Conjugate Match Myths

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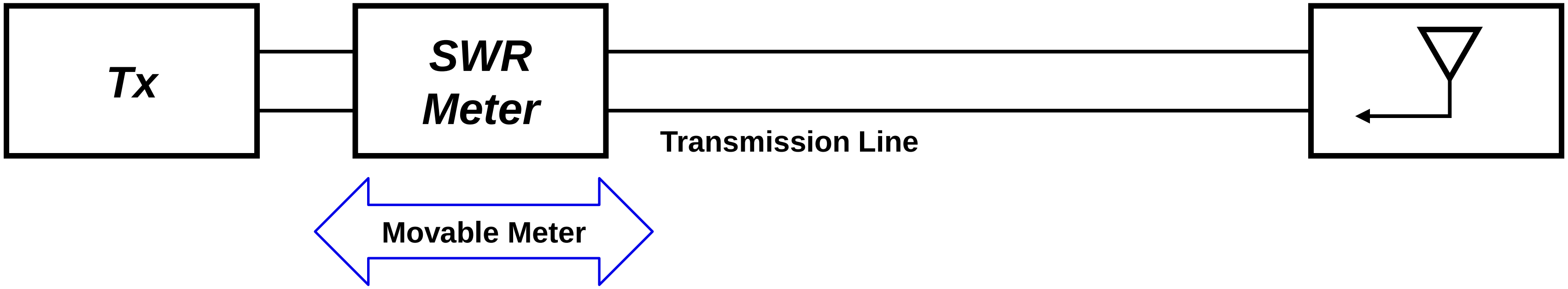
Topics

- **SWR variation on lossy lines**
- **Total line loss with unmatched load**
- **Power transfer and loss using lossy lines**
- **Solution for maximum power transfer through a lossy line**
  - Refutation of W. Maxwell's central claim in his book *Reflections*
- **References**
  - Articles and books

Standing Wave Ratio (SWR)

SWR Varies Along Lossy Transmission Lines



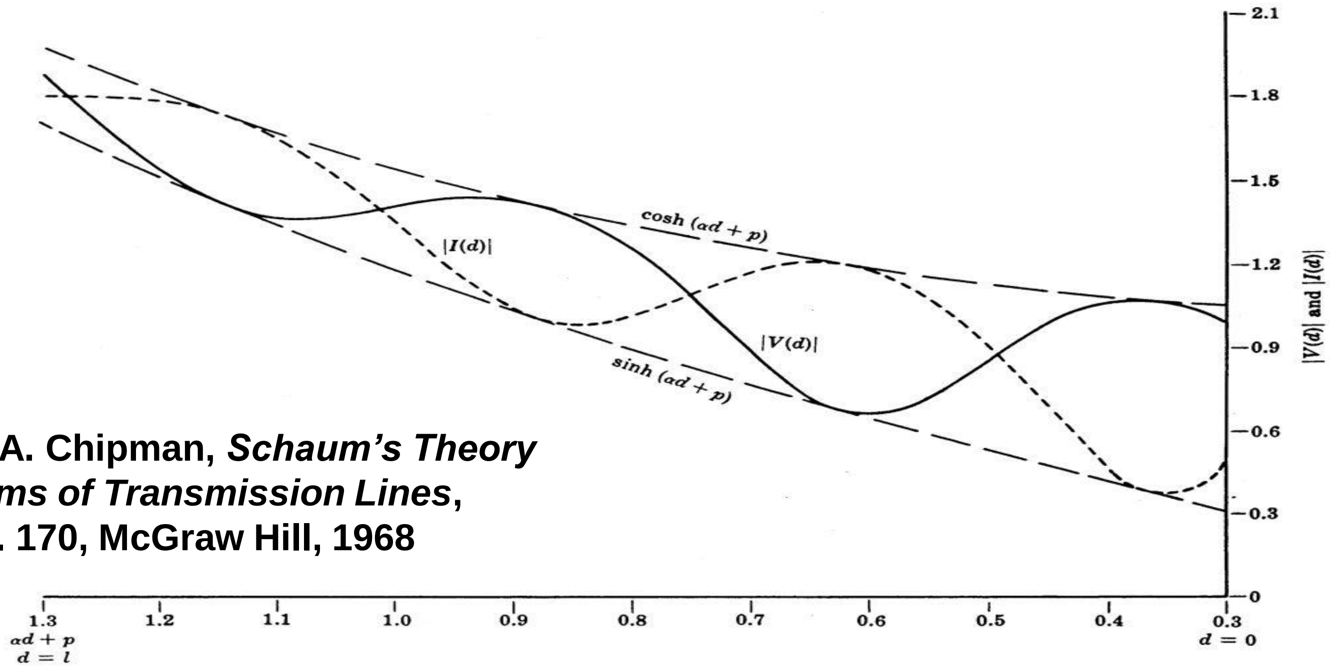
Definition of SWR for General (Lossy) Lines

- **Cannot define SWR using voltage or current "max / min" except for lossless lines**
- **A general definition of SWR that works for all lines is**

$$SWR = \frac{1 + \sqrt{\dfrac{P_R}{P_F}}}{1 - \sqrt{\dfrac{P_R}{P_F}}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

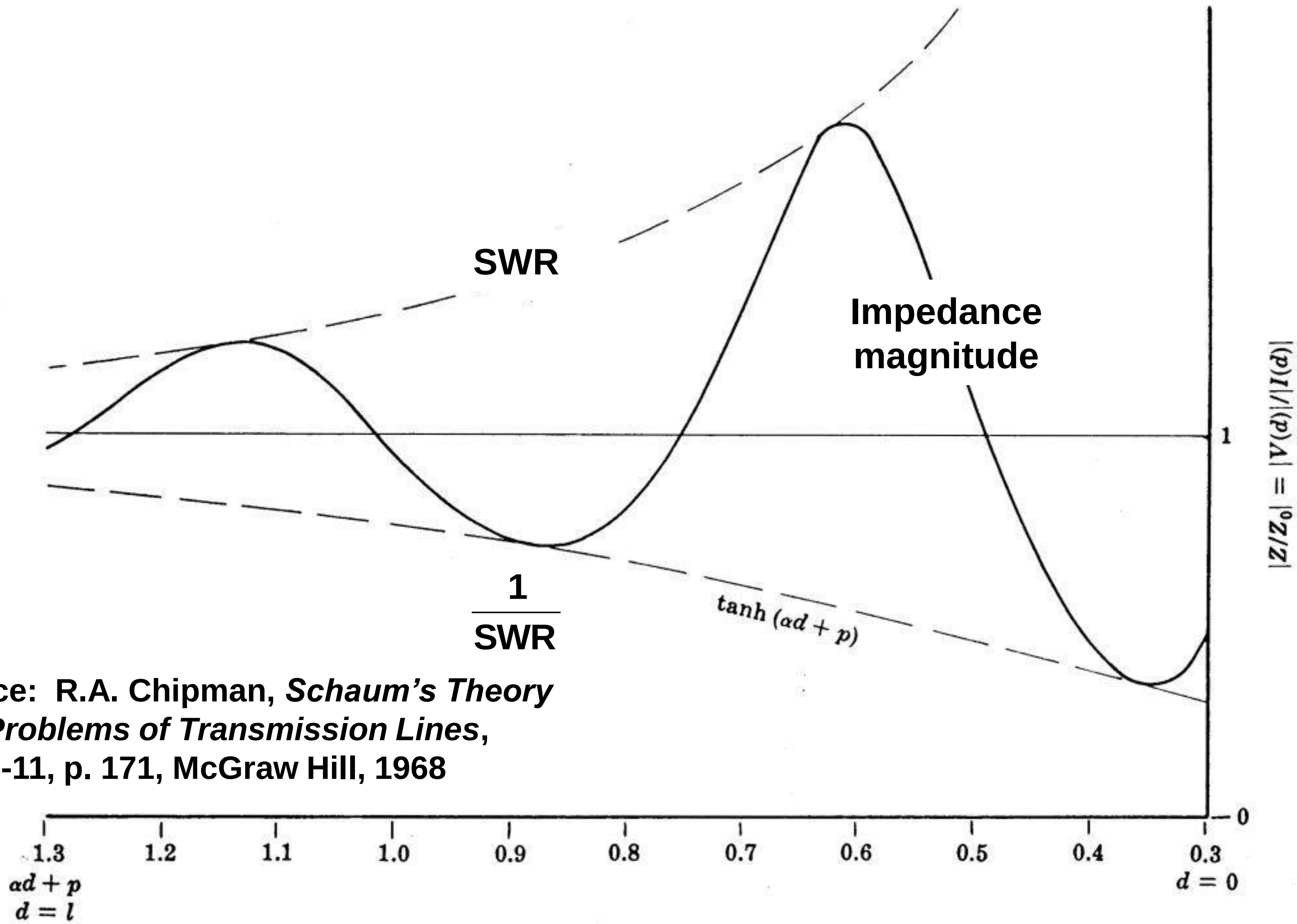
- **Forward and reverse wave amplitudes vary along the line**
- **SWR is maximum at the load and decreases gradually to a minimum at the source**

Voltage and Current Standing Waves



Source: R.A. Chipman, *Schaum's Theory and Problems of Transmission Lines*, Fig. 8-10, p. 170, McGraw Hill, 1968

Impedance and SWR Along a Line

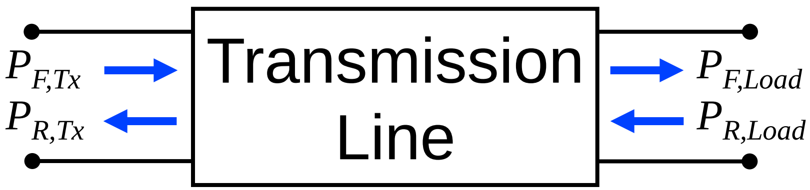


Source: R.A. Chipman, *Schaum's Theory and Problems of Transmission Lines*, Fig. 8-11, p. 171, McGraw Hill, 1968

Loss Graphs

Forward and Reflected Power on a Lossy Line

- **Power at load end in terms of power at transmitter end of line**



$$P_{F,Load} = \frac{1}{a} \cdot P_{F,Tx}$$

$$P_{R,Load} = a \cdot P_{R,Tx}$$

- **a is the power attenuation ratio or matched loss in linear units, a real constant greater than unity, expressible in terms of the line's attenuation constant and scattering parameters as**

$$a = \begin{cases} e^{2\alpha l} & \text{for } \alpha \text{ in nepers/meter and } l \text{ in meters} \\ \quad \text{or} & \\ 10^{\alpha l/1000} & \text{for } \alpha \text{ in dB /100 feet and } l \text{ in feet} \end{cases}$$

Latin a and Greek α should not be confused

$$a = \frac{1}{|s_{21}|^2}$$

Additional Loss Due to SWR at Load or Transmitter

- **Additional loss can be expressed either in terms of the line's input or output SWR**

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$$10\log_{10}\frac{1-|\Gamma_{in}|^2}{1-a^2|\Gamma_{in}|^2} = 10\log_{10}\frac{(SWR_{Tx}+1)^2-(SWR_{Tx}-1)^2}{(SWR_{Tx}+1)^2-a^2(SWR_{Tx}-1)^2}$$

$$\text{Additional Loss (dB)} \;=$$

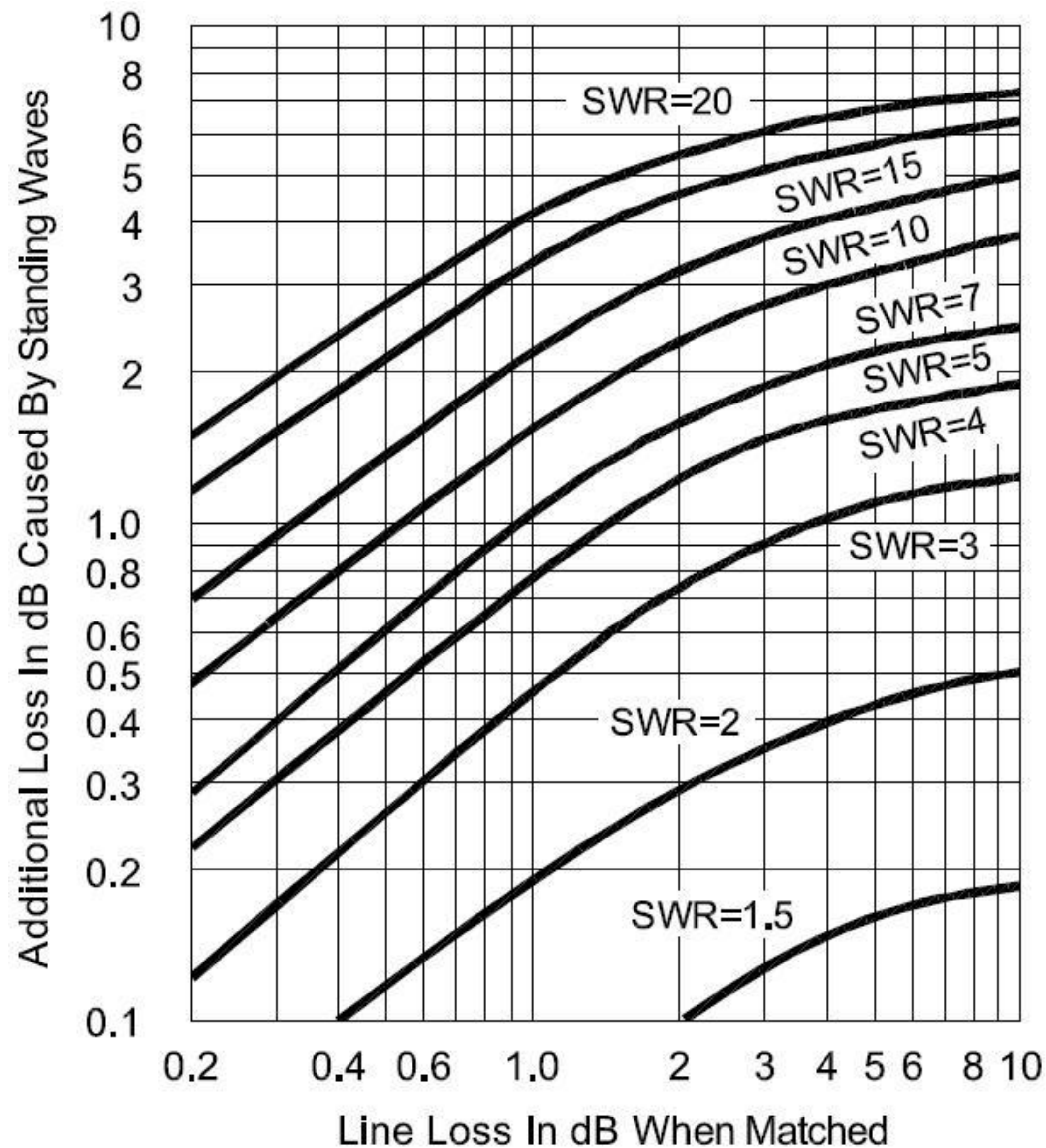
ARRL

$$10\log_{10}\frac{1-\dfrac{1}{a^2}|\Gamma_{Load}|^2}{1-|\Gamma_{Load}|^2} = 10\log_{10}\frac{(SWR_{Load}+1)^2-\dfrac{1}{a^2}(SWR_{Load}-1)^2}{(SWR_{Load}+1)^2-(SWR_{Load}-1)^2}$$

- **The next slides show the loss graph both ways**

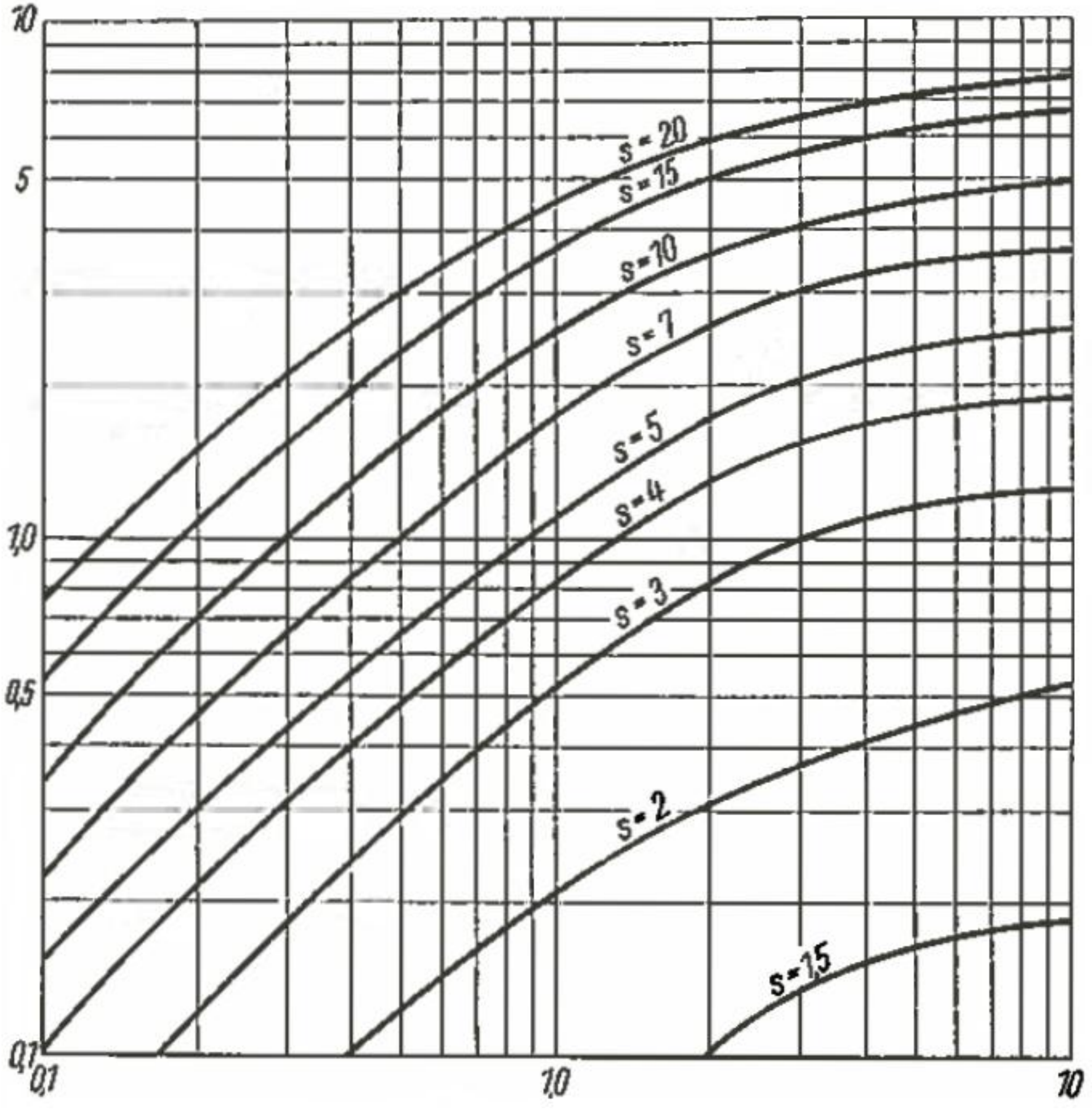
Graph 1: "Additional Loss Due to SWR"

- Published in every *ARRL Antenna Book* since 1949
- Published in every *ARRL Handbook* since 1985
- Published in J. Devoldere, *ON4UN's Low-Band DXing*

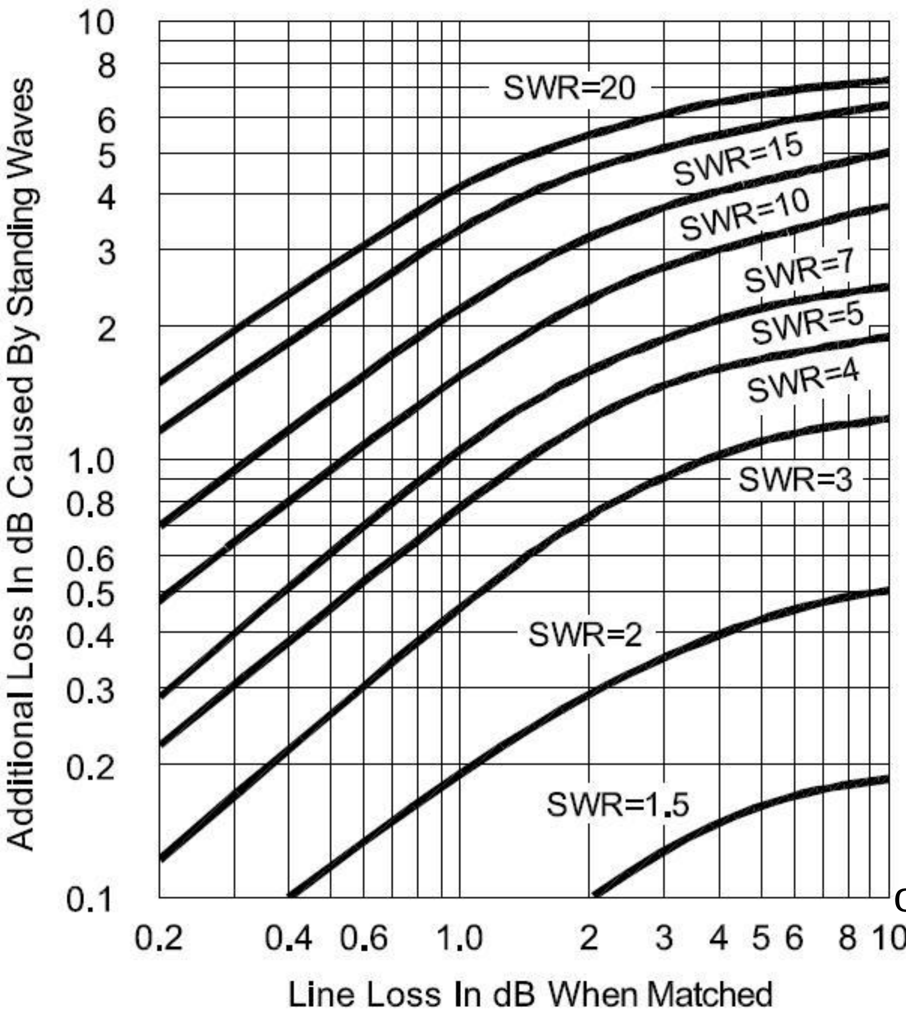


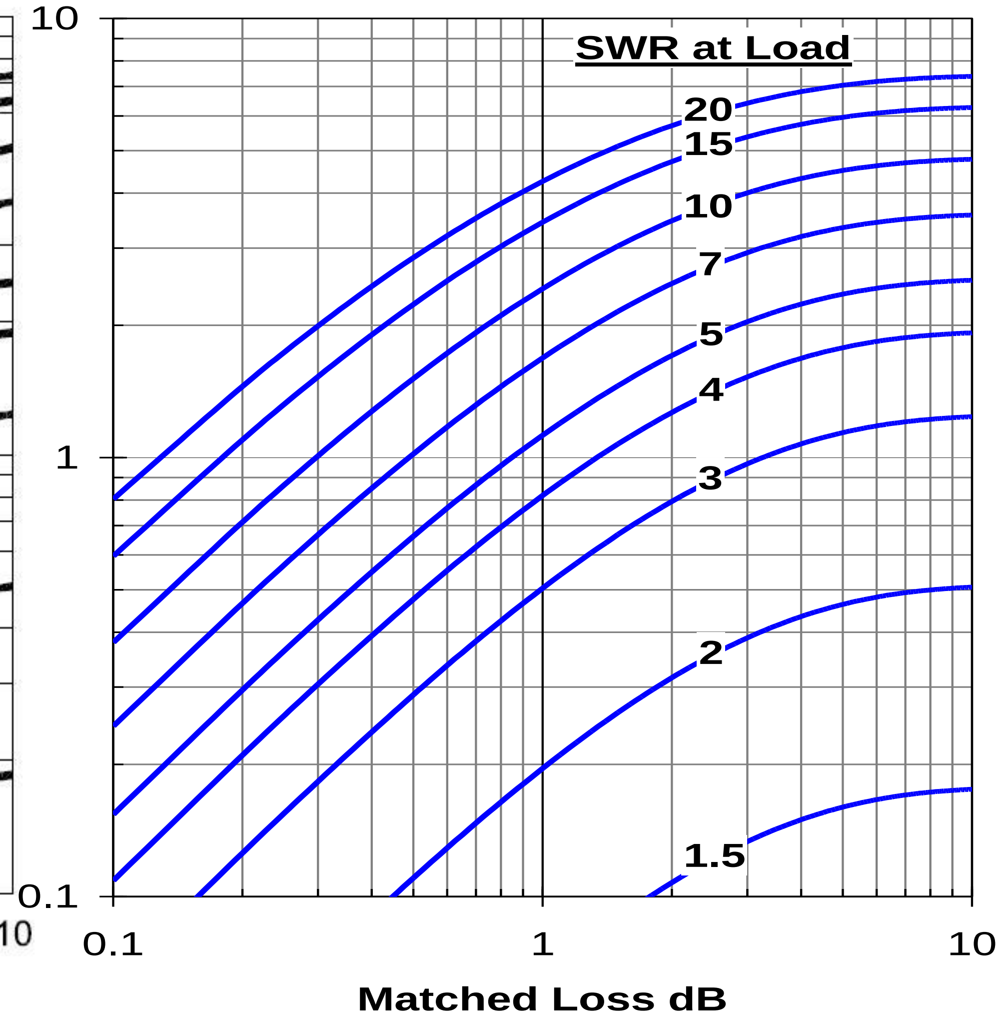
Published in German

- **K. Rothammel (Y21BK), *Antennenbuch*, Fig. 5.25, p. 98, 1981**



Additional Loss in Terms of SWR at Load

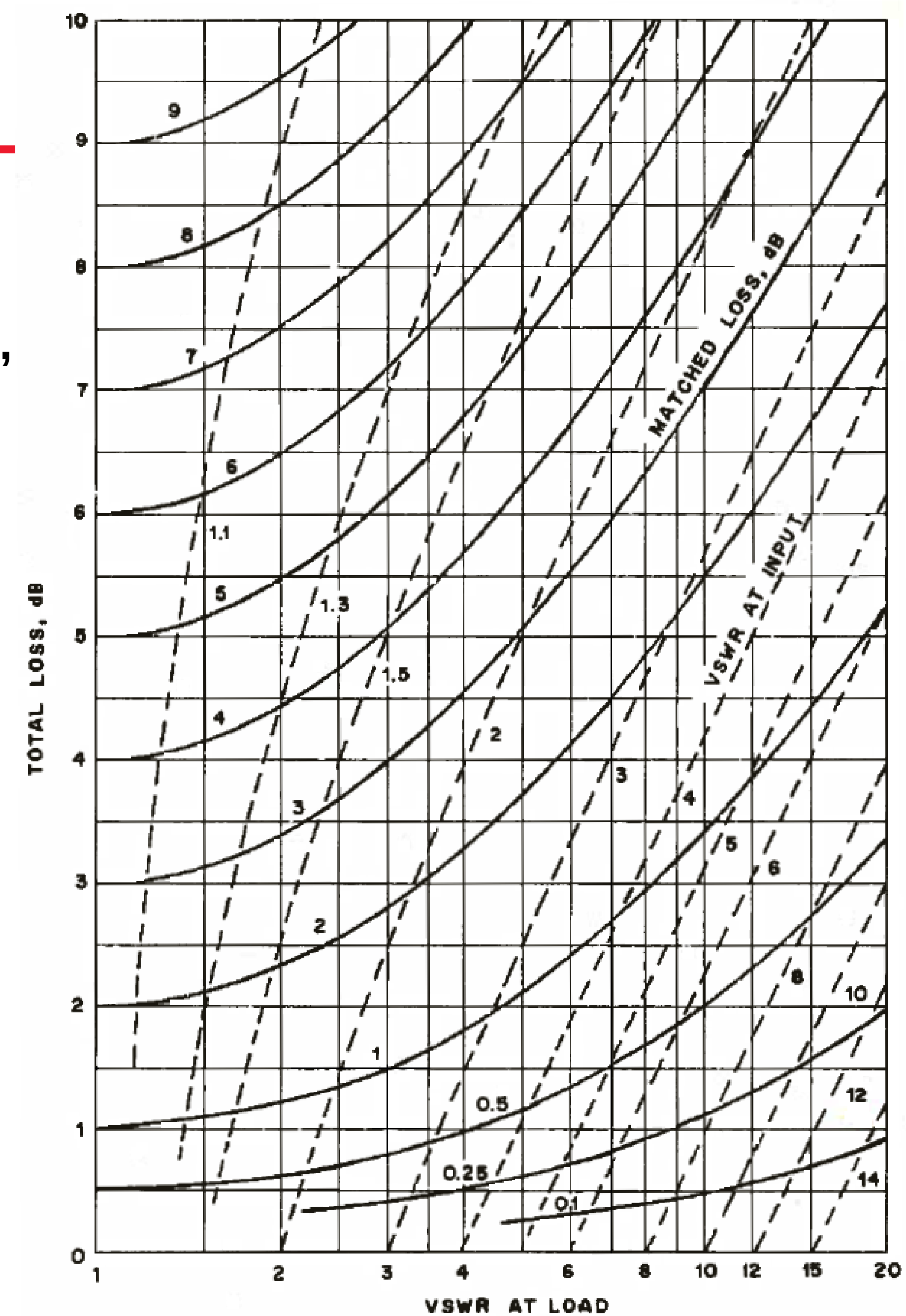




ARRL Handbook, 87th ed., Fig. 20.4, p. 20.5
ARRL Antenna Book, 21st ed., Fig. 14, p. 24-10

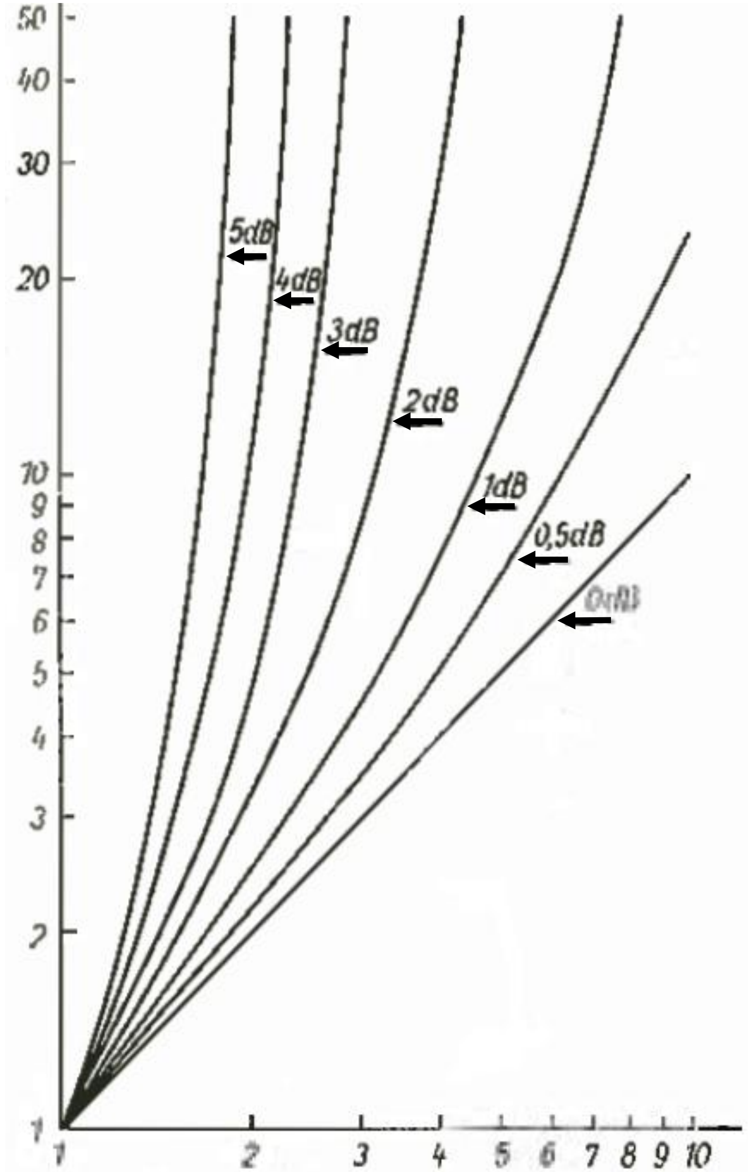
Graph 2: "Total Loss Due to SWR at Load"

- **Published in *ARRL Antenna Book*, 15th ed., 1988, and 16th ed., 1991**
- **Published in *ARRL Handbook* 1981 through 1984**

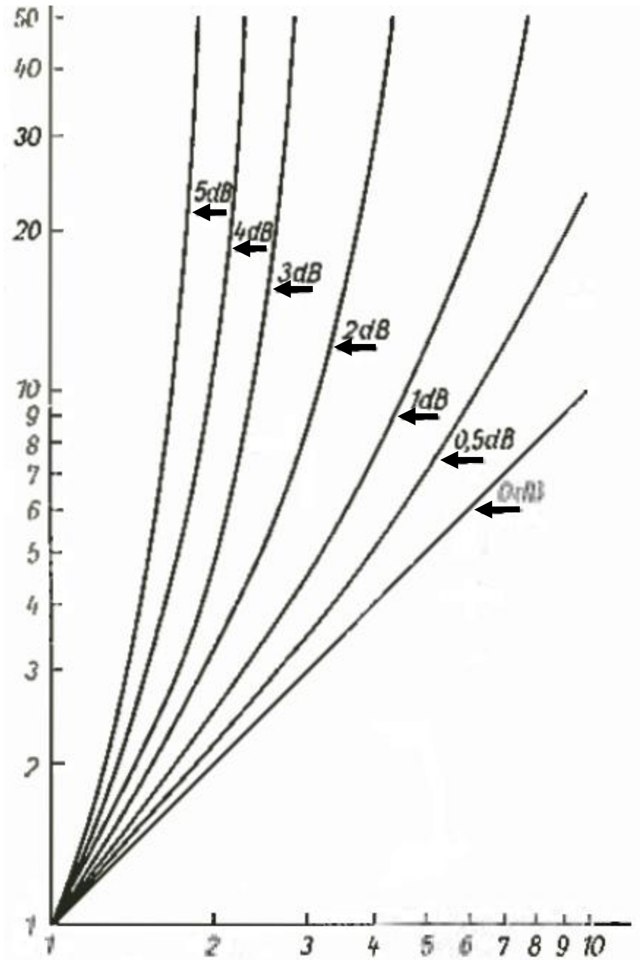


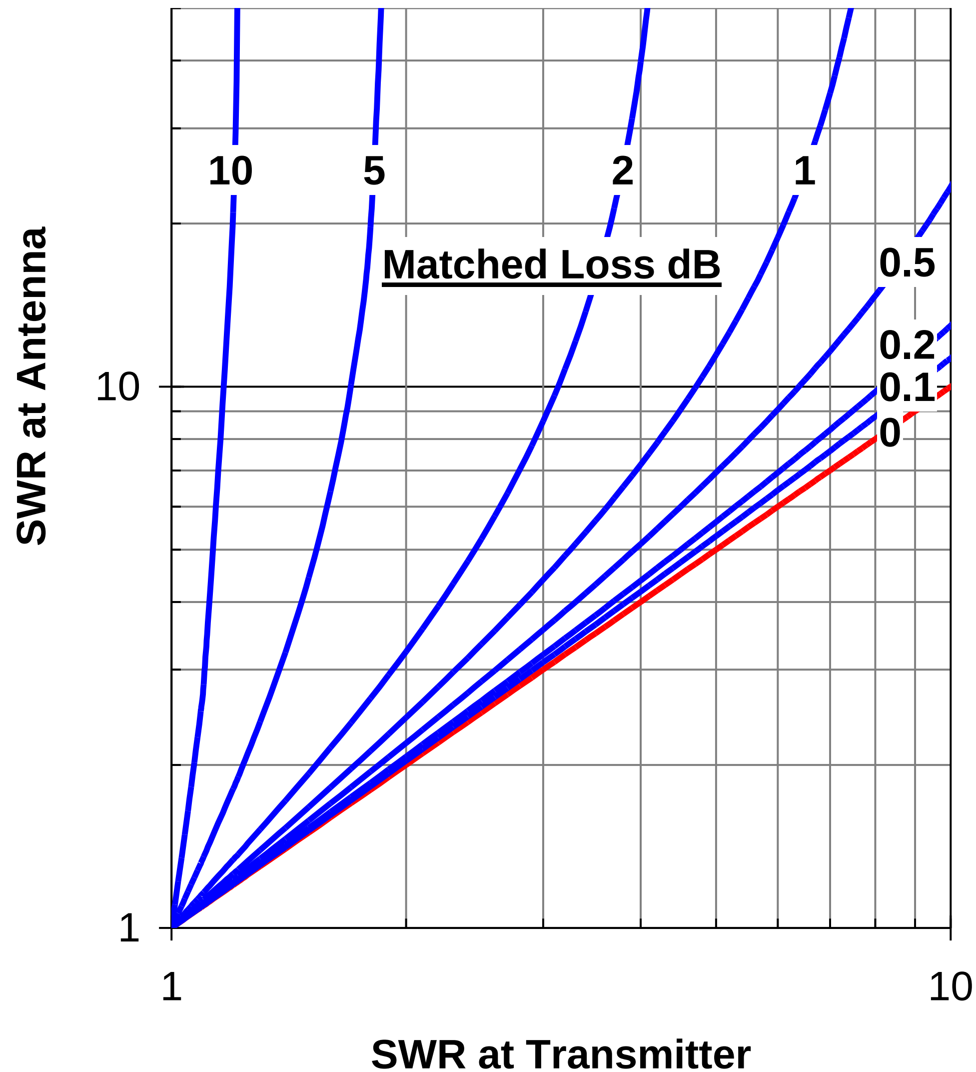
Graph 3: "SWR at Antenna vs SWR at Transmitter"

- **Published in *ARRL Antenna Book* 13th ed., 1974, and 14th ed., 1982**
- **Published in *ARRL Handbook* from 1985 to 1987 or later**
- **Also K. Rothammel (Y21BK), *Antennenbuch*, Fig. 5.26, p. 99, 1981**



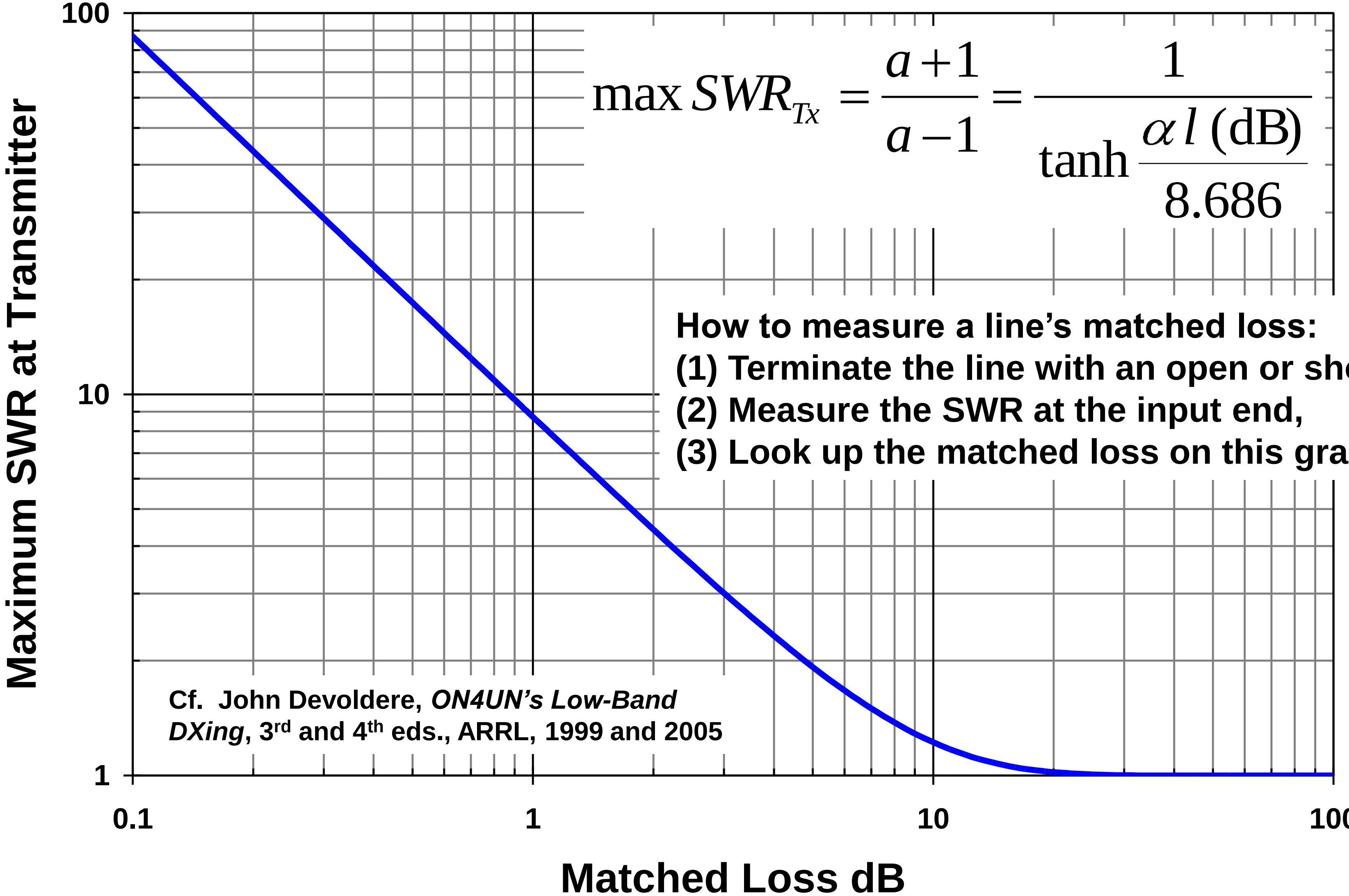
SWR at Antenna versus SWR at Transmitter





Source: K. Rothammel (Y21BK),
***Antennenbuch*, Fig. 5.26, p. 99, 1981**

Graph 4: Measuring a Line's Matched Loss via SWR

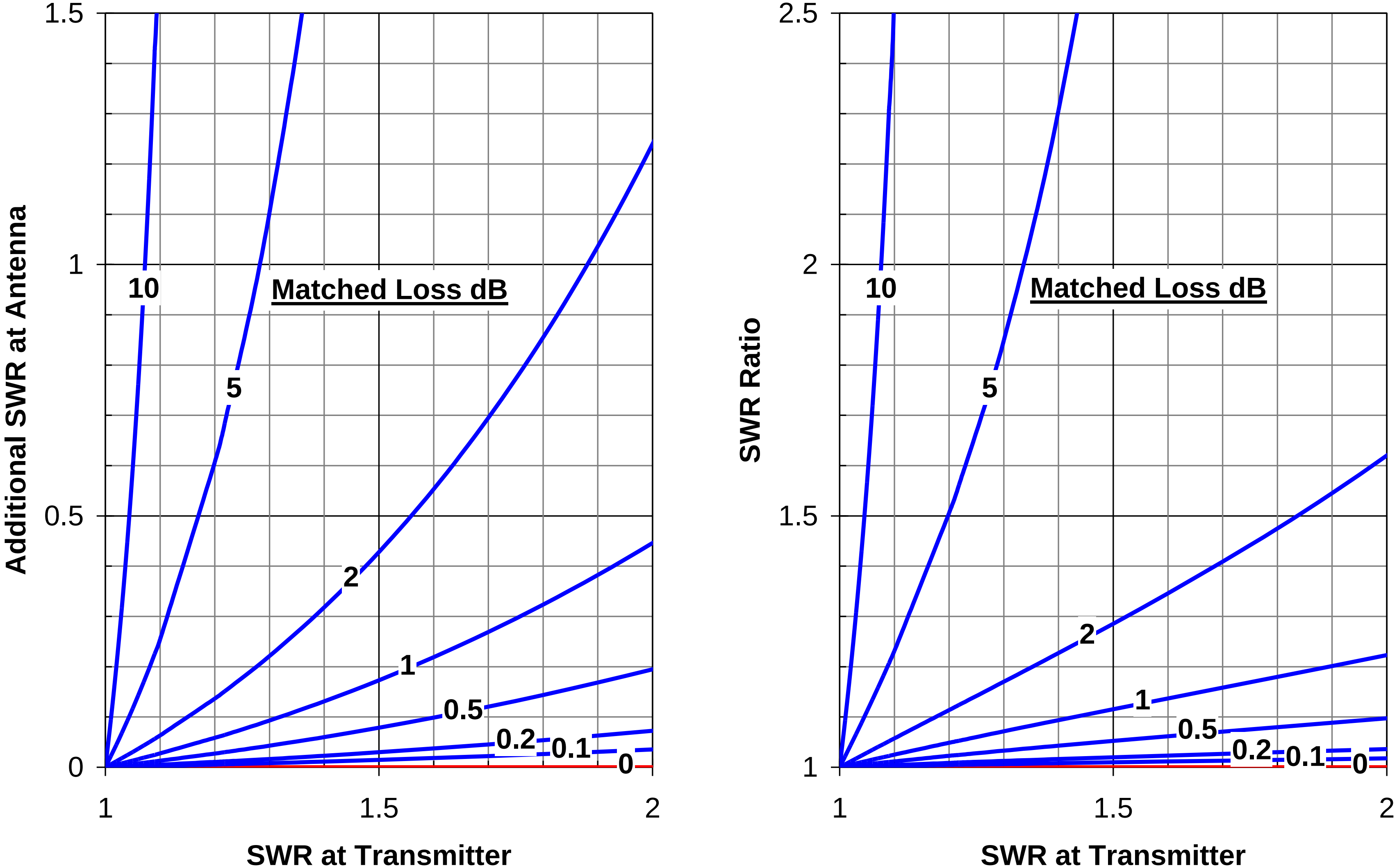


$$\max SWR_{Tx} = \frac{a+1}{a-1} = \frac{1}{\tanh \dfrac{\alpha l\,(\mathrm{dB})}{8.686}}$$

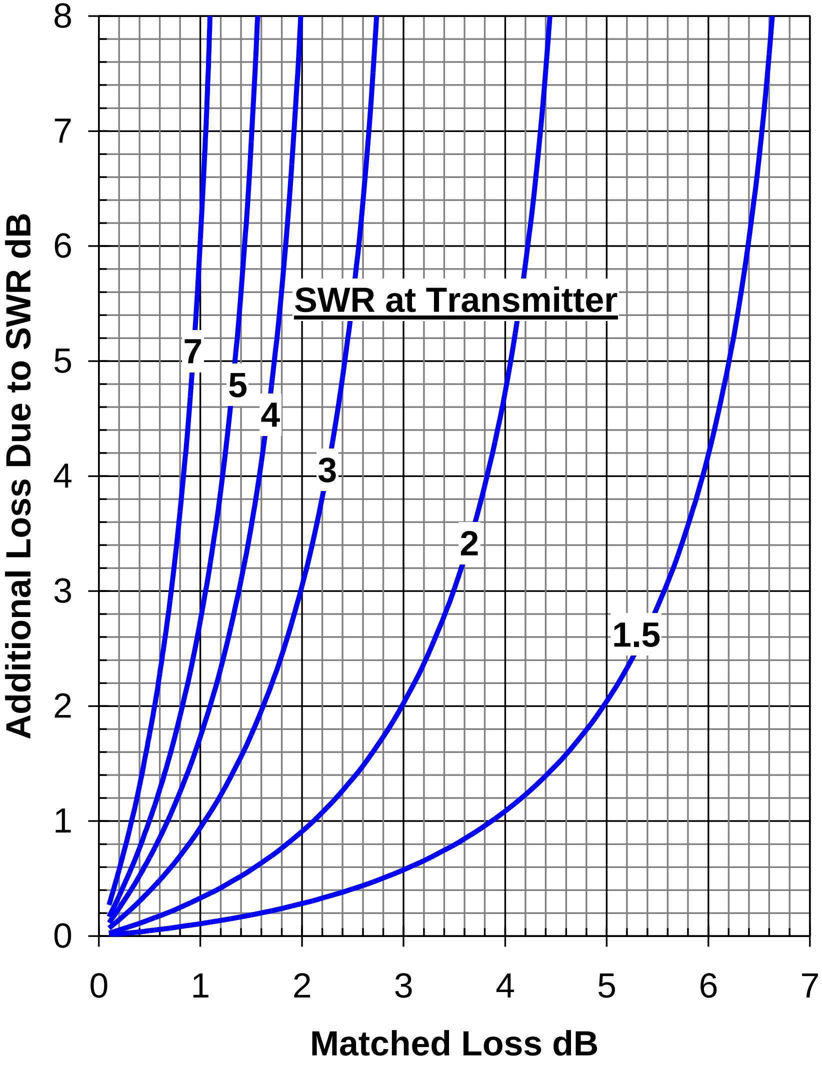
How to measure a line's matched loss:
(1) Terminate the line with an open or short,
(2) Measure the SWR at the input end,
(3) Look up the matched loss on this graph

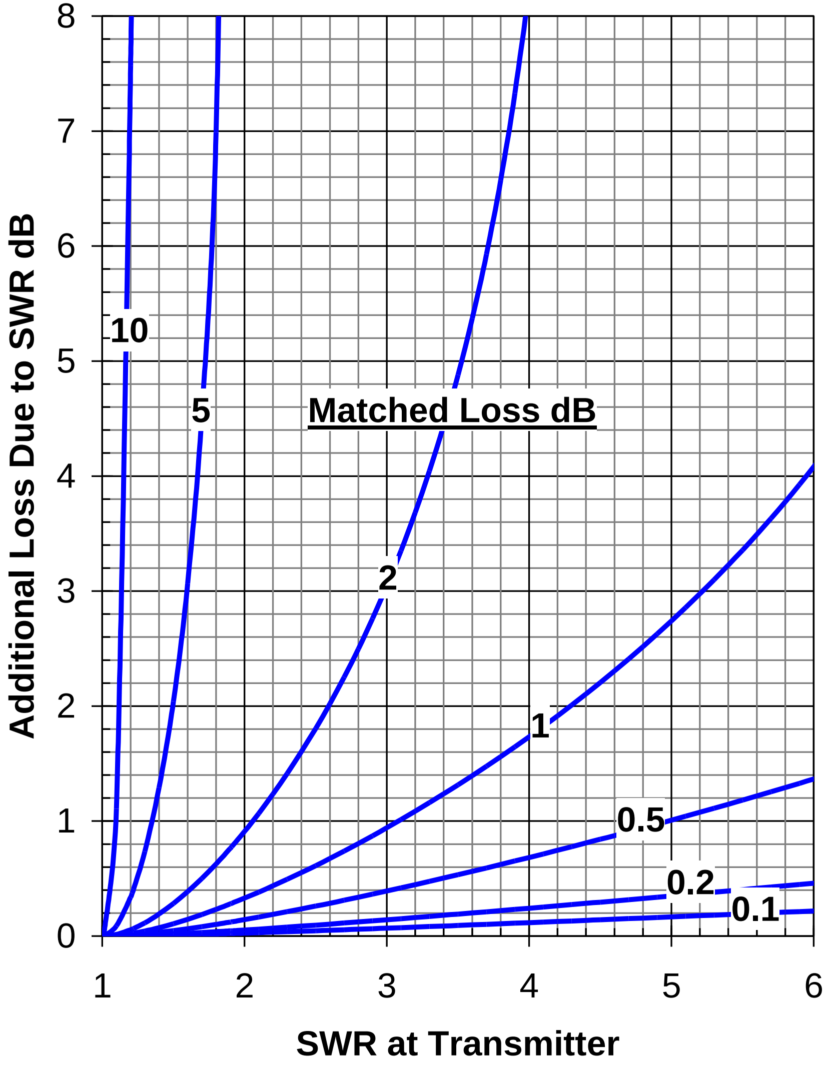
Cf. John Devoldere, *ON4UN's Low-Band DXing*, 3rd and 4th eds., ARRL, 1999 and 2005

Graph 5: Additional SWR at Load Due to SWR



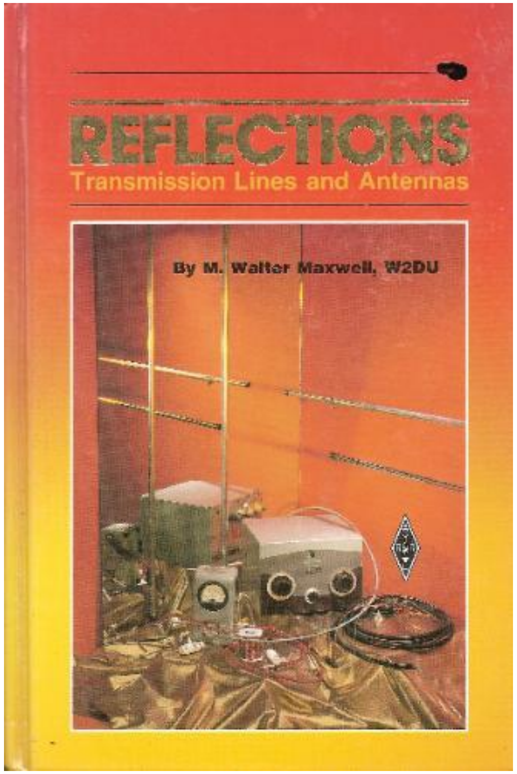
Graph 6: Additional Loss vs SWR at Transmitter



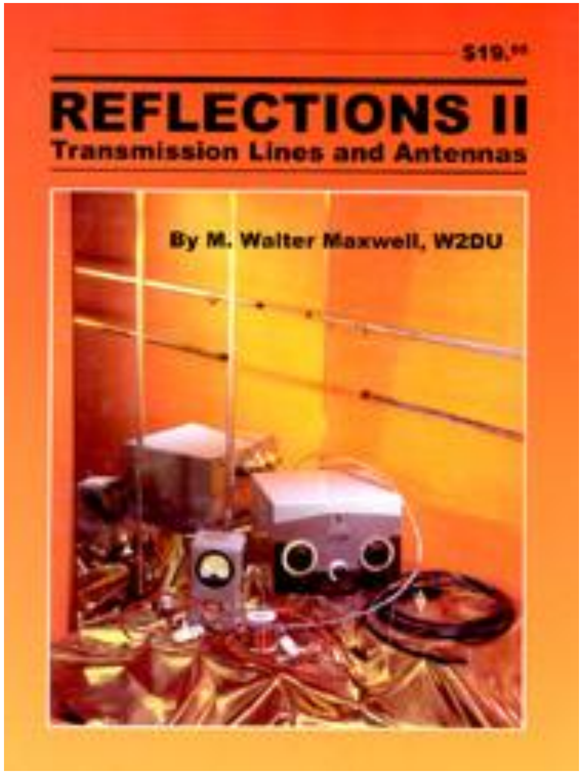


Myths About Conjugate Matching

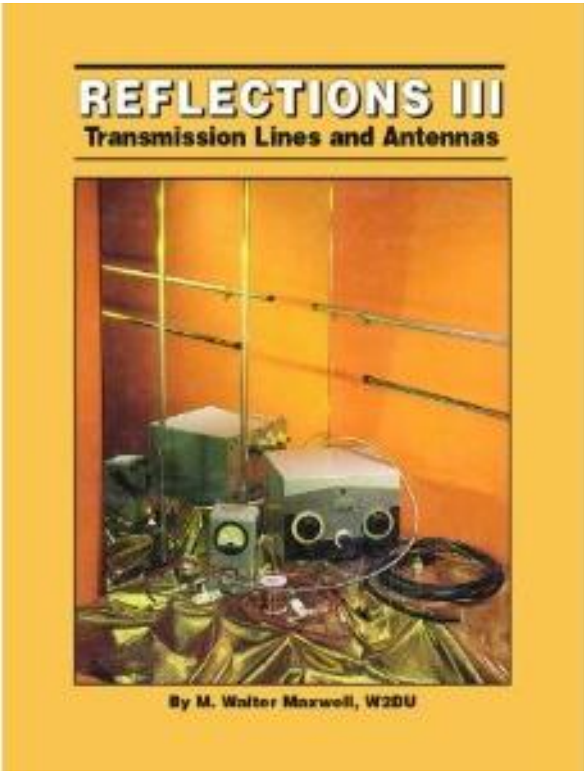
Reflections



1st ed., ARRL, 1990



2nd ed., WorldRadio Books, 2001

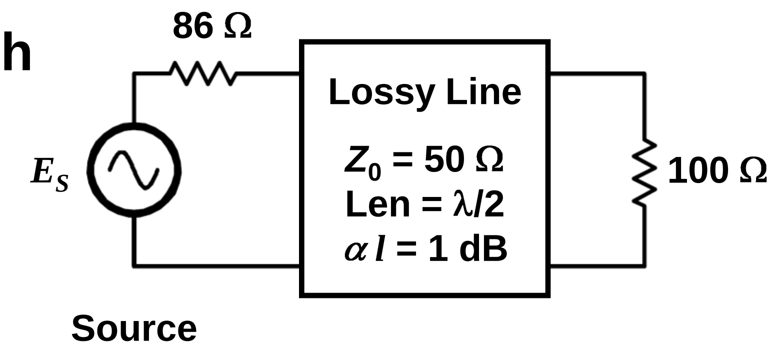


3rd ed., CQ Communications, 2010

Based on a 7-part series published sporadically in *QST* from April 1973 through August 1976

Myths and Bloopers

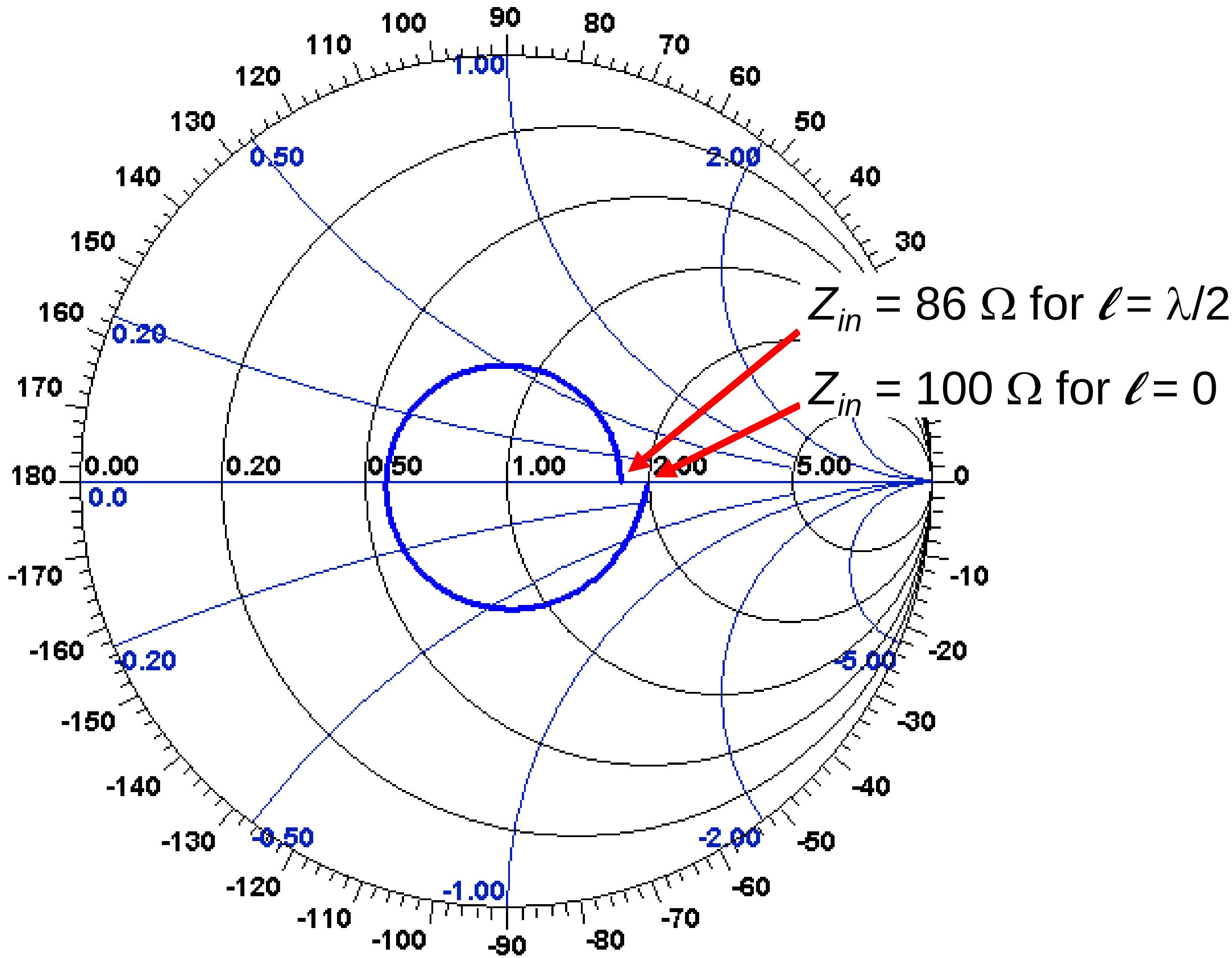
- **Conjugate match**



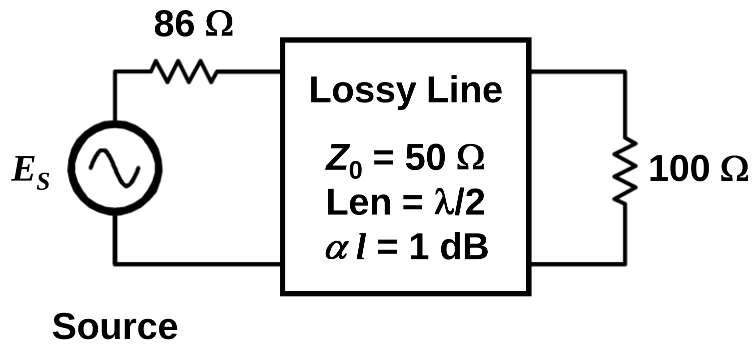
  - "Consequently, the source impedance is matched to the input impedance of the line, and the output impedance of the line is matched to its 100-ohm load. ... Thus the output of the line ... is delivering to the load all of the power that is available at the line output. Ergo, there is a conjugate match by definition between the source and the line input and between the output impedance of the line and the load impedance (Axioms 1 and 2) despite the 1.0-dB attenuation in the line." Walter Maxwell, W2DU, *Reflections II*, p. A9-8, Worldradio Books, 2001. Also in *Reflections III*, sec. A9A.5, CQ Communications, 2010.

- **Facts**
  - Circuit analysis reveals that the load is not conjugately matched to the line, only the source is conjugately matched
  - A single-end conjugate match (at source or load) does not deliver maximum power to the load if the line is lossy
  - Maxwell mistakenly believes otherwise

Line Input Impedance versus Line Length 0 to λ/2



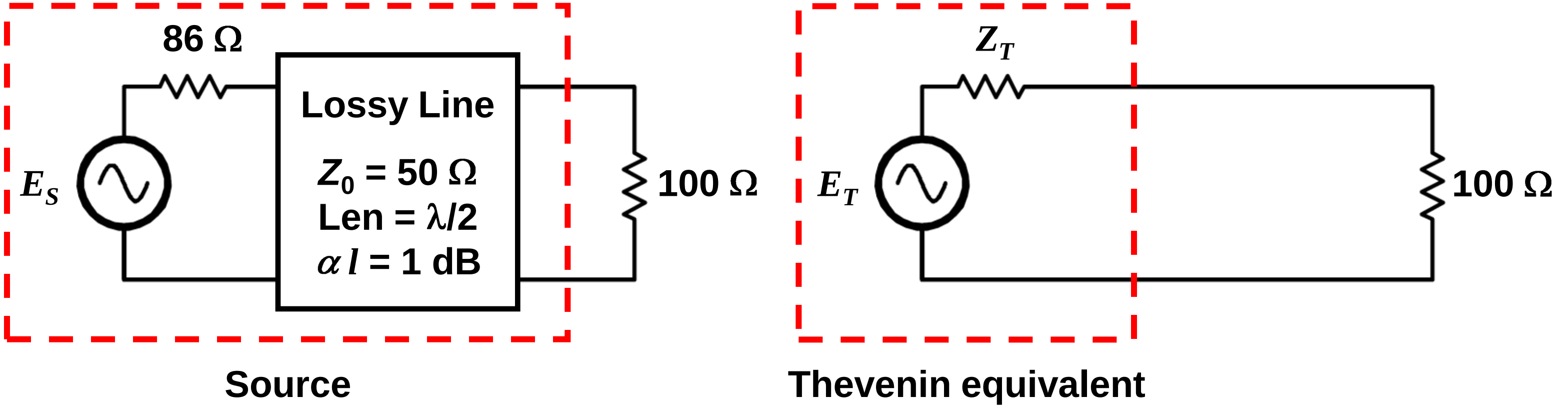
Questions



- **The source is conjugately matched to the line's input impedance**
- **But are Walt Maxwell's claims true?**
  - The line's output impedance is conjugately matched to the load
  - Maximum power is delivered to the load
  - All of the source's available power is delivered to the load

Analysis

- Determine the Thevenin equivalent source



$$E_T = E_{open\ circuit}$$

$$Z_T = \frac{E_{open\ circuit}}{I_{short\ circuit}}$$

Thevenin Equivalent Source

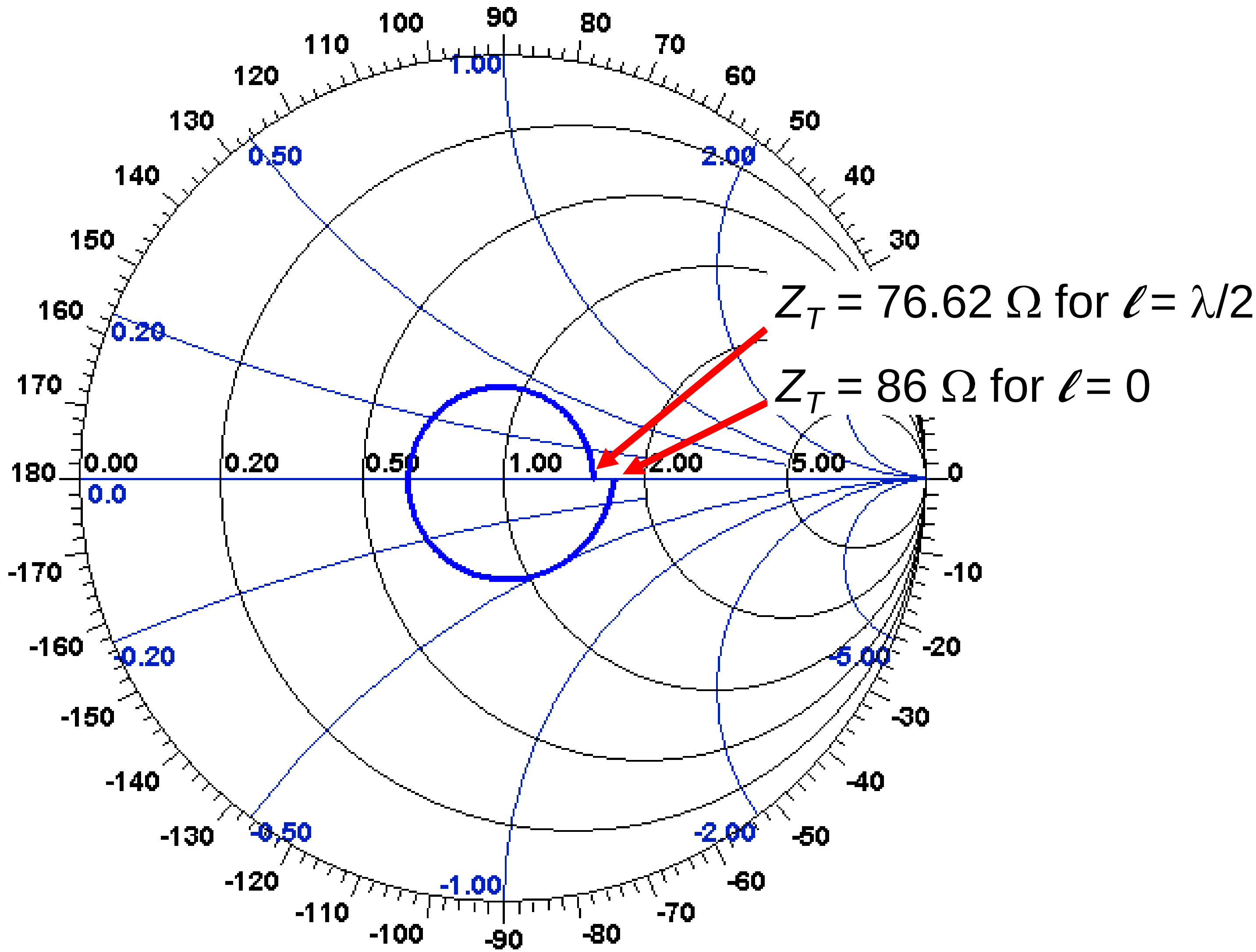
- **Thevenin voltage and impedance**

$$E_T = E_{open\ circuit} = E_S \left[\frac{\dfrac{1}{\cosh \gamma l}}{1 + \dfrac{Z_S}{Z_0} \tanh \gamma l} \right] = E_S \left[\frac{\dfrac{-1}{\cosh \alpha l}}{1 + \dfrac{86}{50} \tanh \alpha l} \right] = -0.8298 \times E_S$$

$$\underbrace{Z_T = \frac{E_{open\ circuit}}{I_{short\ circuit}} = Z_0 \left[\frac{\dfrac{Z_S}{Z_0} + \tanh \gamma l}{1 + \dfrac{Z_S}{Z_0} \tanh \gamma l} \right]}_{\text{General equations}} = \underbrace{50 \left[\frac{\dfrac{86}{50} + \tanh \alpha l}{1 + \dfrac{86}{50} \tanh \alpha l} \right]}_{\beta \ell = \pi} = \underbrace{76.62 \text{ ohms}}_{\alpha \ell = 0.1152 \text{ Np (1 dB)}}$$

- **100 Ω load is not Z_0 matched to 50 Ω nor conjugately matched to 76.6 Ω**
- **SWR = 2 at load means 0.2 dB of additional, avoidable loss is present**
- **All available power is NOT delivered to the load**

Thevenin Source Impedance vs Line Length 0 to $\lambda/2$



Why Walt Maxwell is Wrong About Conjugate Matching

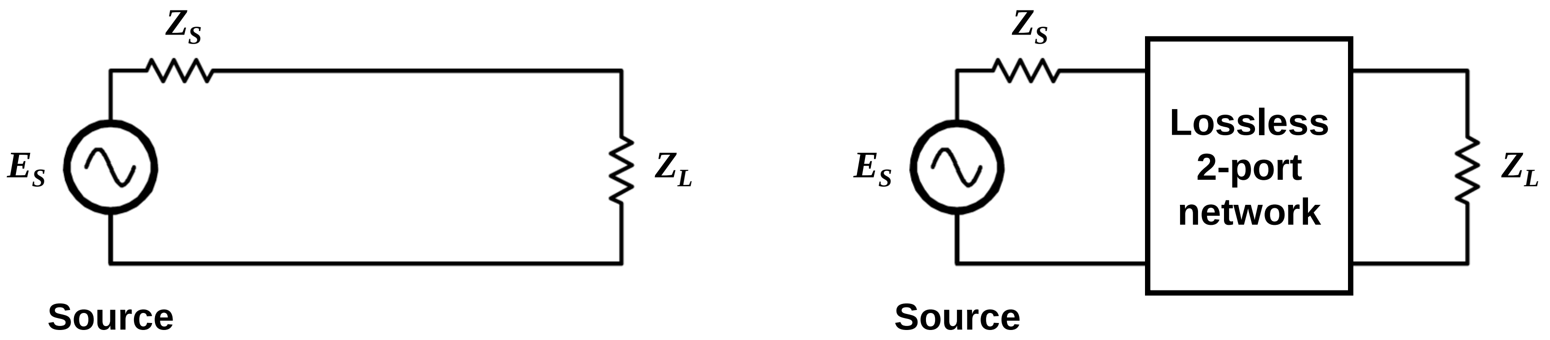
- Walt Maxwell's theory of transmission lines is correct only for mathematically ideal lossless lines
- Maxwell's theory of conjugate matching is incorrect for physical transmission lines that have loss
- Maxwell is wrong because his reasoning is based on "axioms" instead of fundamental principles of circuit theory, and the axioms are stated badly and misunderstood
- Maxwell's Axiom 2 and Axiom 3 are untrue in general
- Maxwell's assertions in his Appendix 9A imply that an impedance transforms on the Smith chart by clockwise or counter-clockwise revolution depending on which end of the transmission line the impedance is connected to
- Pursuing Maxwell's concept further, transmission lines are not symmetric, reciprocal devices and can be shown to lead to violations of R.M. Foster's Reactance Theorem (and so cannot be passive 2-port devices) and violations of energy conservation
- Walt Maxwell's understanding of lossy transmission line behavior is poor

What's Wrong with Walt Maxwell's "Axioms" ?

- **Axiom 1 – There is a conjugate match in an RF power transmission system when the source is delivering all of its available power to the load**
  - The statement would be a theorem if it were proved from first principles. However, terms such as conjugate match and available power must be defined first.
- **Axiom 2 – There is a conjugate match if the delivery of power decreases whenever the impedance of either the source or load is changed in either direction. This follows from the Maximum Power-transfer Theorem**
  - The statement is not true. If a Thevenin voltage source impedance is real and decreases from a conjugate matched value, the delivery of power to the load increases not decreases.
- **Axiom 3 – If there is a conjugate match at any junction in the system, and if there are no active or 'pseudo active' sources within the network, there is a conjugate match everywhere in the system. (The phasors at any point along a transmission line are conjugates)**
  - The statement is not true and misstates what Everitt proved
- **Axiom 4 – The term 'conjugate match' means that if in one direction from a junction the impedance is $R + jX$, then in the opposite direction the impedance will be $R – jX$**
  - The statement defines "conjugate match" in terms of the undefined term "junction." The author uses "junction" when he means "port."

Maximum Power Transfer

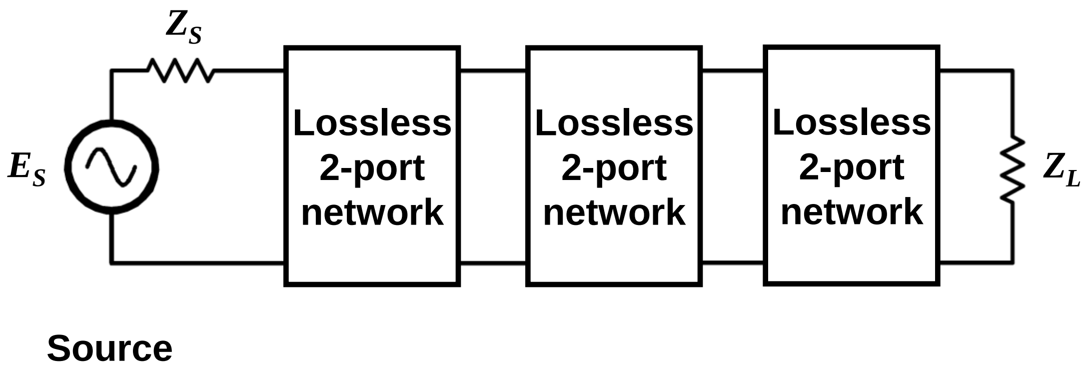
Maximum Power Transfer Theorem



- For a given source voltage E_S and source impedance Z_S, maximum power is delivered to the load when the load impedance is the conjugate of the source impedance
- The theorem does NOT state for a given voltage E_S and load impedance Z_L, maximum power is delivered to the load when the source impedance is the conjugate of the load impedance
- However, if a lossless 2-port network is inserted between source and load, then for a given voltage E_S and load impedance Z_L, maximum power is delivered to the load when the network presents conjugate impedances to the source and load
- This follows from W.L. Everitt's theorem

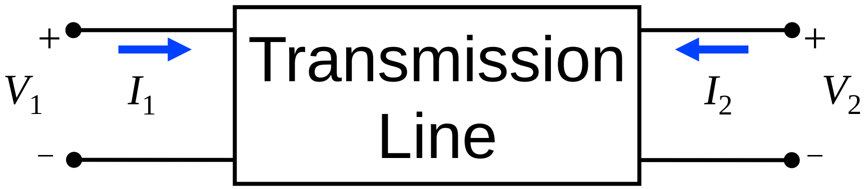
William Littell Everitt, 1900-1986

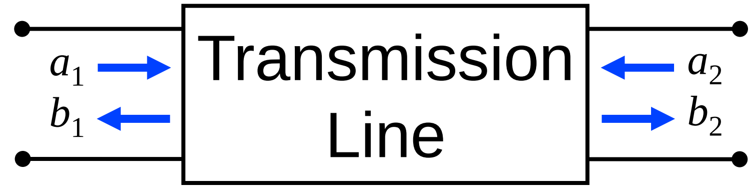
Everitt's Conjugate Match Theorem (1932)



- Consider a series of lossless 2-port networks connected in cascade between a source and a load
- Theorem: If a conjugate match exists at any port in the cascade, then a conjugate match exists at every port in the cascade, including the input and output ports connected to the source and load
- All available power is delivered to the load
- Example: Consider a transmitter, a lossless coupling network, and a transmission line. If the coupling network is conjugately matched, then the transmission line receives all available power from the transmitter

Transmission Line Representations
Z, Y, *ABCD*, and S Parameters





$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = Z_0 \begin{bmatrix} \coth \gamma l & \dfrac{1}{\sinh \gamma l} \\ \dfrac{1}{\sinh \gamma l} & \coth \gamma l \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = Y_0 \begin{bmatrix} \coth \gamma l & \dfrac{-1}{\sinh \gamma l} \\ \dfrac{-1}{\sinh \gamma l} & \coth \gamma l \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}$$

$$\gamma = \alpha + j\beta$$

$$e^{-\gamma l} = e^{-(\alpha l + j\beta l)}$$

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_0 \sinh \gamma l \\ Y_0 \sinh \gamma l & \cosh \gamma l \end{bmatrix} \begin{bmatrix} E_2 \\ -I_2 \end{bmatrix}$$

Important Secondary Parameters of 2-Ports

- **Scattering matrix determinant**

$$\Delta = \det S = s_{11}s_{22} - s_{12}s_{21}$$

- **Rollett's *K* factor**

$$K = \frac{1 - |s_{11}|^2 - |s_{22}|^2 + |\Delta|^2}{2|s_{12}s_{21}|}$$

- **Bodway's *B* factors**

$$B_1 = 1 + |s_{11}|^2 - |s_{22}|^2 - |\Delta|^2$$

$$B_2 = 1 - |s_{11}|^2 + |s_{22}|^2 - |\Delta|^2$$

- ***C* factors**

$$C_1 = s_{11} - \Delta s_{22}^*$$

$$C_1 = s_{22} - \Delta s_{11}^*$$

For lossy lines

$$\Delta = -e^{-2(\alpha l + j\beta l)}$$

$$|\Delta| = e^{-2\alpha l} < 1$$

$$K = \cosh \alpha l > 1$$

$$B_1 = 1 - e^{-4\alpha l} > 0$$

$$B_2 = 1 - e^{-4\alpha l} > 0$$

$$C_1 = 0$$

$$C_2 = 0$$

Transducer Power Gain

- **Maximum power delivery from a given source through a general 2-port to a load is achieved by maximizing “Transducer Power Gain”**

$$G_T = \frac{\text{Power delivered to load}}{\text{Power available from source}}$$

$$= \frac{(1-|\Gamma_S|^2)\,|s_{21}|^2\,(1-|\Gamma_L|^2)}{\left|(1-s_{11}\Gamma_S)(1-s_{22}\Gamma_L)-s_{12}s_{21}\Gamma_L\Gamma_S\right|^2}$$

- **For a general transmission line**

$$G_T = \frac{(1-|\Gamma_S|^2)\,e^{-2\alpha l}(1-|\Gamma_L|^2)}{\left|1-e^{-2(\alpha l+j\beta l)}\Gamma_L\Gamma_S\right|^2}$$

Maximum Transducer Power Gain

- **Question: For a given 2-port network, what is the maximum transducer gain G_T relative to all source and load impedances?**

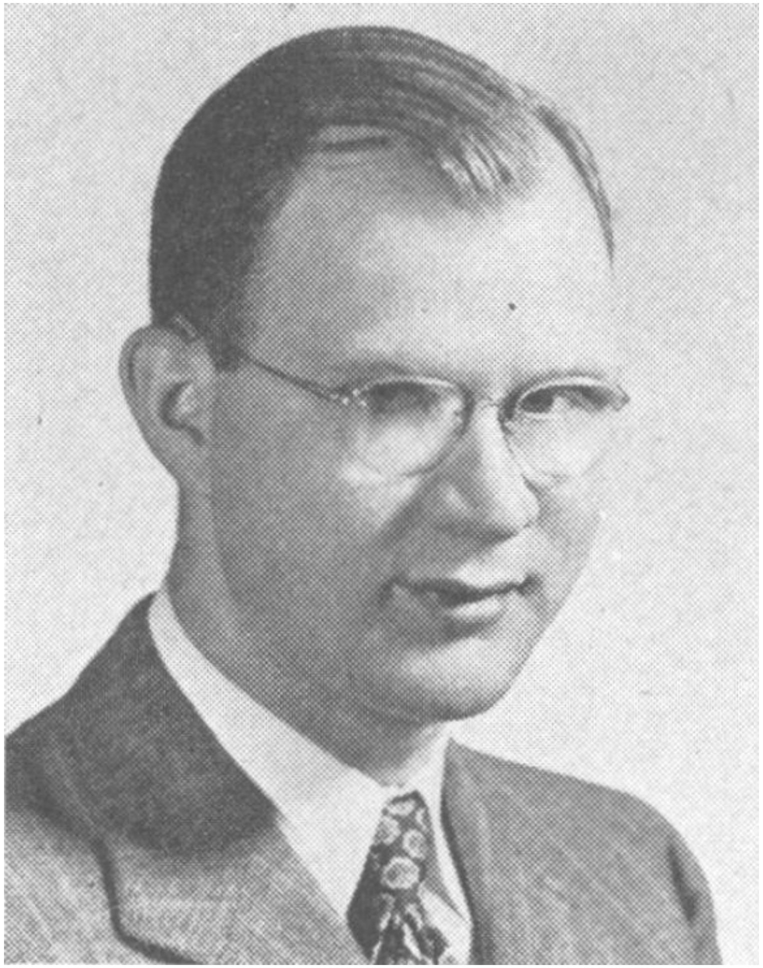
$$G_{MAX} = \max_{|\Gamma_S| \text{ and } |\Gamma_L|} G_T$$
$$= \frac{|s_{21}|}{|s_{12}|}[K - \sqrt{K^2 - 1}]$$

- **For a general transmission line, we can show**

$$G_{MAX} = e^{-2\alpha l} = \frac{1}{a} = \frac{1}{\text{matched loss}}$$

- **How do we get this maximum gain (minimum loss)?**

Shepard Roberts



- S. Roberts, “Conjugate-Image Impedances,” *Proc. IRE*, April 1946.

Simultaneous Equations for Maximum Power Transfer

- **First solved by S. Roberts (1946) using Z or Y parameters**

$$\Gamma_S^* = \Gamma_{in} = s_{11} + \frac{s_{12}s_{21}\Gamma_L}{1 - s_{22}\Gamma_L} = \frac{s_{11} - \Delta\Gamma_L}{1 - s_{22}\Gamma_L}$$

$$\Gamma_L^* = \Gamma_{out} = s_{22} + \frac{s_{12}s_{21}\Gamma_S}{1 - s_{11}\Gamma_S} = \frac{s_{22} - \Delta\Gamma_S}{1 - s_{11}\Gamma_S}$$

Simultaneous Conjugate Match Equations

$$\Gamma_S^* = e^{-2(\alpha l + j\beta l)}\Gamma_L$$

$$\Gamma_L^* = e^{-2(\alpha l + j\beta l)}\Gamma_S$$

Lossy Transmission Line

- **Solution using S parameters is in modern books on amplifier design**
  - G.D. Vendelin, 1982
  - C. Bowick, 1982
  - R.E. Collin, 2nd ed., 1992
  - W. Hayward, 1994
  - G. Gonzalez, 1997
  - D.M. Pozar, 1999
  - R. Ludwig and P. Brechtko, 2000

The Solution for Maximum Power Transfer

- **Solution for transmission line is evident by inspection**

$$\Gamma_S^* = e^{-2(\alpha l + j\beta l)}\Gamma_L \quad \Rightarrow \quad |\Gamma_S| = e^{-2\alpha l}|\Gamma_L| \quad \Rightarrow \quad |\Gamma_S| \le |\Gamma_L|$$

$$\Gamma_L^* = e^{-2(\alpha l + j\beta l)}\Gamma_S \quad \Rightarrow \quad |\Gamma_L| = e^{-2\alpha l}|\Gamma_S| \quad \Rightarrow \quad |\Gamma_L| \le |\Gamma_S|$$

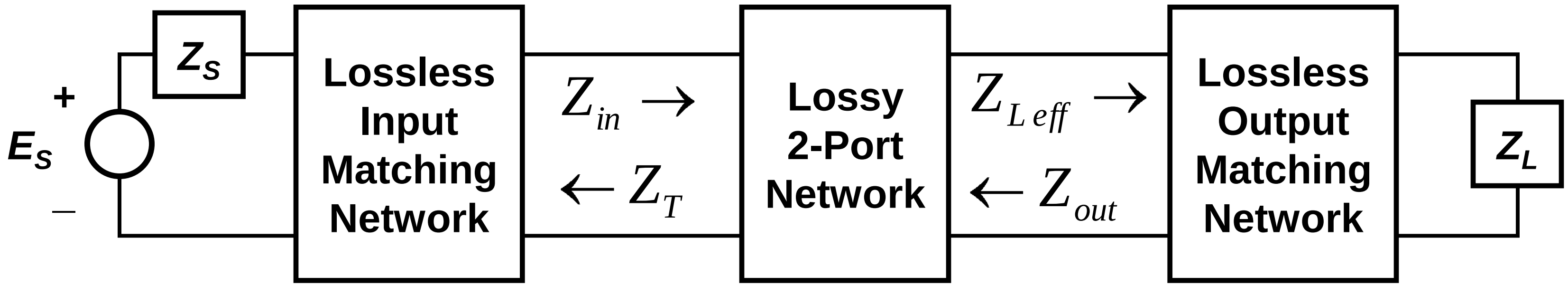
- **Unique solution**

$$\boxed{\Gamma_S = \Gamma_L = 0}$$

- **The solution specifies a pair of lossless match networks at both transmission line ports**
- **Together, the networks give a "simultaneous conjugate match"**
- **But, they do this by implementing double Z_0 matchs**
  - Input network transforms source impedance to Z_0
  - Output network transforms load impedance to Z_0

Maximum Power Transfer Through a 2-Port

- General case



$$Z_{in} = Z_T^* \qquad\qquad Z_{L\,eff} = Z_{out}^*$$

- If the 2-port is a transmission line with real characteristic impedance Z_0, then the max power solution requires that

$$Z_T = Z_{in} = Z_0 = Z_{out} = Z_{L\,eff}$$

Comments

- **A single conjugate match network at source or load does NOT result in maximum power transfer through a physical transmission line, in refutation of Walt Maxwell !**
- **Power transfer to a load through a lossy line is maximized by simultaneous conjugate matching at both ends**
  - Maximizes "transducer power gain" of the transmission line
  - Technique is well known in solid-state RF amplifier design
- **Maximum power transfer requires two match networks, one at each transmission line end which, for real (non-reactive) characteristic impedance Z_0, are ordinary Z_0 match networks**
  - Input network transforms source impedance to Z_0
  - Output network transforms load impedance to Z_0
  - SWR on the line = unity $\Rightarrow$ no reflected wave $\Rightarrow$ no additional loss
- **The output network half of the solution should be used**
- **The input network should not be used with a solid-state amplifier unless the amplifier is unconditionally stable as it can move the load impedance on the transistors outside the stable region of operation**

Comments on the Single-End Conjugate Match

- **The Maximum Power Transfer Theorem is about power delivery to 1-port impedances, not about power delivery through 2-port devices**
- **Single-end conjugate matching at either end of a general lossy line does NOT maximize power transfer from source to load in general**
  - Does NOT give maximum power transfer from source to load through an intervening 2-port, e.g. a line, except in special cases
  - A conjugate match at the input does NOT imply a conjugate match at the output (load) and vice versa, except in special cases
- **Conjugate matching at the load permits reflected waves on the line**
  - Total loss = Matched loss + Additional loss due to SWR
  - Maximum power is not delivered because Total Loss is not minimum
- **Conjugate matching at the source can damage solid-state amplifiers**
  - Conjugate match network between amplifier and transmission line interferes with the amplifier's own coupling network and can make the amplifier unstable unless the design is "unconditionally" stable
  - Transistor gain can be unwittingly altered to exceed maximum stable gain (MSG), causing operation outside of the stable region on Smith chart

Late-Breaking News from antenneX, October 2011

- **NEW METHOD OF MEASURING R_{OS} DISPROVES HFTPA CONJUGATE MATCH CLAIM: Part I**
  By Dave Gordon-Smith, G3UUR
- **This article describes a new technique for measuring the output impedance of HF tuned power amplifiers, which differs radically from methods previously described in the course of the HFTPA conjugate match debate. It measures only dissipative resistance and the results show quite conclusively that not only is the real part of the output impedance dissipative, but it is also substantially less than the value of the load when adjusted for maximum power.**
- **This method is straightforward and the interpretation of the results only requires simple RLC theory. It clears up a major stumbling block in this long-running debate about the nature of the output impedance of HF tuned power amplifiers. Previously described techniques are reviewed in the light of this new evidence and the results of the reverse SWR (RPG) and load-variation methods are shown to be due to the directional nature of tube operation and the existence of an optimum-load mechanism, which is governed by the sensitivity of the plate current to plate voltage variations. This explains how a peak in output can occur without a conjugate match.**

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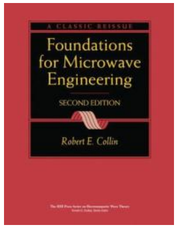
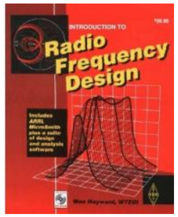
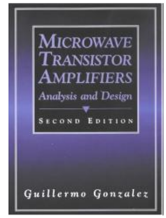
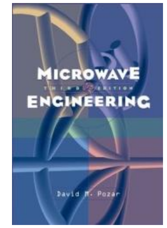
- W.W. Gartner, "Maximum Available Power Gain of Linear Four-Poles," *IRE Trans. Circuit Theory*, Dec. 1958.
- C. Shulman, "Conditions for Maximum Power Transfer," *IEEE Trans. Microw. Theory and Techniques*, Sept. 1961. Discussion Mar. 1963.
- H.F. Mathis, "Comment on 'Conditions for Maximum Power Transfer'," *IEEE Trans. Microw. Theory and Techniques*, Sept. 1963.
- H.F. Mathis, "Power Transfer Through a 4-Terminal Network," *Proc. IEEE*, June 1967.
- C.A. Desoer, "The Maximum Power Transfer Theorem for *n*-Ports," *IEEE Trans. Circuit Theory*, May 1973.
- C.A. Desoer, "A Maximum Power Transfer Problem," *IEEE Trans. Circuits and Syst.*, Oct. 1983.
- J. Rahola, "Power Waves and Conjugate Matching," *IEEE Trans. Circuits and Syst.*, Jan. 2008.

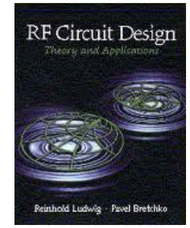
Books on Matching for Maximum Power Transfer

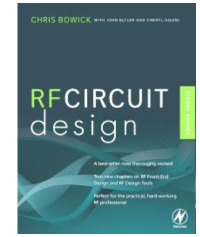












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- **R.E. Collin, *Foundation for Microwave Engineering*, 2nd ed., Wiley, 1992, ISBN 0780360311. Pages 730-733.**
- **W. Hayward, W7ZOI, *Introduction to Radio Frequency Design*, ARRL, 1994, ISBN 0872594920. Pages 196-197.**
- **G. Gonzalez, *Microwave Transistor Amplifiers: Analysis and Design*, 2nd ed., 466-468, Prentice-Hall, 1997, ISBN 0132543354. Pages 240-252.**
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- **C. Bowick, *RF Circuit Design*, 2nd ed., Newnes, 2007, ISBN 0750685182. Pages 128-131.**

The End

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