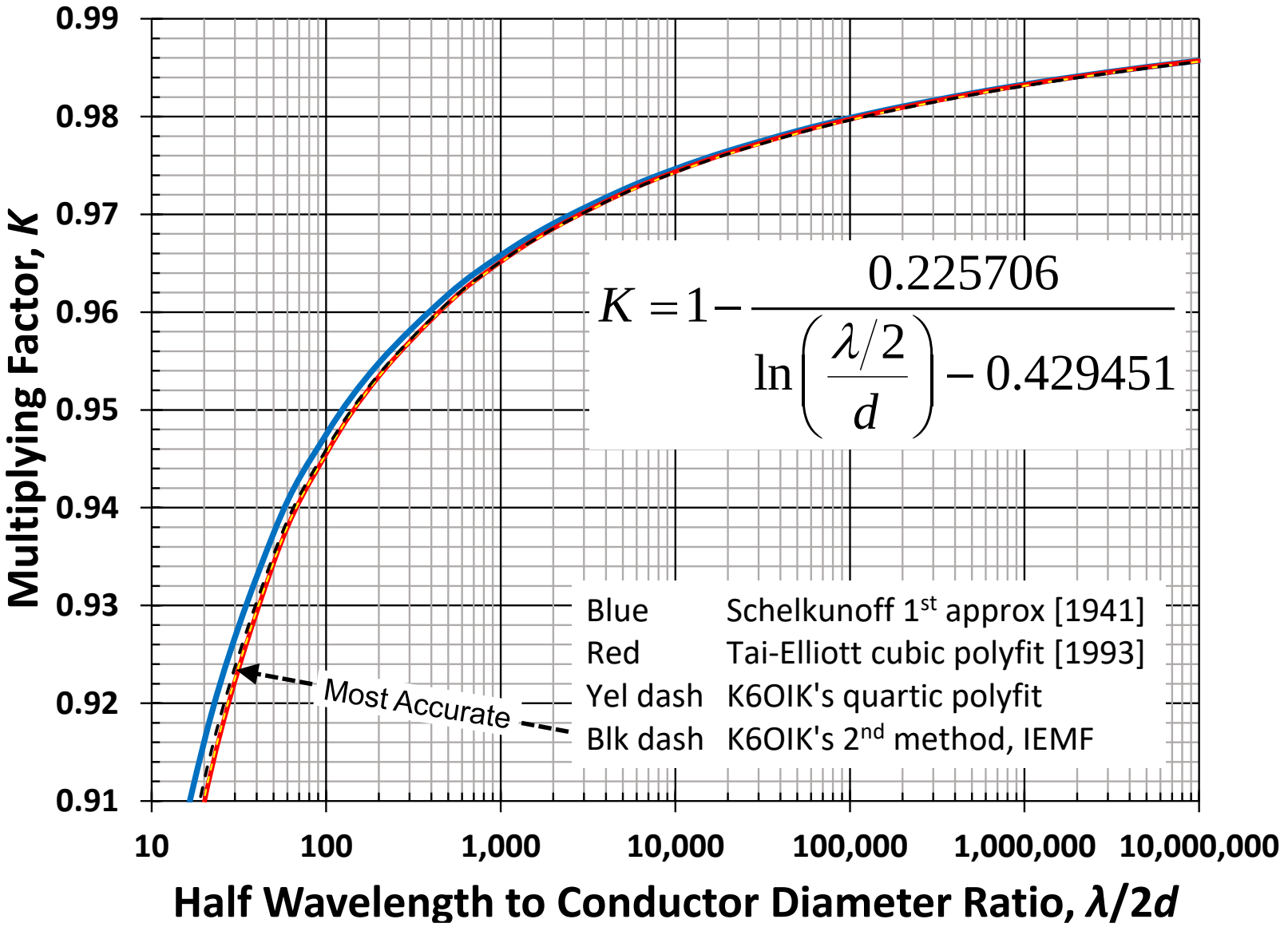
$$K = 1 - \frac{0.225706}{\ln\left(\frac{\lambda}{2d}\right) - 0.306853}$$

King-Blake (1942)

$$K = 1 - \frac{0.225706}{\ln\left(\frac{\lambda}{2d}\right) + 1.38629}$$

Worst

The Real *K* – for a Dipole in Free Space



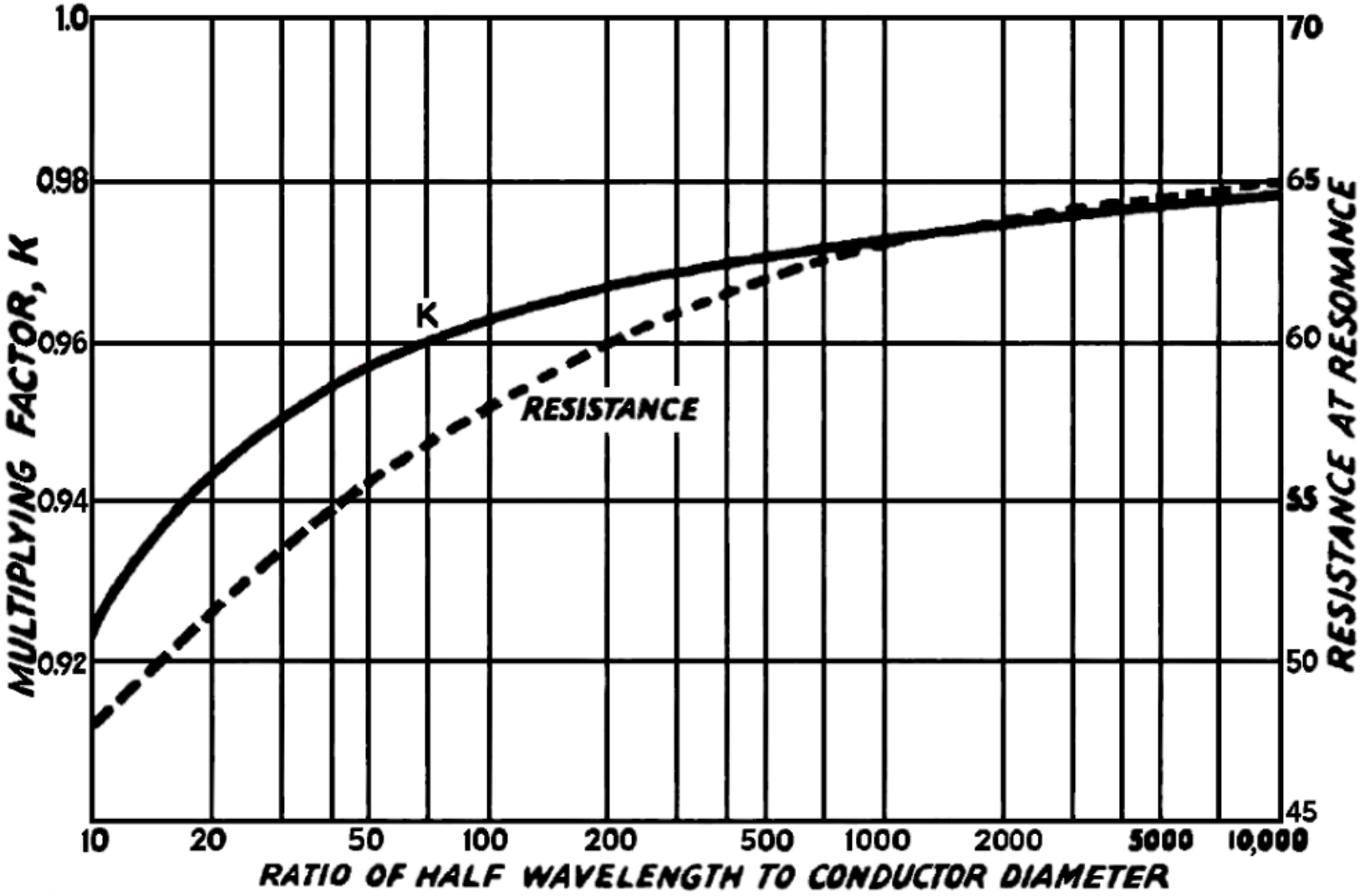
Comments on the Multiplying Factor *K*

- **Most popular expositions on *K* are partly correct at best**
  - *K* is not a velocity factor
  - *K* is not an "end" effect
  - *K* is not due to departure from sinusoidal current distribution
- ***K* can be determined by rigorous methods**
  - Induced e.m.f. method gives *K* accurately for the 1st resonance
  - Best method: Analyze a dipole as a boundary value problem
  - Solve Pocklington's or Hallén's equations for the current on the antenna
  - *K* is found from the dipole length for which the feedpoint reactance is zero
- **Numerical methods are fine, but MoM has a caveat**
  - Antenna models that use MoM and delta-gap sources do not predict resonance or *K* accurately

Antenna models that use MoM with delta-gap sources do not predict resonance or *K* accurately.

Dipole Resistance at Resonance

Resonant Resistance: ARRL 1947-1997



Solution Method

- **Given an impedance function of the form**

$$Z(kl, ka) = R(kl, ka) + jX(kl, ka)$$

- **The resonant length is defined as the solution *kl** that satisfies**

$$X(kl^*, ka) = 0$$

- **The resulting *K* factor is**

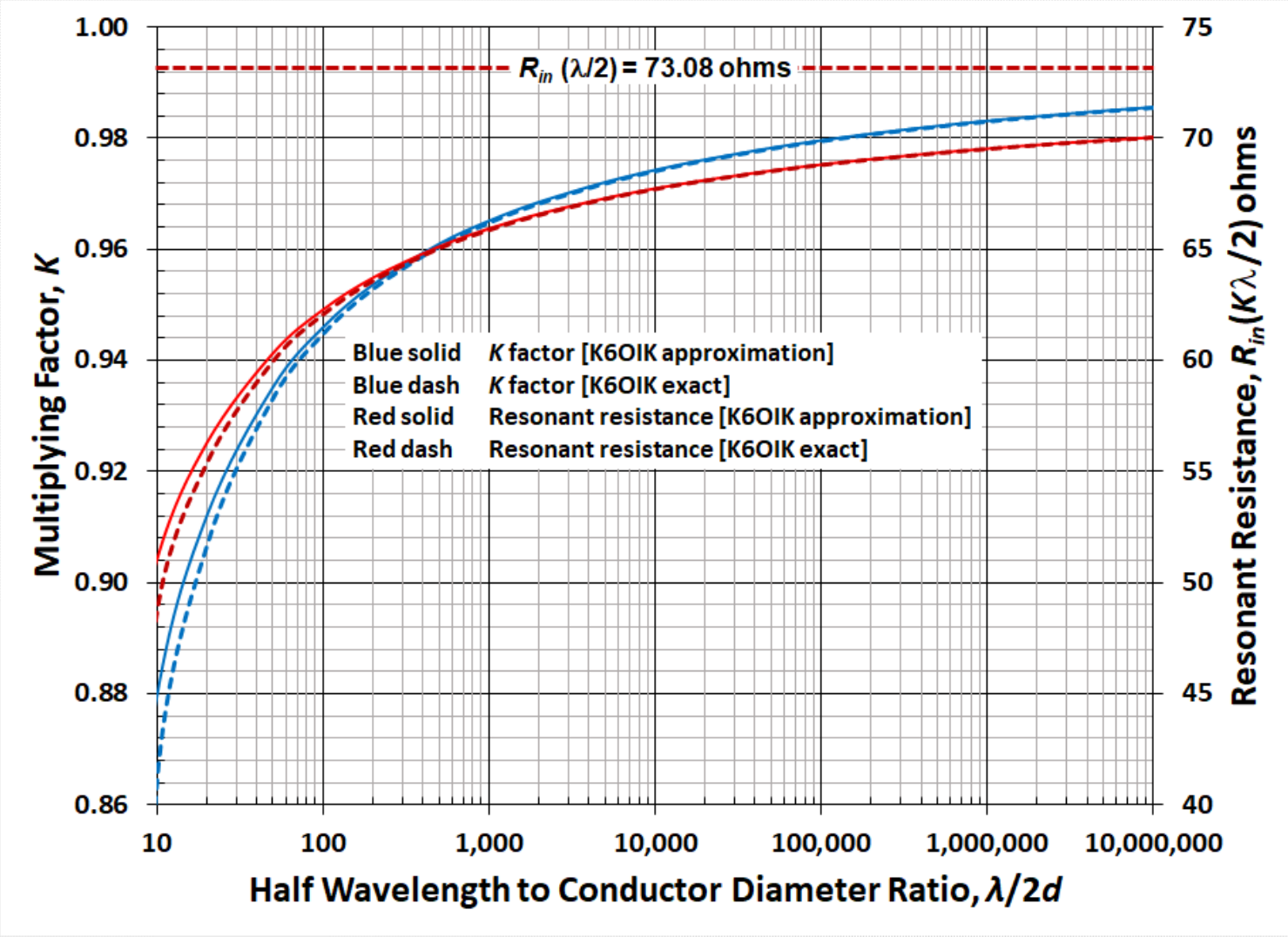
$$K = \frac{kl^*}{\pi/2}$$

- **And the resonant resistance is**

$$R^* = R(kl^*, ka)$$

$$= R(K\frac{\pi}{2}, ka)$$

K and Resonant Resistance per IEMF



K and Resonant Resistance per NEC-5 with Delta-Gap Source

Antenna analyzed with delta-gap source excitation.

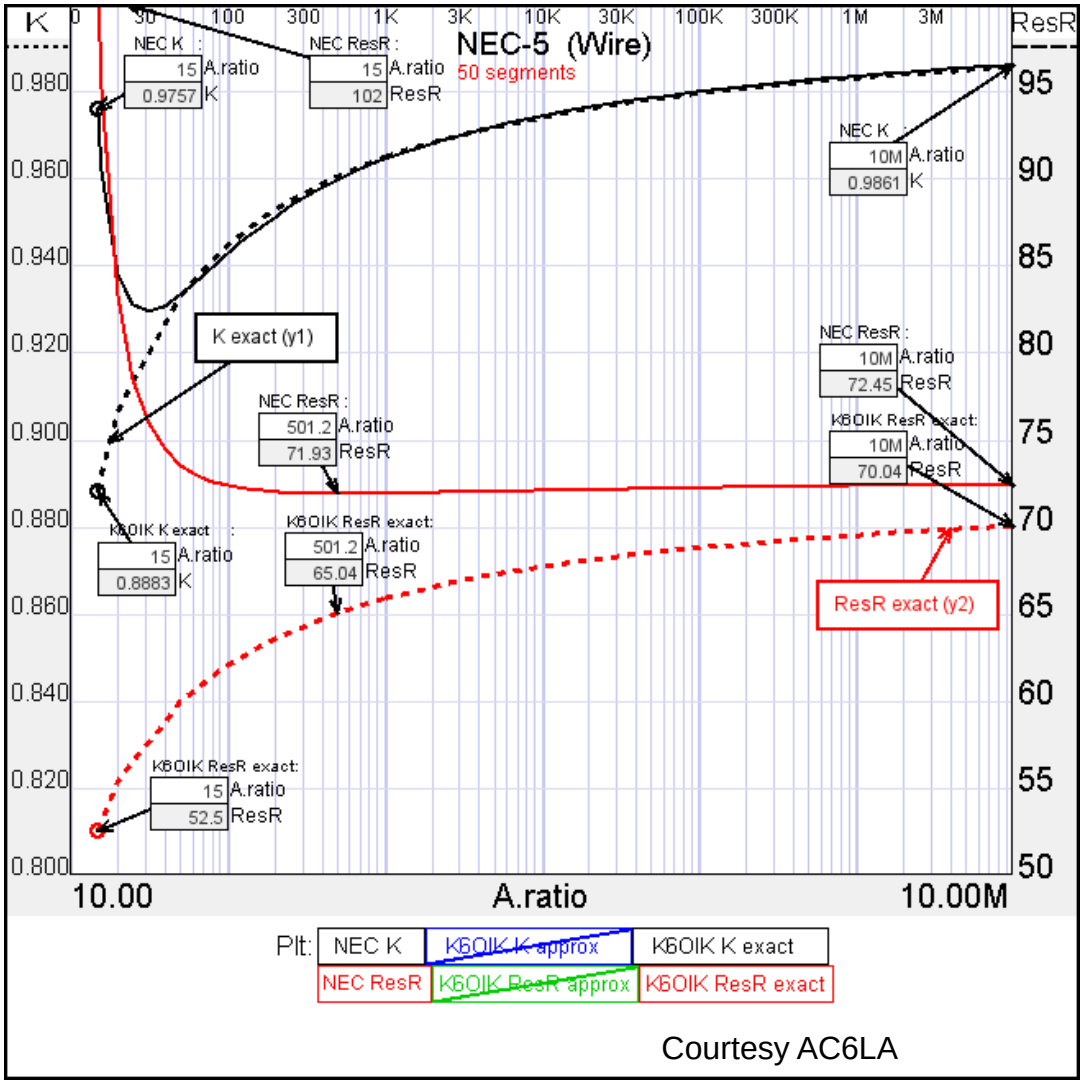
Feedpoint impedance obtained directly.

Anomalous behavior as dipole becomes fatter.

K and *R* curves are not monotonic.

K curve diverges for length to diameter ratios < 40

R curve diverges for length to diameter ratios < 200



Courtesy AC6LA

K and Resonant Resistance per NEC-5 with Different Source Type (Plane Wave)

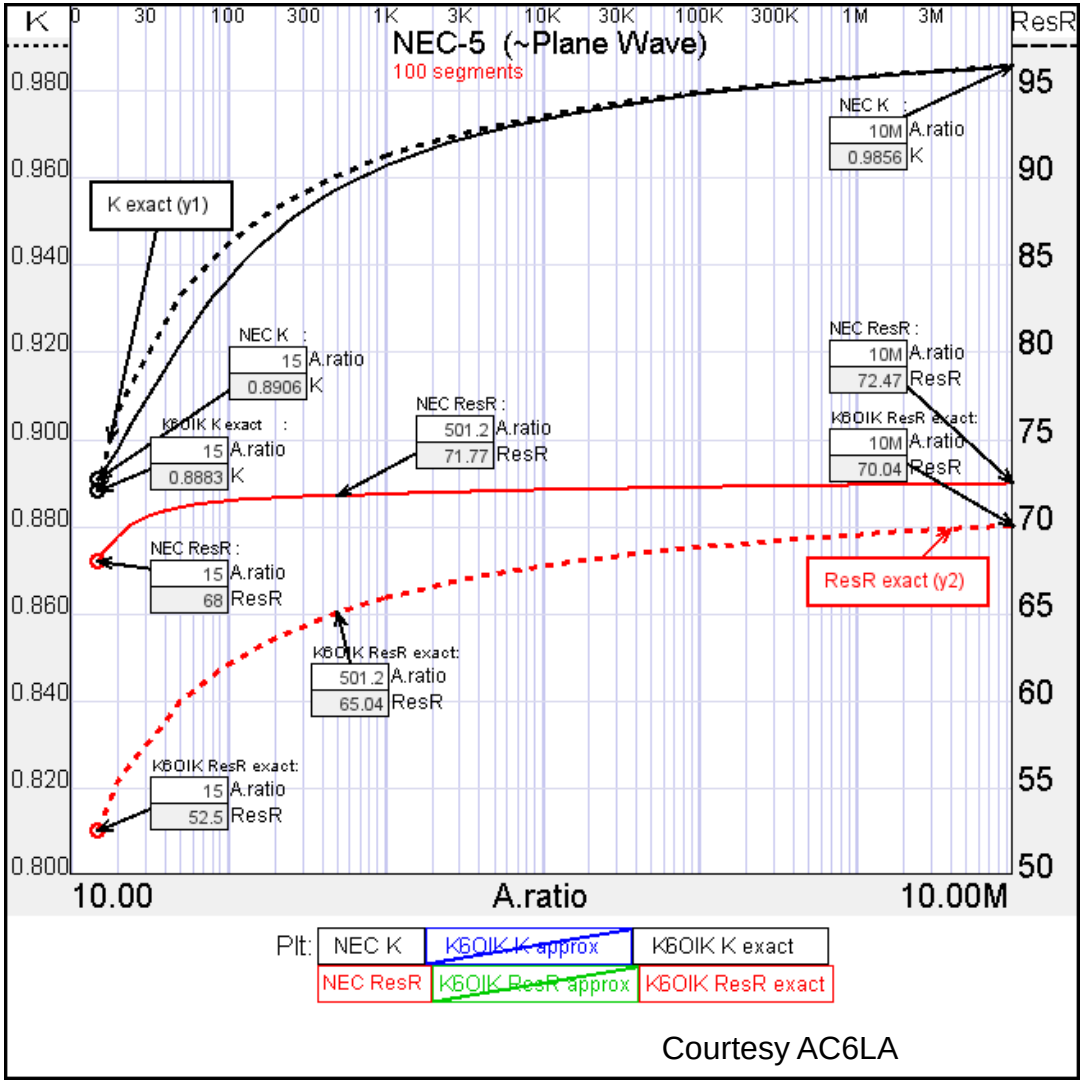
Antenna reanalyzed in scattering mode with plane wave excitation.

Feedpoint impedance obtained as Thévenin impedance from different loads.

Anomalous behavior disappears as dipole becomes fatter.

Delta-gap source used in MoM codes is the apparent problem.

Problem is well known for thin-wire MoM codes.



Courtesy AC6LA

Summary

1917-1932	The induced e.m.f. method is introduced for solving linear cylindrical antennas by B. van der Pol, A.A. Pistolkors, and P.S. Carter
1938	E. Hallén introduced a rigorous method for solving linear cylindrical antennas
1941	S.A. Schelkunoff of Bell Telephone Laboratories published two impedance formulas for cylindrical dipoles, one simple (less accurate) and one complicated (more accurate)
1942	R.W.P. King and F.G. Blake of Harvard University, solving Hallén's equation, published a solution of low accuracy. Simultaneously, C.J. Bouwkamp of Philips Labs published a near identical but more accurate solution of Hallén's equation
1943	C.W. Harrison, under R.W.P. King at Harvard, continued to work the problem
1944	M.C. Gray of Bell Labs published a more accurate solution of Hallén's equation
1946	D. Middleton at Harvard published a yet more accurate solution of Hallén's equation and compared the competing solution methods
1947	ARRL published a curve based on the King-Blake-Harrison (1942,1943) results
1949	R.A. Smith of Cambridge University obtained *K* from Schelkunoff's 2nd formula. Smith's math was correct, but his calculations had only 2.5 digit "slide-rule" accuracy
1962	RSGB published Smith's calculated *K* as a table in 1962 and as a graph in 1985
1996	L.B. Cebik published an article in *QEX* that showed NEC2 calculations did not agree well with the ARRL's published curve
1998	ARRL, under Dean Straw N6BV, published a revised curve based on L.B. Cebik W4RNL that agreed better with RSGB. RSGB, however, simultaneously published its revised curve that agreed with King-Blake !
2019	ARRL published a 2nd revised, curve based on the author's (K6OIK) work on dipole resonance
2026	AC6LA and AE6TY discover MoM thin wire codes disagree with IEMF for fat dipoles. *K* and *R* curves are non-monotonic, increasing as a dipole becomes fatter, in direct contradiction of IEMF. Problem is associated with delta-gap source model used in thin-wire antenna modeling programs

Conclusion

- **Popular method of moment antenna modeling programs are not perfect**
- **Wrong answers can be produced even for the simplest of antennas – a simple dipole in free space**
- **Don't trust modeling programs blindly**
- **Always confirm calculations by an independent method**
  - Theory where possible
  - A different type of modeling program (FEM or FDTD), different from MoM
  - Experimental test and measurement

Don't trust antenna modeling programs blindly.

Resources

References, Software, Books

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Free or Low Cost Antenna Modeling Software

- **Thin Wire MoM Codes**
  - ANSim – by Mark Tilson, Multiradius Bridge Current method is more accurate than NEC-4. No longer available.
  - AN-SOF (2010) – by Tony Golden, similar accuracy to NEC-5, uses exact kernel and integral equation. https://antennasimulator.com
  - AntennaKNoBs – by Steven Burns, KK7KNB, thin-wire code with web based GUI permits modeling in the cloud using computer or phone app. https://antennaknobs.dev
  - AWAS 2.0 (2001) – by Tony Djordjević, predecessor thin-wire formulation to that in WIPL-D and HOBBIES, has exact kernel, higher-order polynomial basis functions, minimal geometry restrictions, high numerical efficiency. Student version: https://mtt.etf.bg.ac.rs/download.htm
  - MiniNEC (1980) – by Jay Rockway and Jim Logan, N6BRF, different algorithms from NEC, used inside MMANA-GAL
  - NEC-5 (2019) – Improved accuracy, replaced kernels with exact kernel and integral equation, fewer artifacts, improved numerical stability, less strict geometry rules, similar accuracy to AN-SOF. https://softwarelicensing.llnl.gov/product/nec-v50
  - NEC-4 (1992) – Improved accuracy for stepped-radius wires and electrically-small segments, end caps and insulated wires, catenary-shaped wires, improved error detection
  - NEC-2 (1981) – Sommerfield-Norton ground interaction for wire structures above lossy ground; numerical Green's function allows modifying without repeating whole calculation
- **User Interface Programs**
  - AutoEZ v. 2.0.30 – by Dan Maguire, AC6LA. Low-cost GUI for EZNEC that adds automatic sweeping/stepping and an optimizer
  - EZNEC Pro+ v. 7.0.4 – by Roy Lewallen, W7EL. Free GUI for NEC-2, NEC-4, and NEC-5
  - 4nec2 – by Arie Voors. Free GUI for NEC-2 and NEC-4
  - MMANA-GAL – GUI for MiniNEC (popular in the UK)
- **Yagi Design**
  - QY4 (Quick Yagi) – by Sidney Smith, WA7RAI, a calculator for Yagi-Uda design
  - Yagi Calculator – by John Drew, VK5DJ, a calculator for DL6WU VHF/UHF Yagi-Uda design
  - YagiCAD – by Paul McMahon, VK3DIP, a calculator for VHF/UHF Yagi-Uda design
  - YO (Yagi Optimizer) – by Brian Beezley, K6STI, a MiniNEC based DOS program for Yagi-Uda antenna design, v6.5.1 archived by IW5EDI
  - YW 2.0 (Yagi for Windows) – by Dean Straw, N6BV, for monoband Yagi-Uda design, included with the *ARRL Antenna Book*
- **Surface MoM Codes**
  - HOBBIES (2010) – Similar to WIPL-D except has out-of-core solver. Development led by T.K. Sarkar, Syracuse University, based on algorithms developed at University of Belgrade. No longer available
  - WIPL-D – Limited demo version of the professional code: https://wipl-d.com/support/download/demo
- **Finite Element Method (FEM) Codes**
  - Ansys HFSS Student Edition – https://www.ansys.com/academic/students/ansys-electronics-desktop-student
  - OpenParEM – https://openparem.org
- **Finite Difference Time Domain (FDTD) Codes**
  - GprMax – Python based. https://www.gprmax.com
  - Meep – Python or Scheme based. https://meep.readthedocs.io/en/master
  - OpenEMS – https://www.openems.de

More links at https://www.fars.k6ya.org/others

Accessory Software

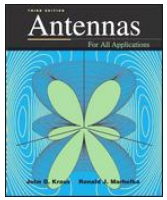
- **AutoEZ by Dan Maguire, AC6LA, https://www.ac6la.com**
  - Recommended accessory software for EZNEC
  - Gives EZNEC parameter sweeping and optimization
  - Reads NEC, AO, and MMANA-GAL files
  - Requires Excel and EZNEC installed on computer
  - Free demo version (30 segment limit). Regular version, $79

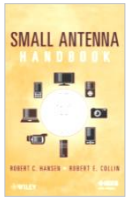
- OPTENNI **https://optenni.com**
  - Automated match network synthesis. Free trial on request

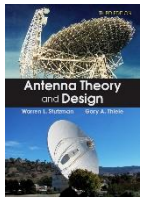
- Ampsa **https://www.ampsa.com**
  - Automated match network synthesis. Free trial on request

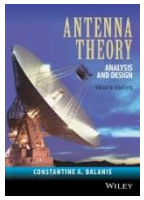
- **QucsStudio: https://qucsstudio.de**
  - General circuit analysis and design, Smith charting, and optimizers. Free

Good Antenna Books





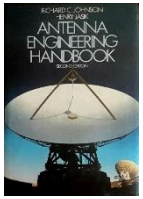




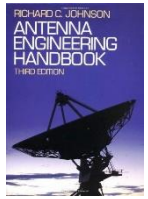
- **Books for antenna engineers and students**
  - C.A. Balanis, *Antenna Theory: Analysis and Design*, 4e, Wiley, 2016
  - W.L. Stutzman and G.A. Thiele, *Antenna Theory and Design*, 3e, Wiley, 2013
  - R.C. Hansen and R.E. Collin, *Small Antenna Handbook*, Wiley, 2011
  - J.D. Kraus and R.J. Marhefka, *Antennas*, 3e, McGraw-Hill, 2001
- **Antenna Engineering Handbooks – 5 editions**



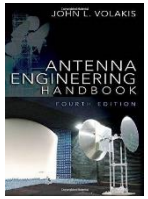
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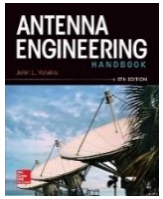
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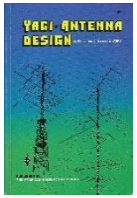
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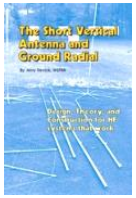
2019

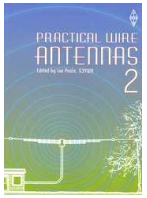
- **Antenna research resources**
  - IEEE Antennas & Propagation Society videos https://resourcecenter.ieeeaps.org
  - IEEE Xplore subscription online archive https://ieeexplore.ieee.org/Xplore/home.jsp
  - Allerton Antenna Applications Symposium DVD archive 1952-present
  - ACES Journal Archives http://www.aces-society.org/journal.php
  - Progress in Electromagnetics Research https://www.jpier.org

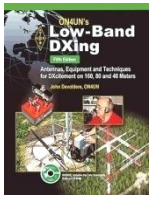
Good Antenna Books continued

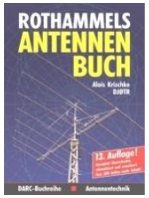


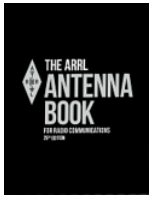




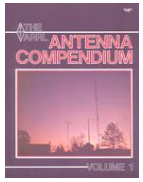


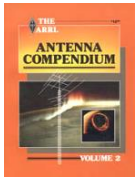


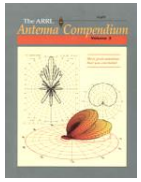


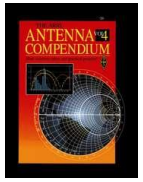


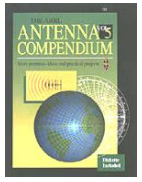
- **Books for Radio Amateurs**
  - H.W. Silver, N0AX, ed., *ARRL Antenna Book*, 25e, ARRL, 2023
  - A. Krischke, DJ0TR, ed., *Rothammel's Antenna Book*, 13e, English, DARC, 2019
  - J. Devoldere, ON4UN, *ON4UN's Low-Band Dxing*, 5e, ARRL, 2011
  - I. Poole, G3YWX, ed., *Practical Wire Antennas 2*, RSGB, 2005
  - J. Sevick, W2FMI, *The Short Vertical Antenna and Ground Radial*, CQ, 2003
  - L. Moxon, G6XN, *HF Antennas for All Locations*, 2e, RSGB, 1983
  - J.L. Lawson, W2PV, *Yagi Antenna Design*, ARRL, 1986
- **ARRL Antenna Compendium series – nine volumes**

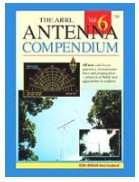


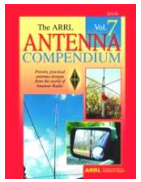


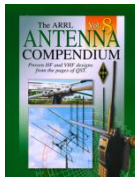






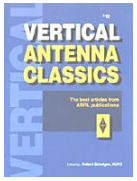


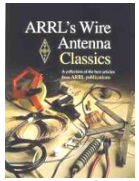


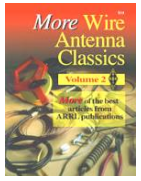


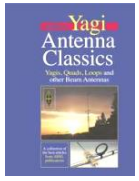


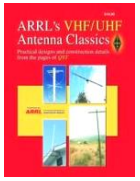
- **ARRL Antenna Classics series – eight titles**

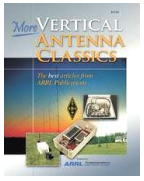


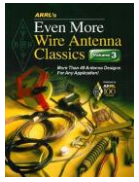


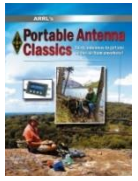








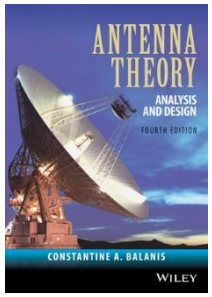




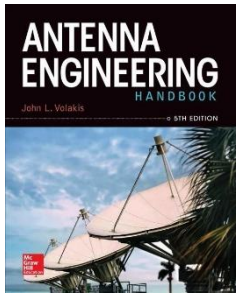
Recent Antenna Books of Interest



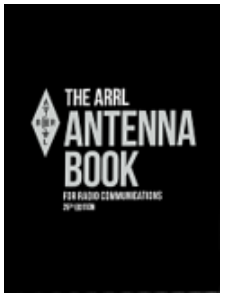
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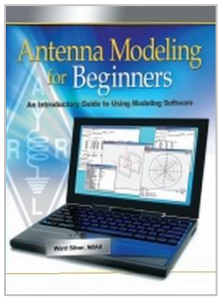
C.A. Balanis, *Antenna Theory: Analysis and Design*, 4e, Wiley, 2016



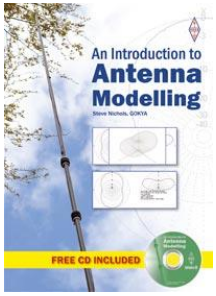
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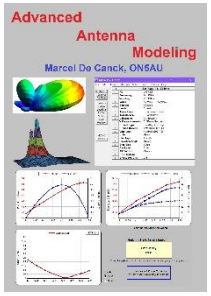
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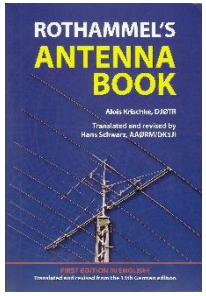
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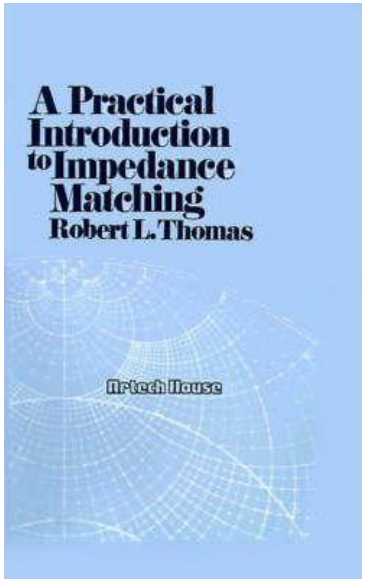


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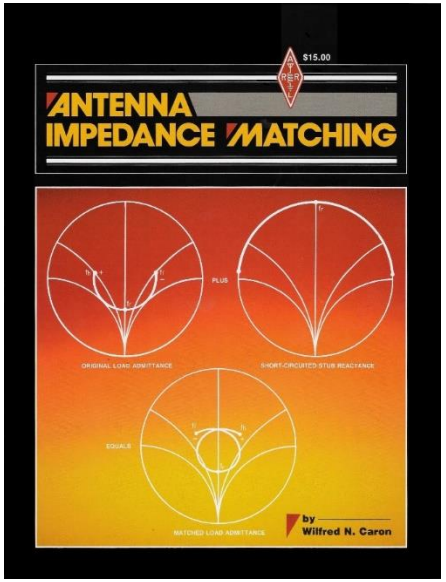


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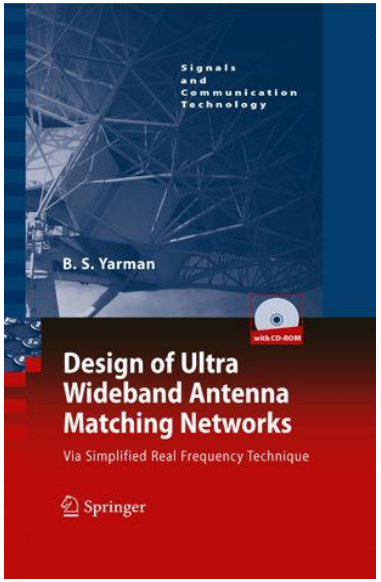
Impedance Matching for Beginner and Professional



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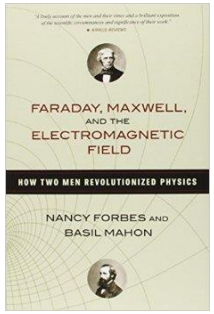


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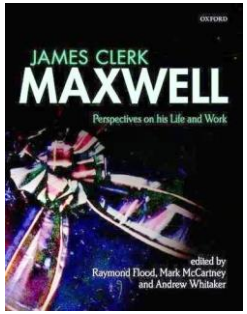


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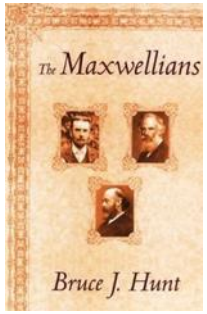
General Interest Books



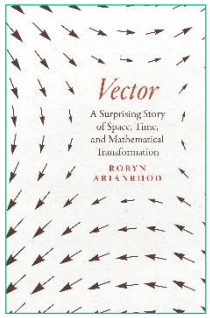
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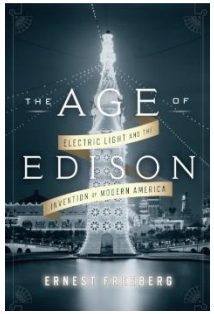
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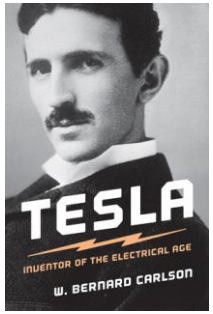
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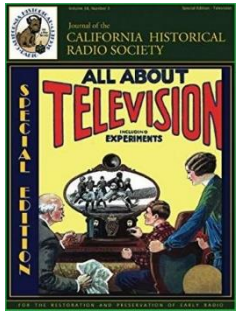
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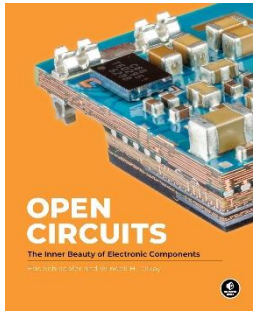
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Eric Schlaepfer and Windell H. Oskay, *Open Circuits*, No Starch Press, 2022

The End

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k6oik@arrl.net