Antenna Potpourri

Random Musings on Cats, Waves, Q, Efficiency, and Ground Effects

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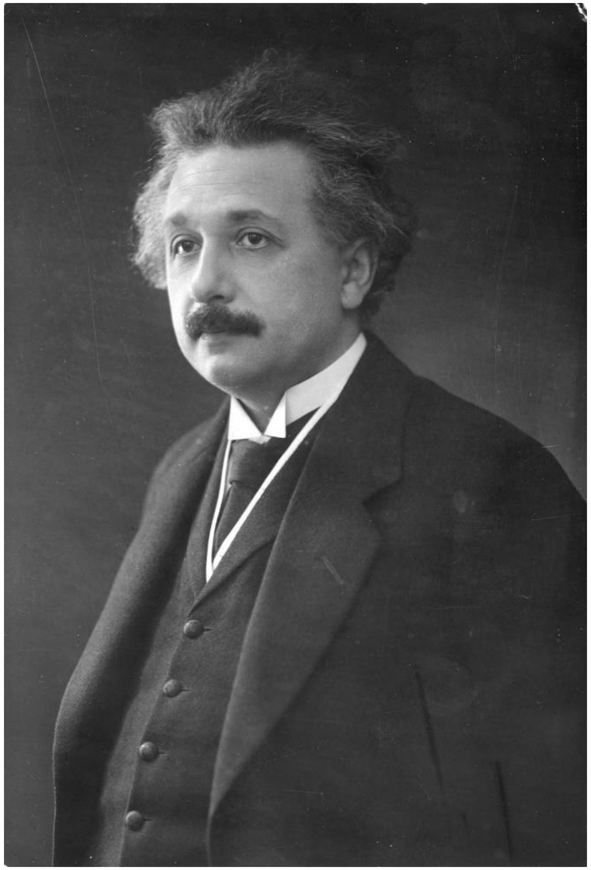
Abstract

Cats feature in some famous explanations of radio. Examples are given. Antenna Q has multiple definitions. A fundamental definition of Q is given. One kind of formula for Q is shown to be wrong. Chu’s famous 1948 result on antenna Q is revisited and shown to be correct but incomplete. A correction is given that generalizes Chu. One kind of formula for Q is shown to be wrong. Finally a comparison of HF dipoles versus verticals versus horizontal and vertical loops over ground shows which is better and why. No, it is not just a matter of pattern shape or noise polarization.

Topics

- **Einstein, Eliot, Webber, and Schrödinger on how radio works**
- **Dispelling a myth about wave propagation**
- **Comments on equivalent circuits for antenna impedance**
- **Antenna Q**
  - Fundamental definition
  - Qs of basic antennas in free space
  - Common formulas for antenna Q
- **Apparent Q=0 antennas**
  - Reflectionless match networks
  - Non-Foster match networks
  - Metamaterial radomes
  - Problems with Q formulas
- **Ground loss**
  - Comparison of dipoles vs verticals vs horizontal and vertical loops
- **References**

Albert Einstein’s Explanation of How Radio Works



Albert Einstein, 1879-1955

- **“You see, wire telegraph is a kind of a very, very long cat. You pull his tail in New York and his head is meowing in Los Angeles. Do you understand this? And radio operates exactly the same way: you send signals here, they receive them there. The only difference is that there is no cat.” – *Albert Einstein***

T.S. Eliot and Andrew Lloyd Webber's Contrary View of How HF Radio Works

Old Deuteronomy sends Jellicle cats "up, up, up, past the Russell Hotel up, up, up, up to the Heaviside Layer"

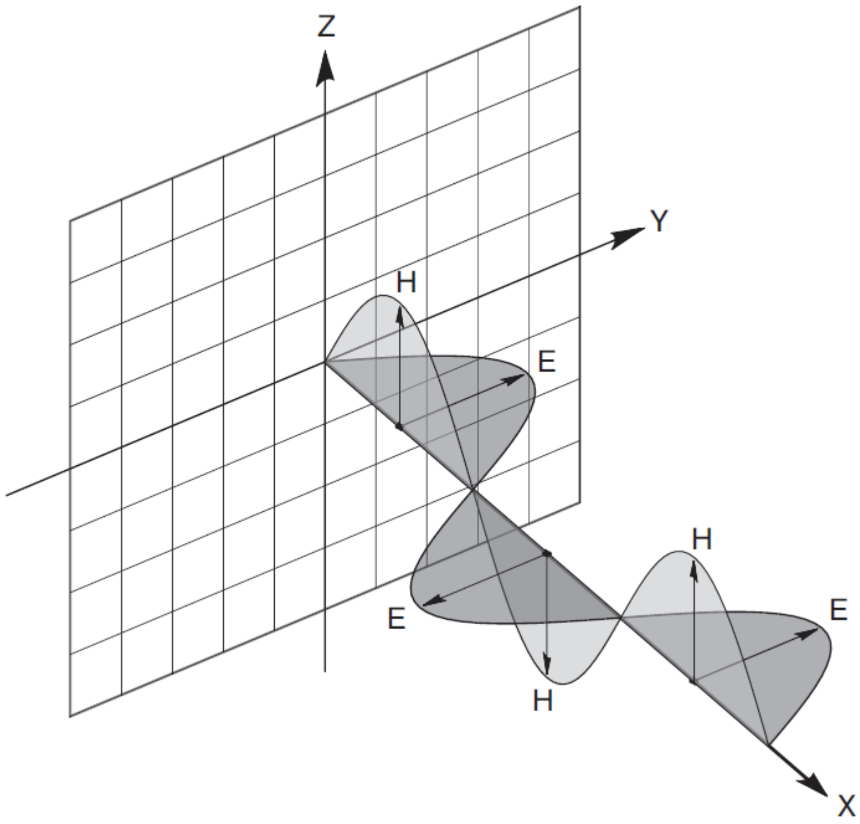
Cats handle HF radio communications in the ionosphere

But what explains band openings?

Schrödinger Cat Hypothesis

Erwin Schrödinger cat’s wave function never collapsed.
The cat is 50% alive.
The cat makes radio work.
He works half time, which explains band openings.

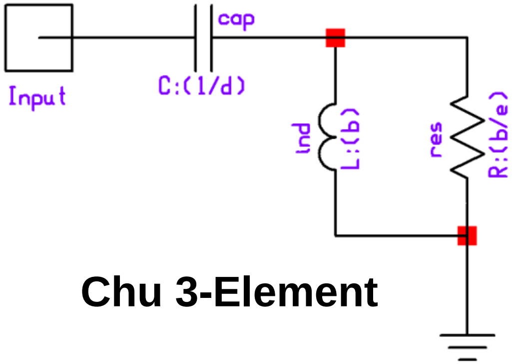
Dispelling a Common Myth About Wave Propagation



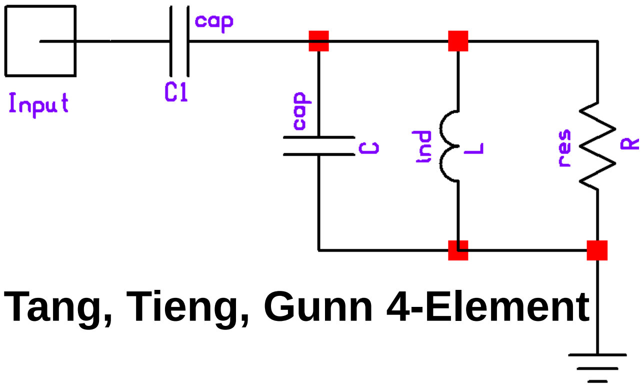
- For radiating TEM waves
- A time-varying electric field does NOT create or produce the magnetic field
- A time-varying magnetic field does NOT create or produce the electric field
- The electric and magnetic waves travel together but do not “cause” each other
- Both fields are created by accelerating charge at the source
- Both fields are given by radiation integrals of the source currents

O.D. Jefimenko, *Electricity and Magnetism*, 2e, Electret Scientific, 1966, 1989.
O.D. Jefimenko, *Causality, Electromagnetic Induction, and Gravitation*, 2e, Electret Scientific, 2000.

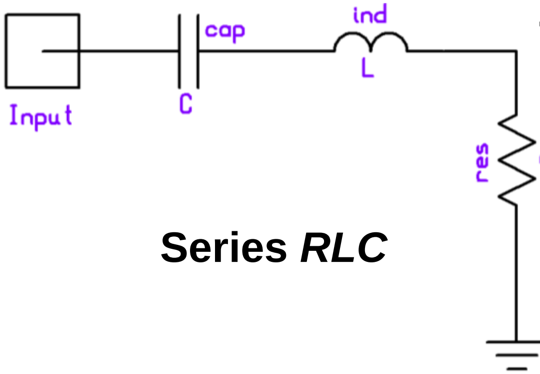
Equivalent Circuits for Electrically-Small Dipoles & Monopoles – Narrowband Models



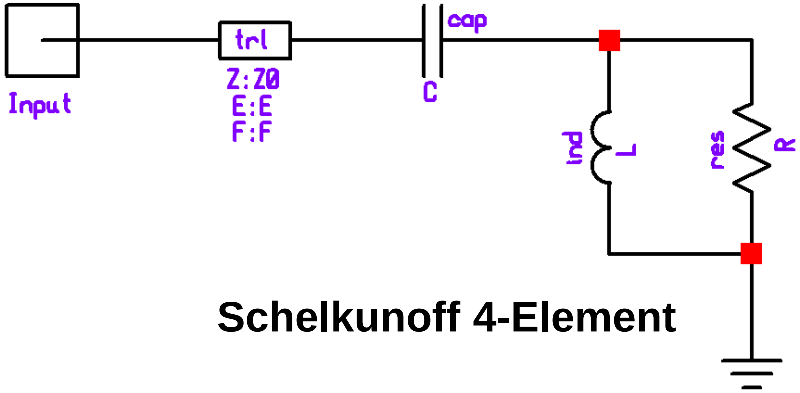
Chu 3-Element



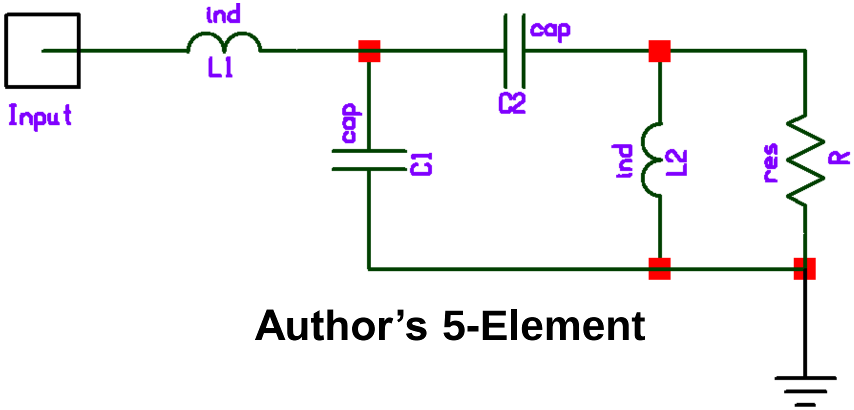
Tang, Tieng, Gunn 4-Element



Series *RLC*

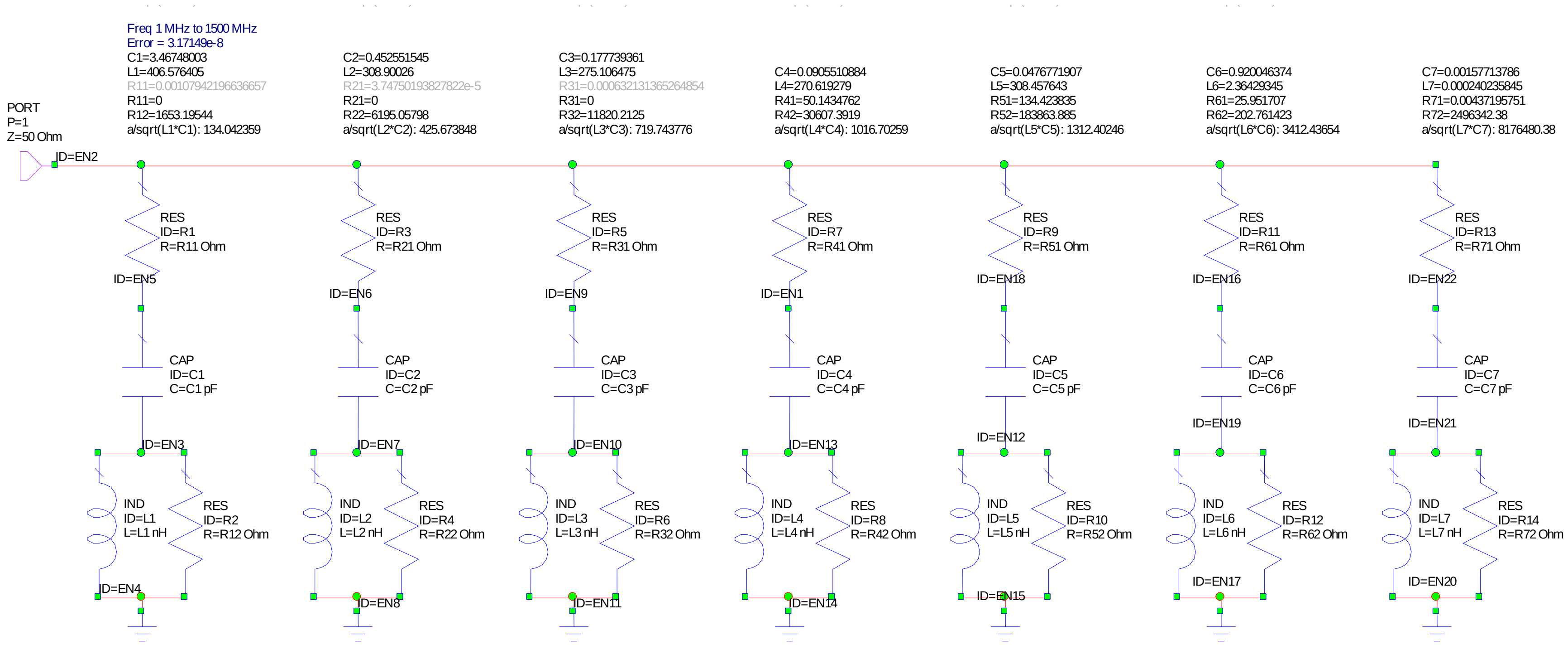


Schelkunoff 4-Element



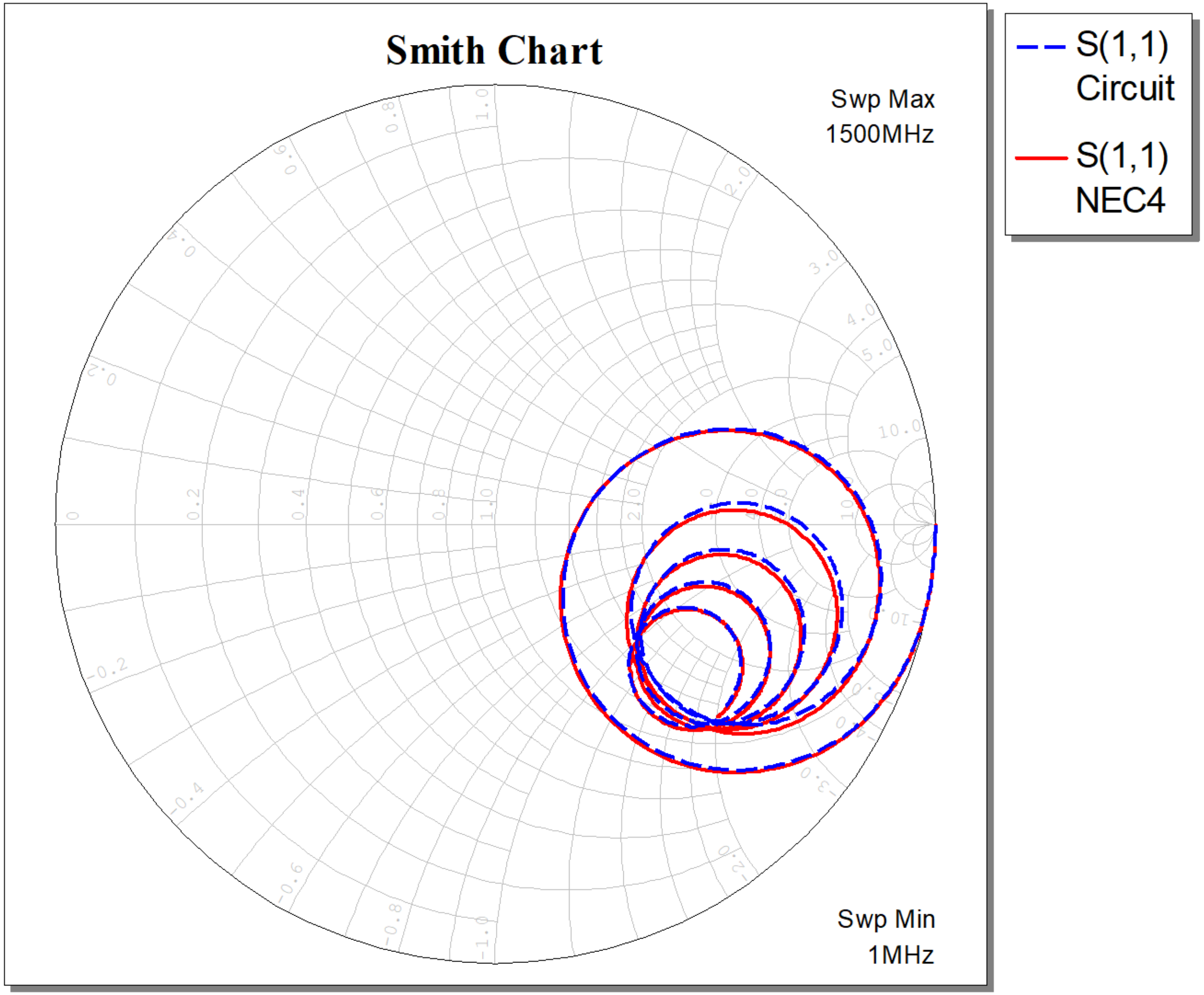
Author's 5-Element

Equivalent Circuit for Electrically-Large Dipole Broadband Model
1-meter Dipole (*L* / *d* = 50) from DC to 1.5 GHz

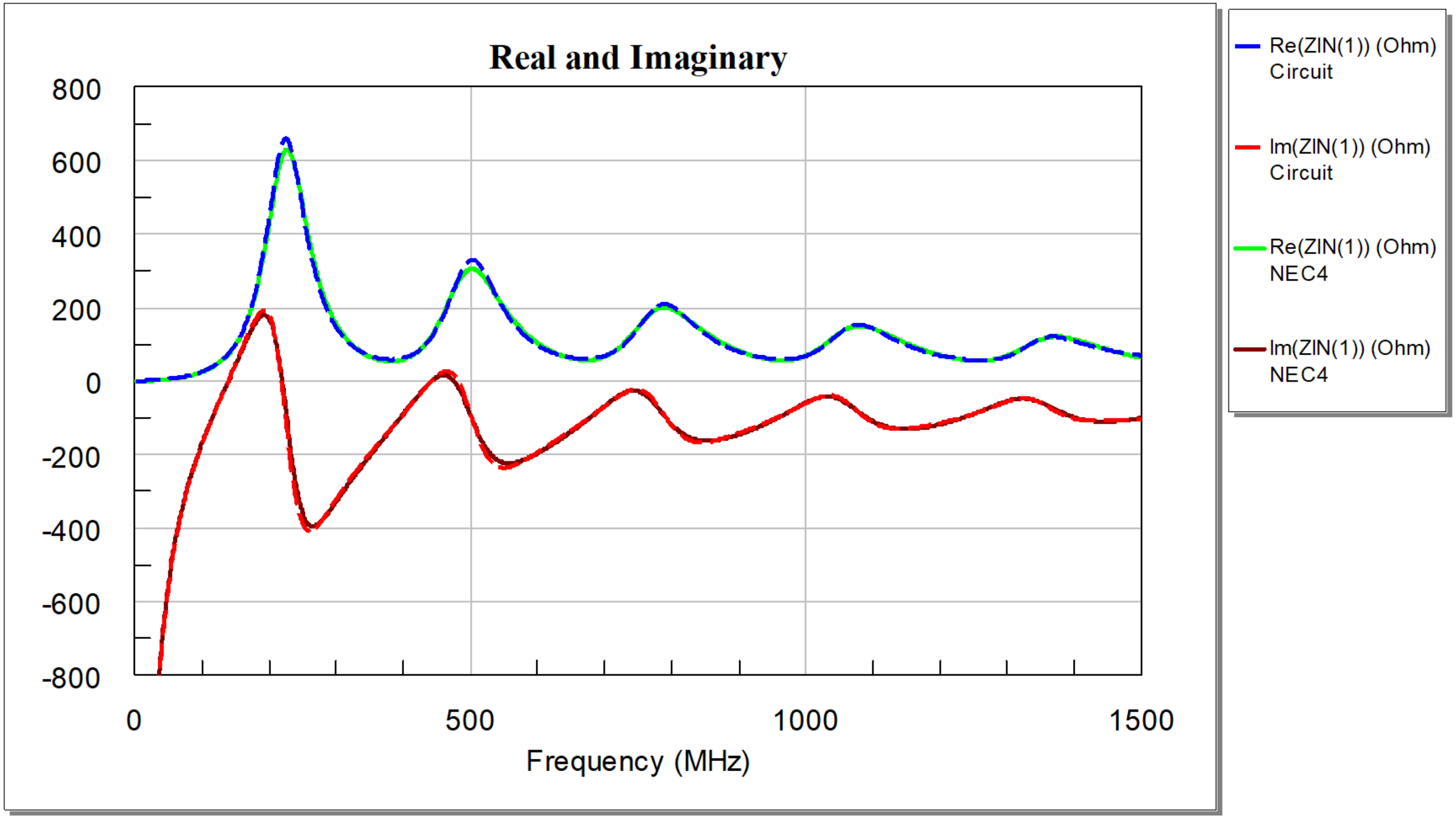


- Introduced by the author in 2007
- Partial fraction expansion of dipole admittance
- A modification of Foster's 2nd canonical form
- More accurate than other broadband equivalent circuits for dipoles, viz. Hamid-Hamid (1997), Rambabu-Ramesh-Kalghatgi (1999), and Streable-Pearson (1981)
- Six stages sufficient to cover *d-c* to 1.5 GHz

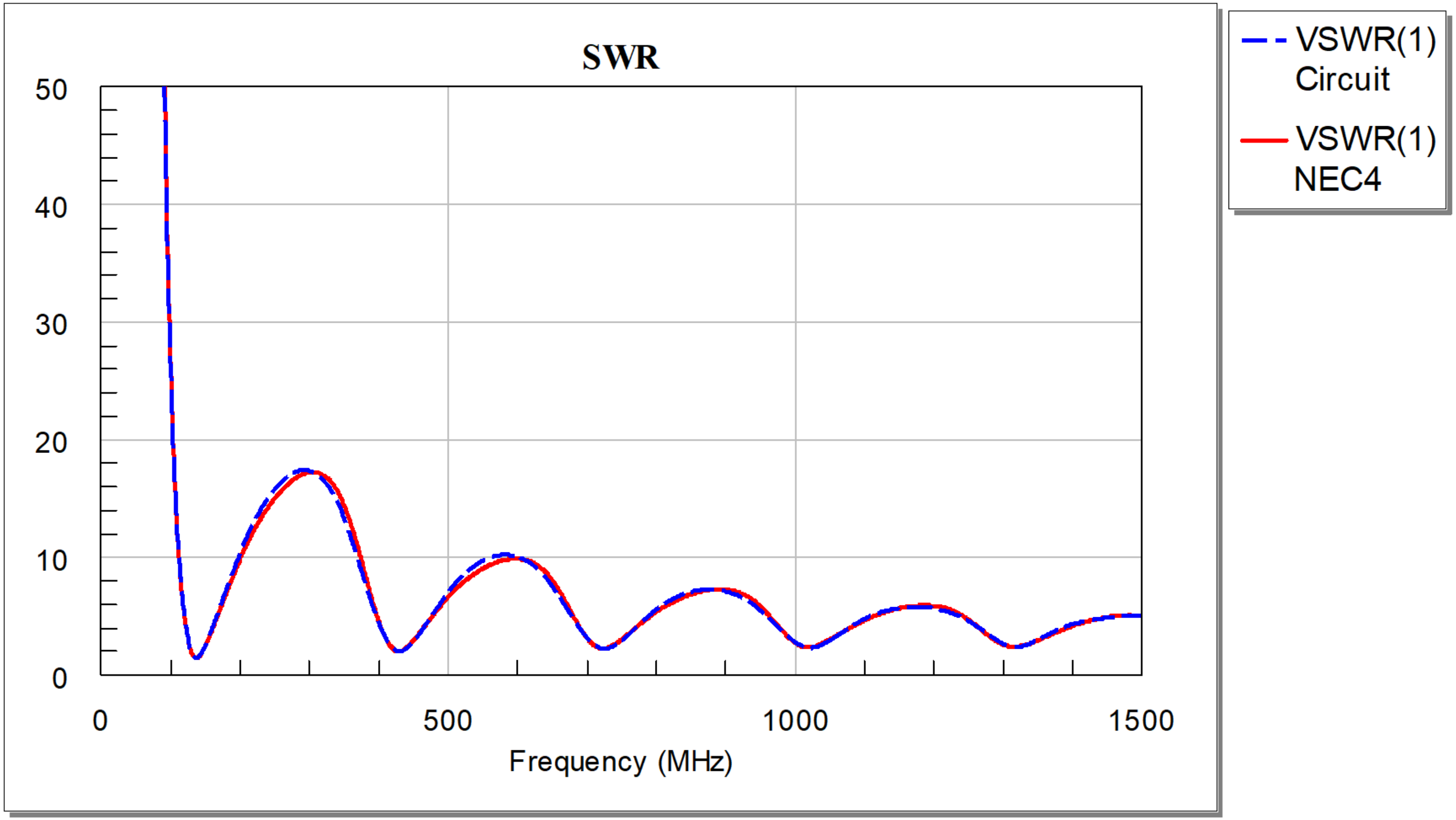
Impedance Comparison on Smith Chart



Impedance Comparison: NEC4 vs Equivalent Circuit



SWR of Equivalent Circuit



Antenna Q

Heinrich Hertz's Drawings of Electric Fields of a Dipole circa 1888

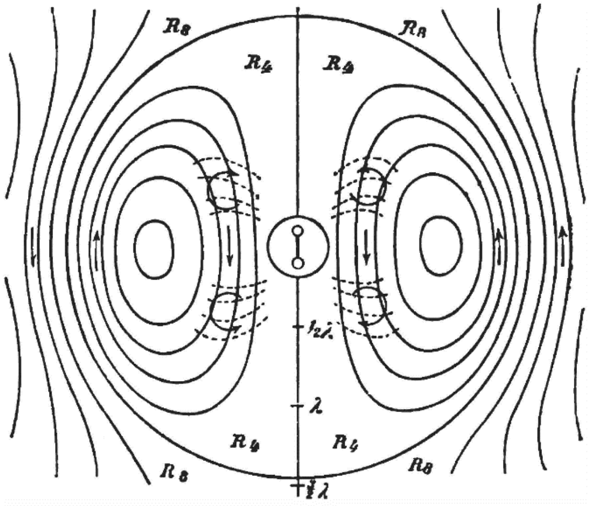


Fig. 27.

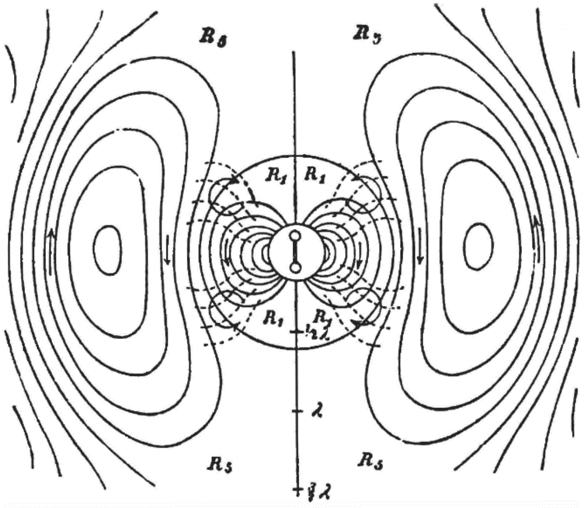


Fig. 28.

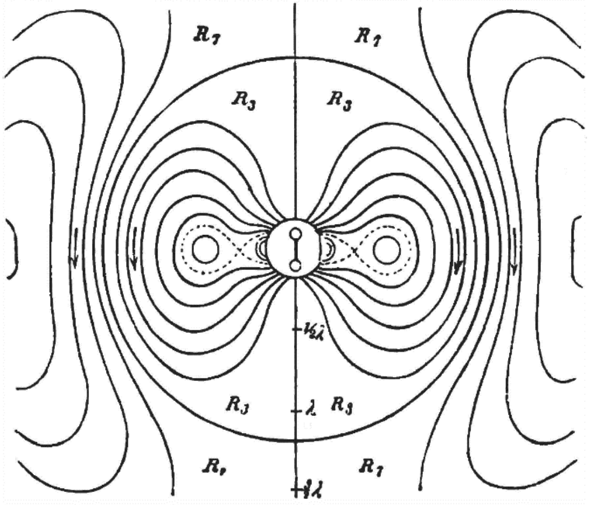


Fig. 30.

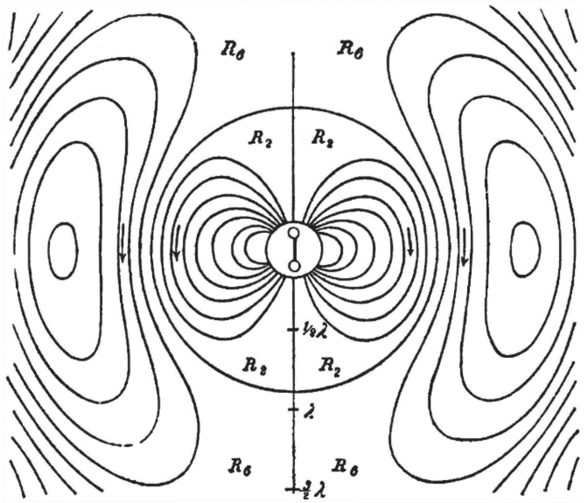
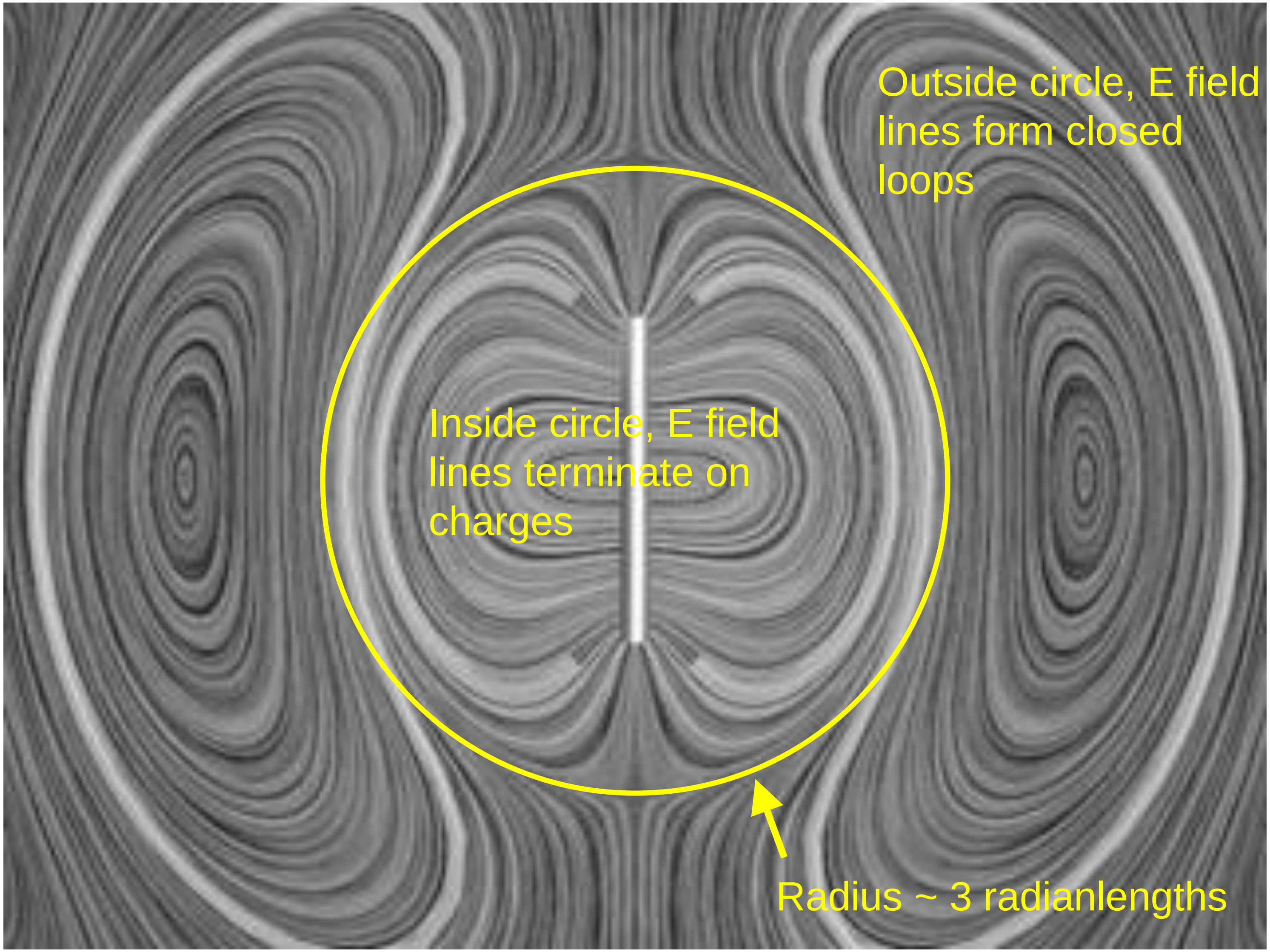


Fig. 29.

Generation of Dipole Fields



Outside circle, E field lines form closed loops

Inside circle, E field lines terminate on charges

Radius ~ 3 radianlengths

Q – Fundamental Definition

$$Q = 2\pi \left(\frac{\text{Energy stored}}{\text{Energy dissipated per cycle}} \right)$$

$$= 2\pi f \left(\frac{\text{Energy stored}}{\text{Power dissipated}} \right)$$

$$= \frac{\omega U_{stored}}{P_{dissipated}}$$

$$\le \frac{2\omega \max\{U_E, U_H\}}{P_{dissipated}}$$

Electric or magnetic field energy, whichever is greater

Ambiguity in the Numerator

- **Does stored energy mean total field energy minus energy that is being transported via radiation?**
  - Includes circulating real power, such as knotted waves

$$U_{stored} = U_{total} - U_{radiation}$$

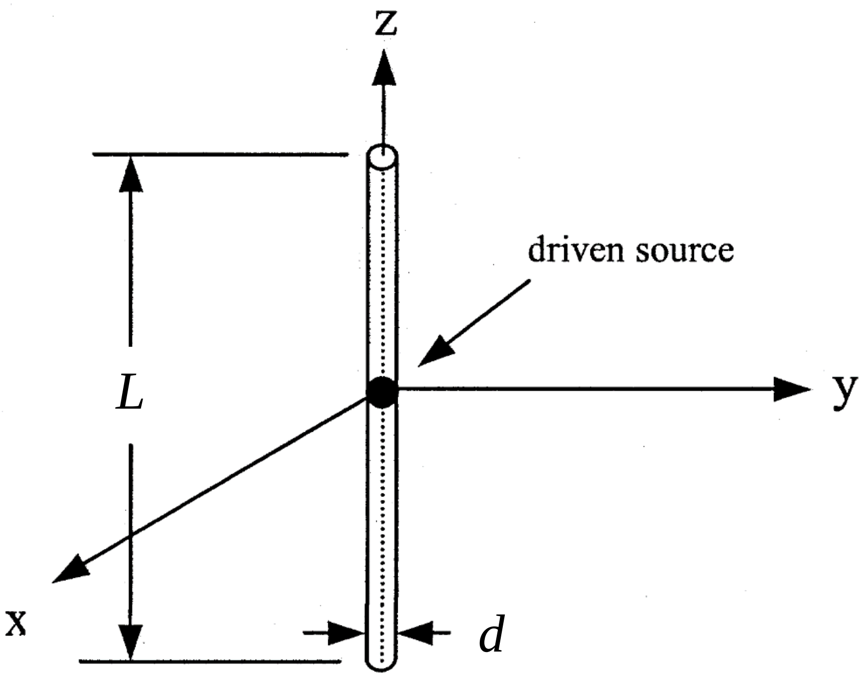
- **Or does stored energy mean reactive energy associated with the imaginary part of the Poynting vector, not total field energy?**
  - Excludes circulating real power
- **Either way, determining the difference between infinite quantities requires care**

$$U_{stored} = \infty - \infty$$

Ambiguity in the Denominator

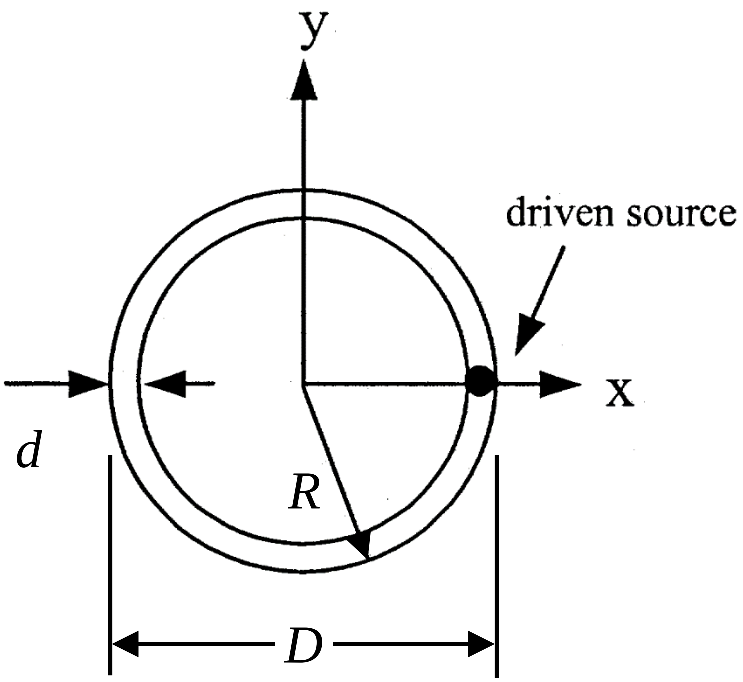
- **Does power lost per cycle mean all dissipated power?**
  - Includes ohmic and ground loss
- **Or does it mean real power that reaches the far field?**
  - Excludes ohmic and ground loss

Q of Small Dipole from Electromagnetic Field Analysis



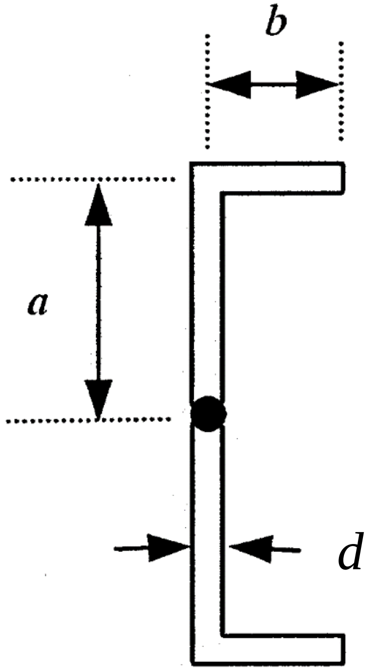
$$Q_{dipole} \approx \frac{6\left[\ln\left(\frac{L}{d}\right)-1\right]}{\pi^3\left(\frac{f\,L}{c}\right)^3}$$

Q of Small Loop from Electromagnetic Field Analysis



$$Q_{loop} \approx \frac{6\ln\left(\frac{D}{d}\right)}{\pi^4\left(\frac{fD}{c}\right)^3}$$

Q of Inverted-L from Electromagnetic Field Analysis



$$Q_{inverted-L} \approx \left(\frac{3}{4}\right) \frac{\left[\ln\left(\frac{2a}{d}\right)-1\right] + \frac{b}{a}\left[\ln\left(\frac{4b}{d}\right)-1\right]}{\pi^3\left(\frac{f}{c}\right)^3 a(a+b)^2}$$

Simple Formulas for Q in Terms of Feedpoint Impedance

- **Series RLC equivalent circuit**

$$Q(f) = \frac{|X(f)|}{R(f)}$$

- **Geyi (2000, 2003)**

$$Q(f) = \frac{f}{2R(f)}\left[\frac{dX(f)}{df} \pm \frac{X(f)}{f}\right]$$

- **Yaghjian and Best (2003, 2005)**

$$Q(f) = \frac{f}{2R(f)}\sqrt{\left(\frac{dR(f)}{df}\right)^2 + \left(\frac{dX(f)}{df} + \frac{|X(f)|}{f}\right)^2}$$

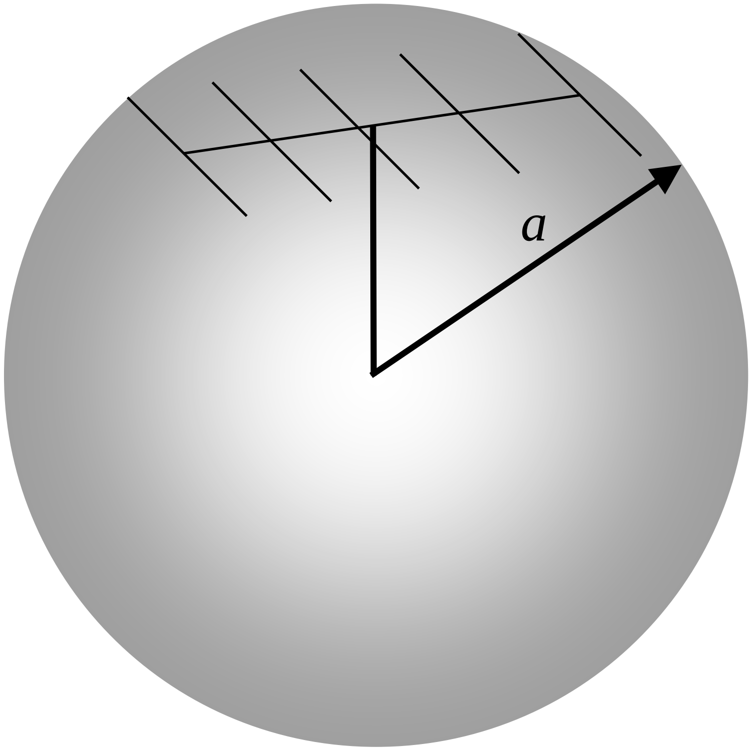
- **Hansen (2007)**

$$Q(f) = \frac{f}{2R(f)}\left|\frac{dX(f)}{df}\right|$$

If feedpoint loading or matching is allowed, Q cannot be computed from Z. Q must be computed directly from field formulas!

Fundamental Bounds on Antenna Q

L.J. Chu, *Physical Limitations of Omni-Directional Antennas*, tech rept. 64, MIT, May 1948. Also in MIT J. Appl. Phys., Dec 1948.



Smallest sphere that circumscribes antenna

- **Chu (1948)**

$$Q_{Chu} \geq \frac{1}{ka} + \frac{1}{(ka)^3}$$

- **Hansen and Collin (2007)**

$$Q_{New} \geq \frac{0.71327}{ka} + \frac{1.49589}{(ka)^3}$$

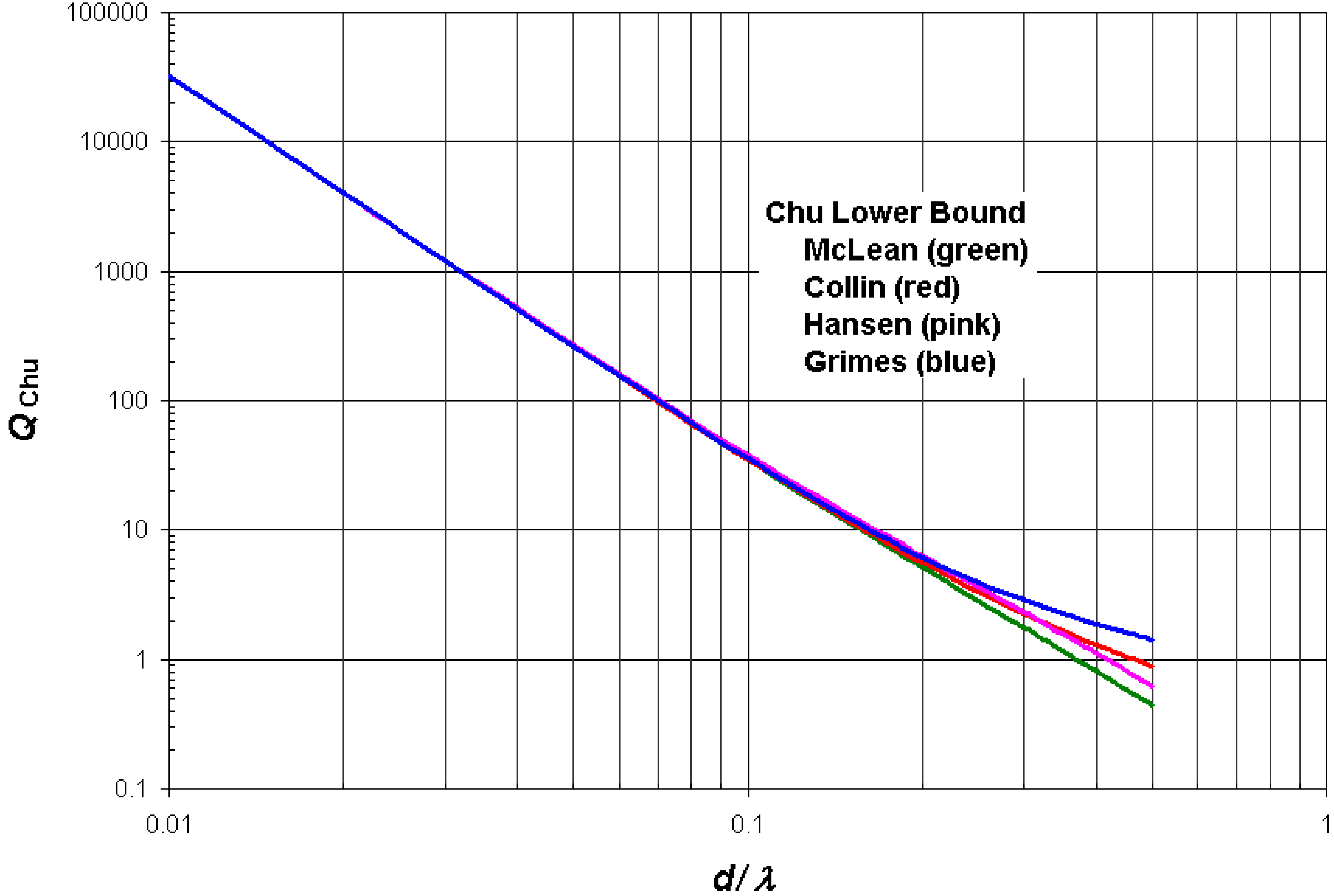
where

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$$

$$a = \text{radius of sphere}$$

Reducing an antenna’s size in half increases its Q by 8 and reduces its bandwidth by 8!

Chu Q



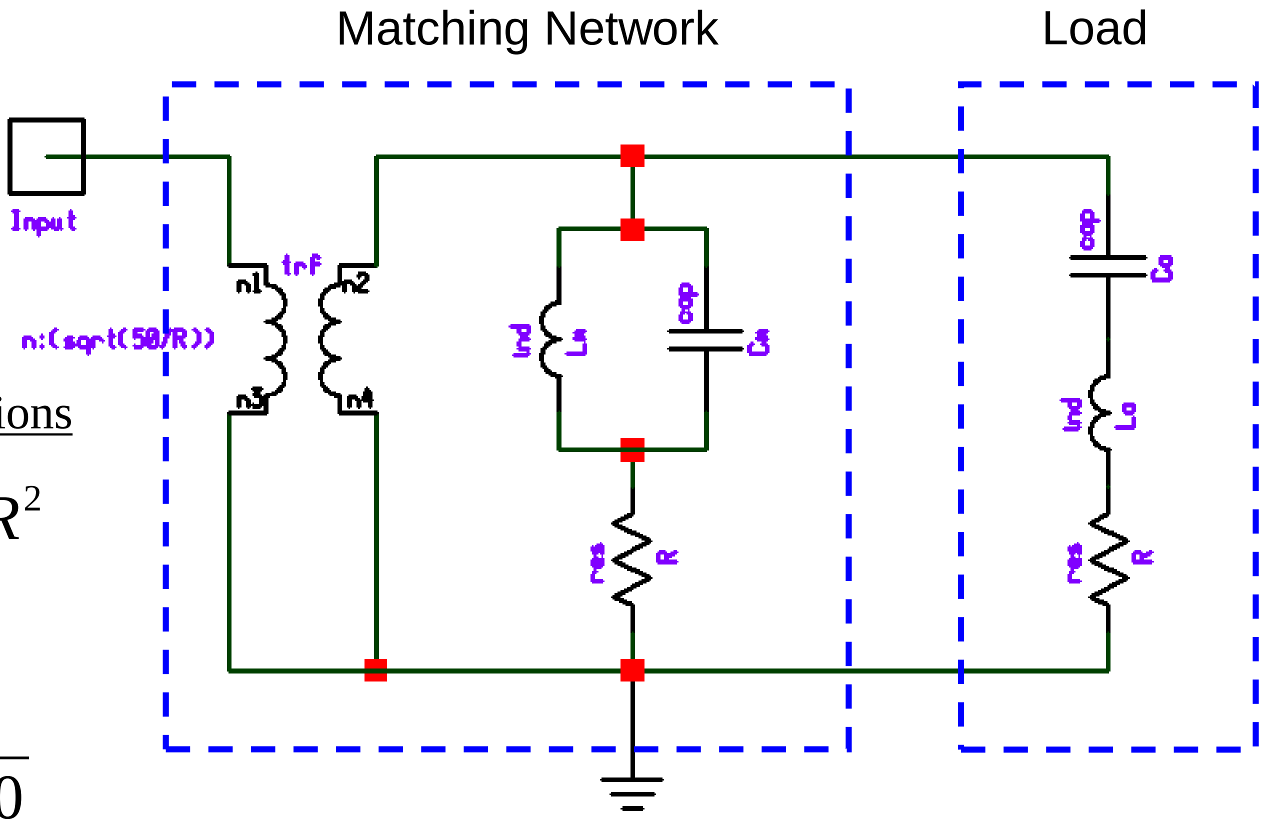
100000
10000
1000
100
10
1
0.1
0.01
0.1
1
Q Chu
d/ λ
Chu Lower Bound
McLean (green)
Collin (red)
Hansen (pink)
Grimes (blue)

Apparent Q=0 Antennas

Four Approaches to Making Q=0 Antennas

- **Use two antennas that have dual impedances, i.e. complementary antennas – small dipole and small loop**
- **Use a single antennas that has**
  - An integrated reflectionless match network
    - Restricted to antennas whose Darlington reactance 2-port is a simple series or shunt reactance connected to load resistor
  - An integrated broadband non-Foster match network
    - Requires clone of Darlington reactance 2-port and two negative impedance converters or inverters
  - A metamaterial radome shell that does external non-Foster matching
    - Shell must encompass reactive near field region

Constant Resistance Network for a Series RLC Load



Design Equations

$$L_M = C_A R^2$$

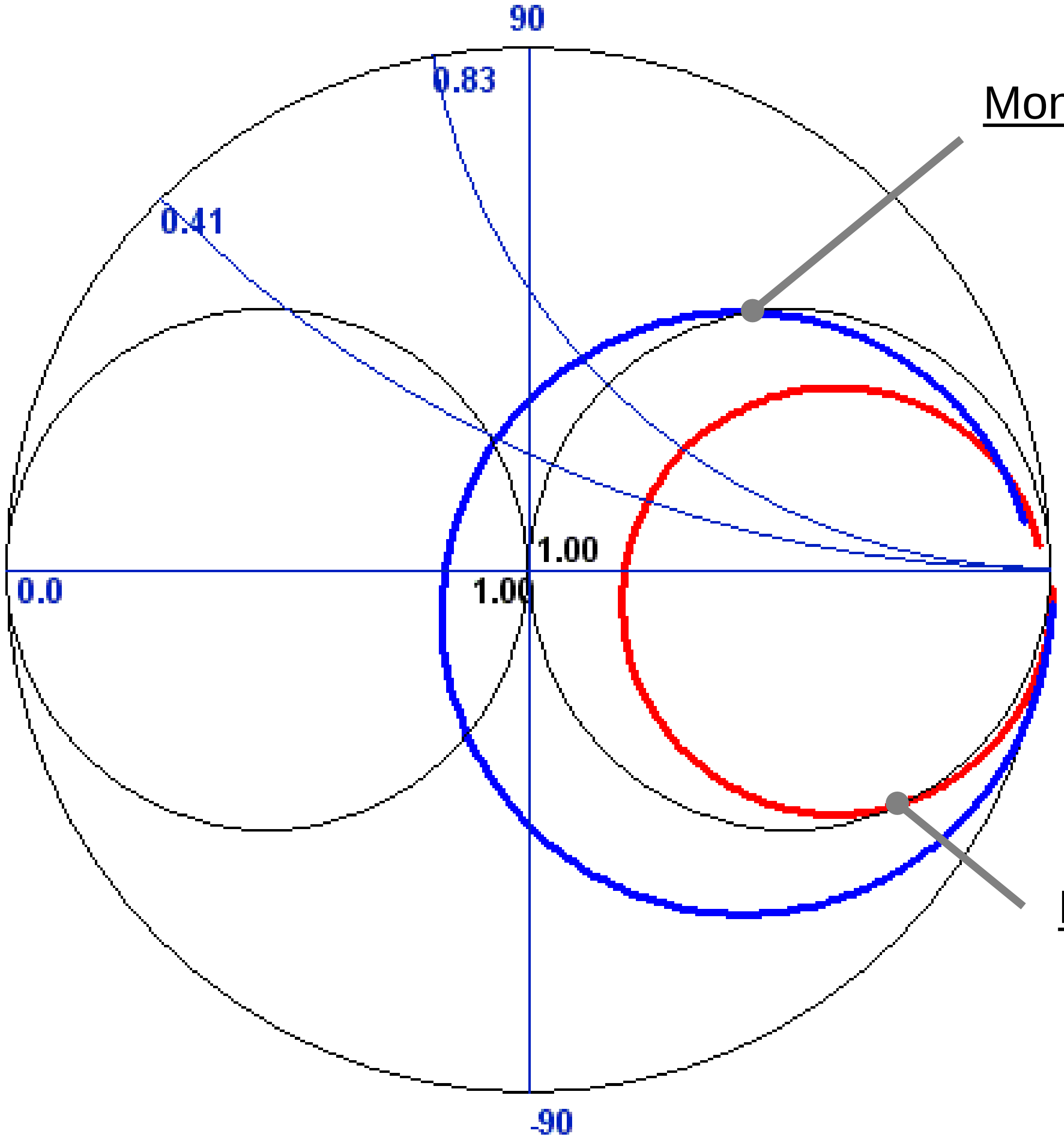
$$C_M = \frac{L_A}{R^2}$$

$$N = \sqrt{\frac{50}{R}}$$

Reflectionless Matching for Dipoles and Monopoles

- **Set length so that R_A = 50 ohms**
  - Dipoles: Use $K \approx 0.86$, or $L \approx 0.43\lambda$
  - Monopoles: Use $K \approx 1.07$, or $L \approx 0.27\lambda$
- **Add a series reactance to cancel feedpoint reactance**
  - Dipoles: Add a series inductor
  - Monopoles: Add a series capacitor
- **Add a shunt network to yield a 50-ohm constant-resistance network**

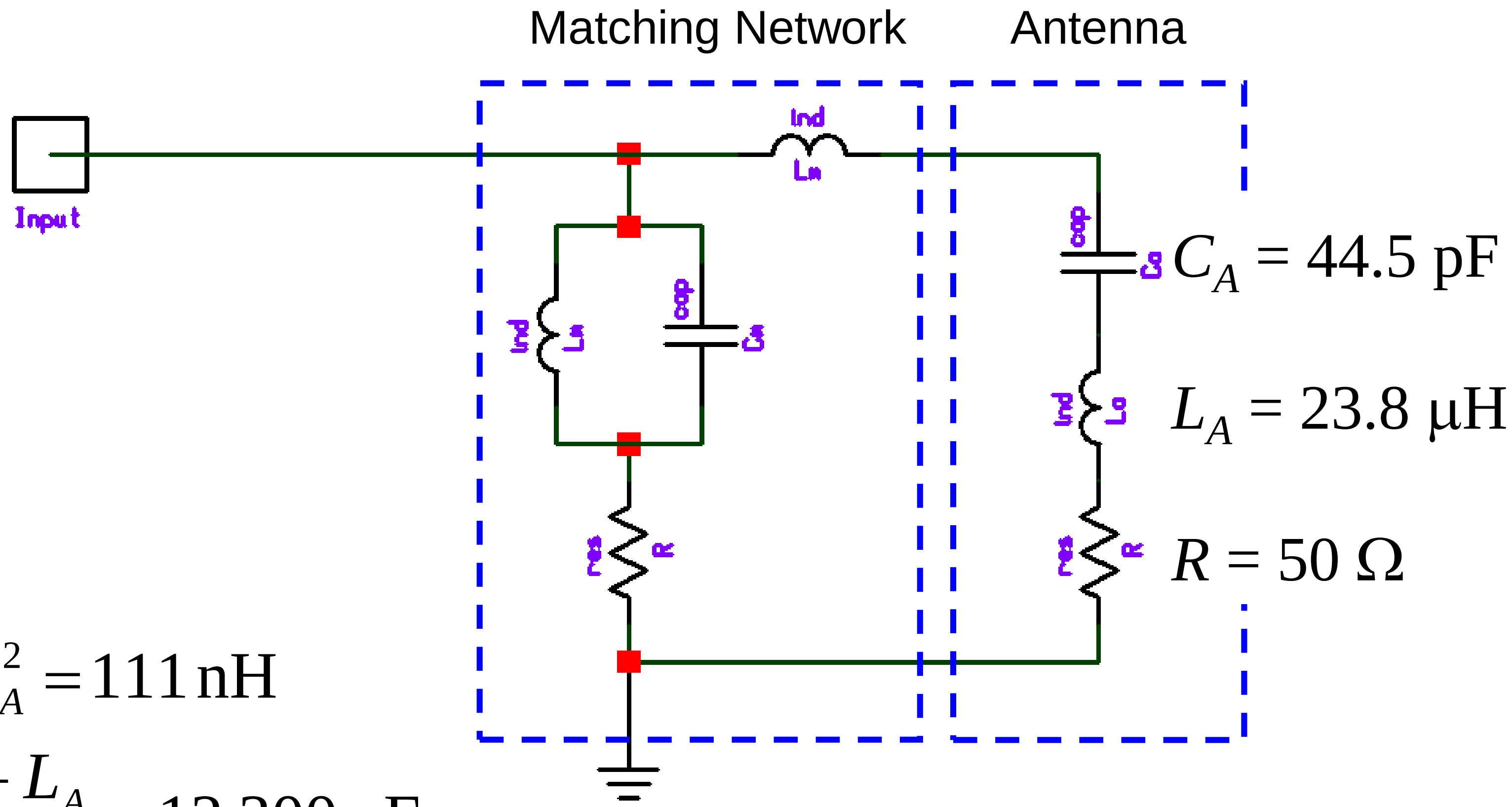
Eliminating the Transformer



Monopole Length
L = 0.27λ

Dipole Length
L = 0.43λ

Example: Reflectionless Match to 0.43λ Dipole



$C_A = 44.5 \text{ pF}$

$L_A = 23.8\ \mu\text{H}$

$R = 50\ \Omega$

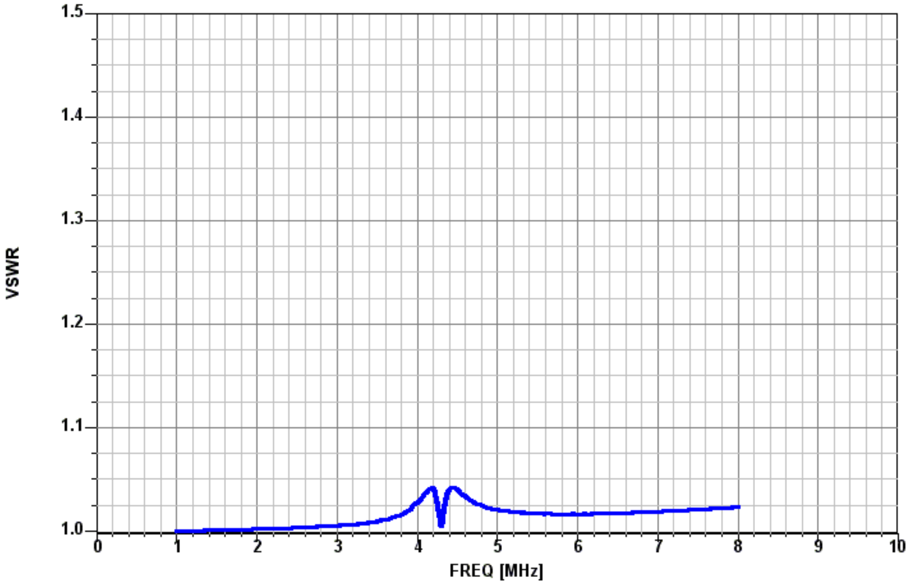
$$L_S = 7\ \mu\text{H}$$

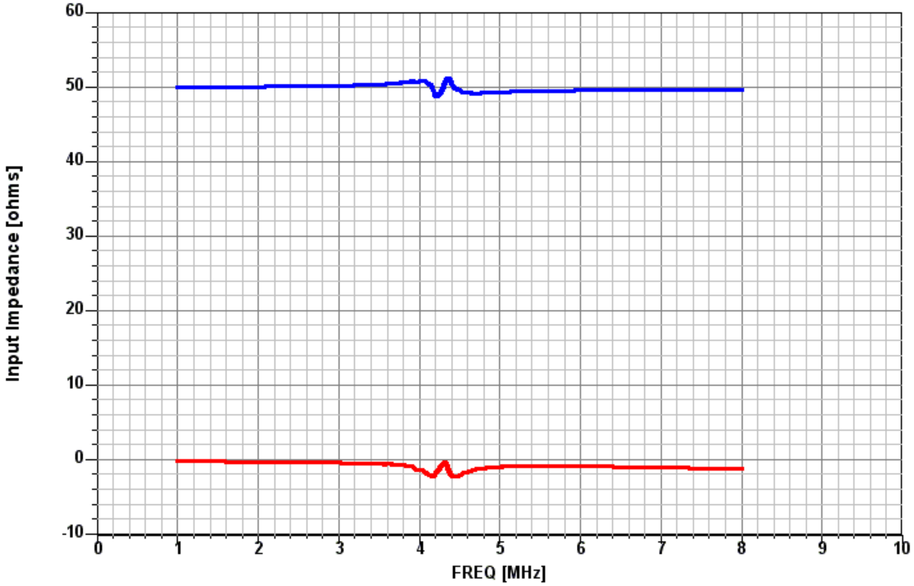
$$L_M = C_A R_A^2 = 111\,\text{nH}$$

$$C_M = \frac{L_S + L_A}{R_A^2} = 12{,}300\text{ pF}$$

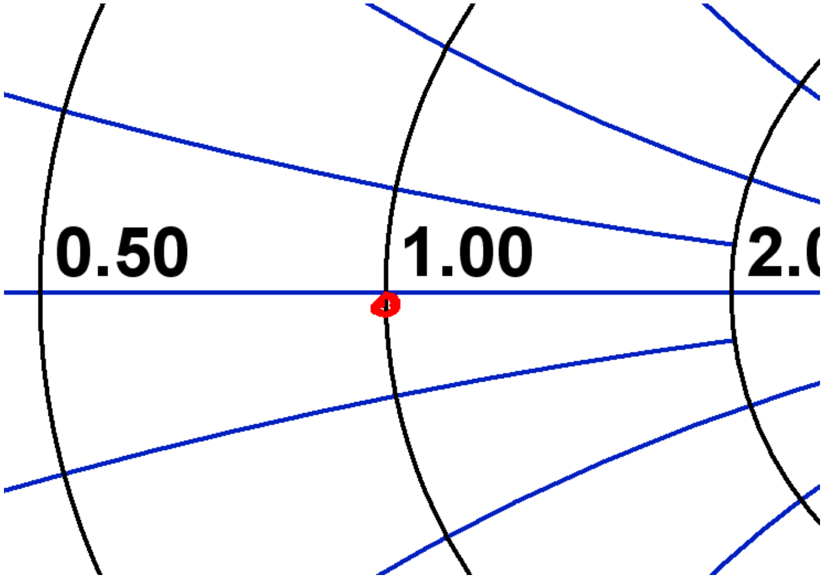
$$R = R_A = 50\ \Omega$$

Network Performance on Dipole Impedance Data

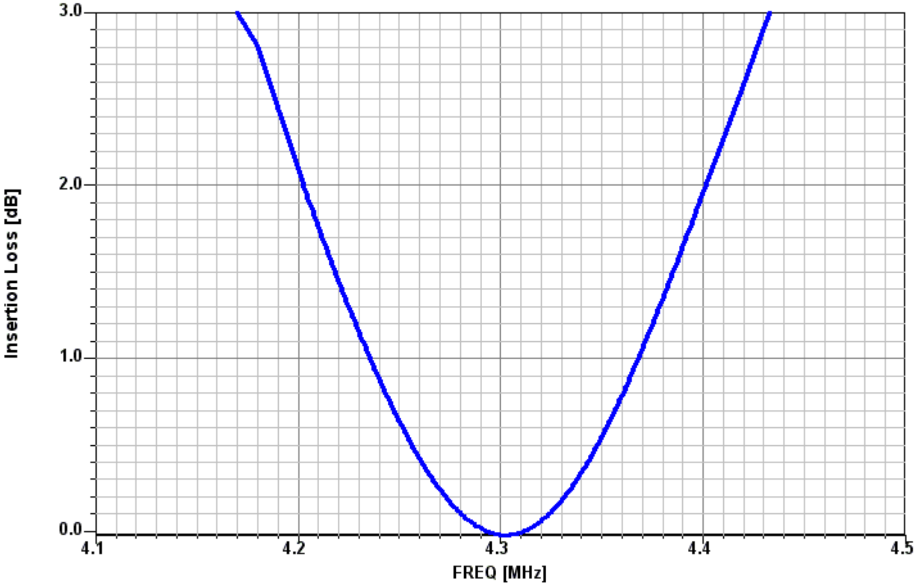




- **Frequency sweep 1 to 8 MHz**
- **Maximum VSWR = 1.04**
- **Input resistance: 48.8 to 51.2 ohms**
- **Input reactance: -2.1 to 0 ohms**



Power Delivered to the 0.43λ Dipole



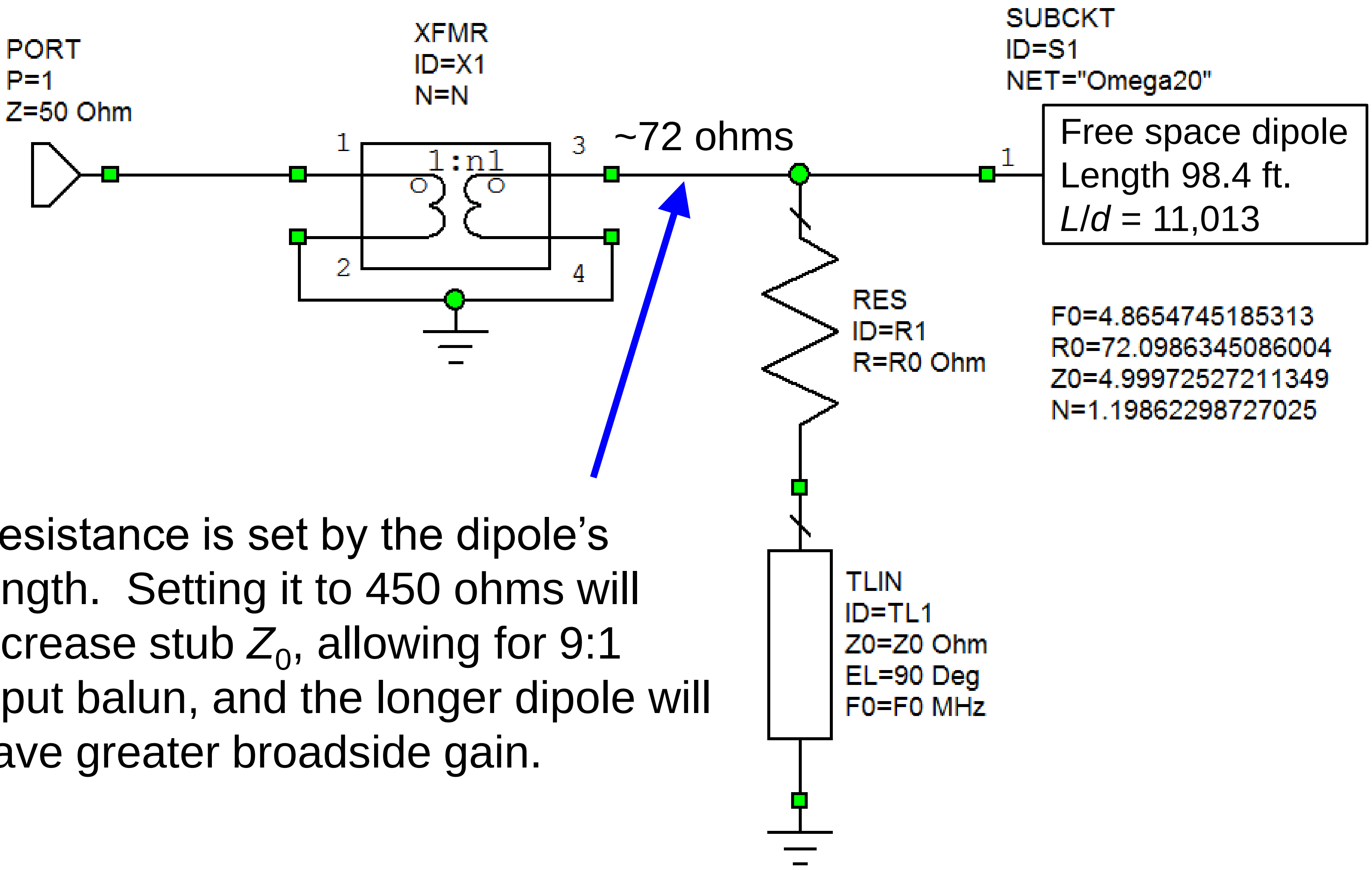
- **Pattern gain = - 0.11 dBd**
- **Minimum insertion loss = 0 dB**
- **100% power delivery at 4.3 MHz**
- **3-dB Bandwidth = 259 kHz (6.0%)**
- **0.51-dB Bandwidth = 91 kHz (2.1%)**

Bandwidths to Compare		
Lossless Networks		**Reflectionless Networks**
$BW_{SWR\ 5.83:1}$	⟷	$BW_{IL\ 3\text{-dB}}$
$BW_{SWR\ 2:1}$	⟷	$BW_{IL\ 0.51\text{-dB}}$

Reflectionless Matching Using a Complementary Stub

- Assume the resistance varies slowly across a band
- A reflectionless match network is obtained by putting a complementary admittance in parallel with the dipole or monopole
- We obtain a constant-resistance (CR) match network made from a stub and a resistor

Parallel Connection of Resistor-Stub and Dipole

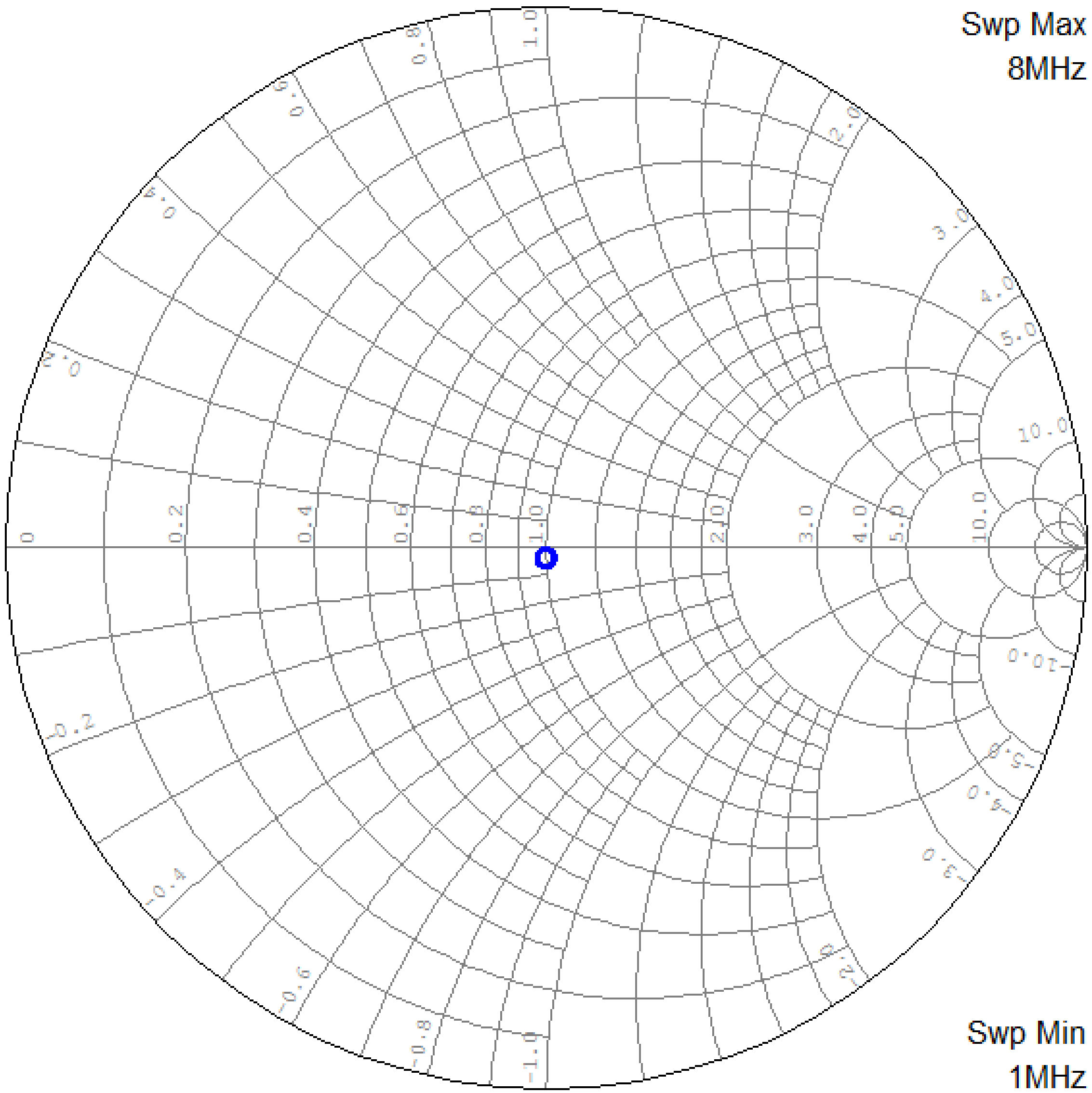


~72 ohms

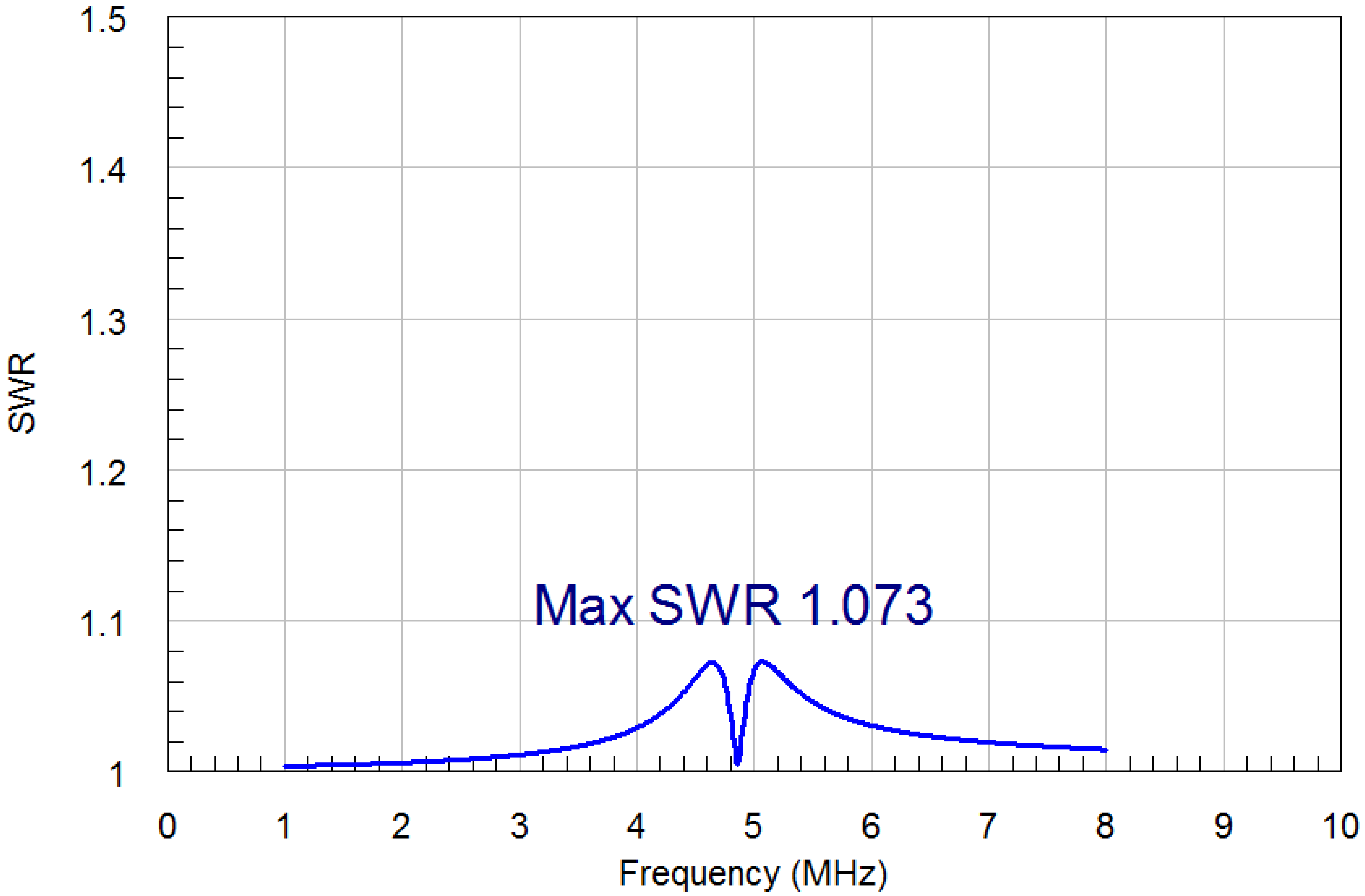
Free space dipole
Length 98.4 ft.
L/*d* = 11,013

Resistance is set by the dipole’s length. Setting it to 450 ohms will increase stub Z_0, allowing for 9:1 input balun, and the longer dipole will have greater broadside gain.

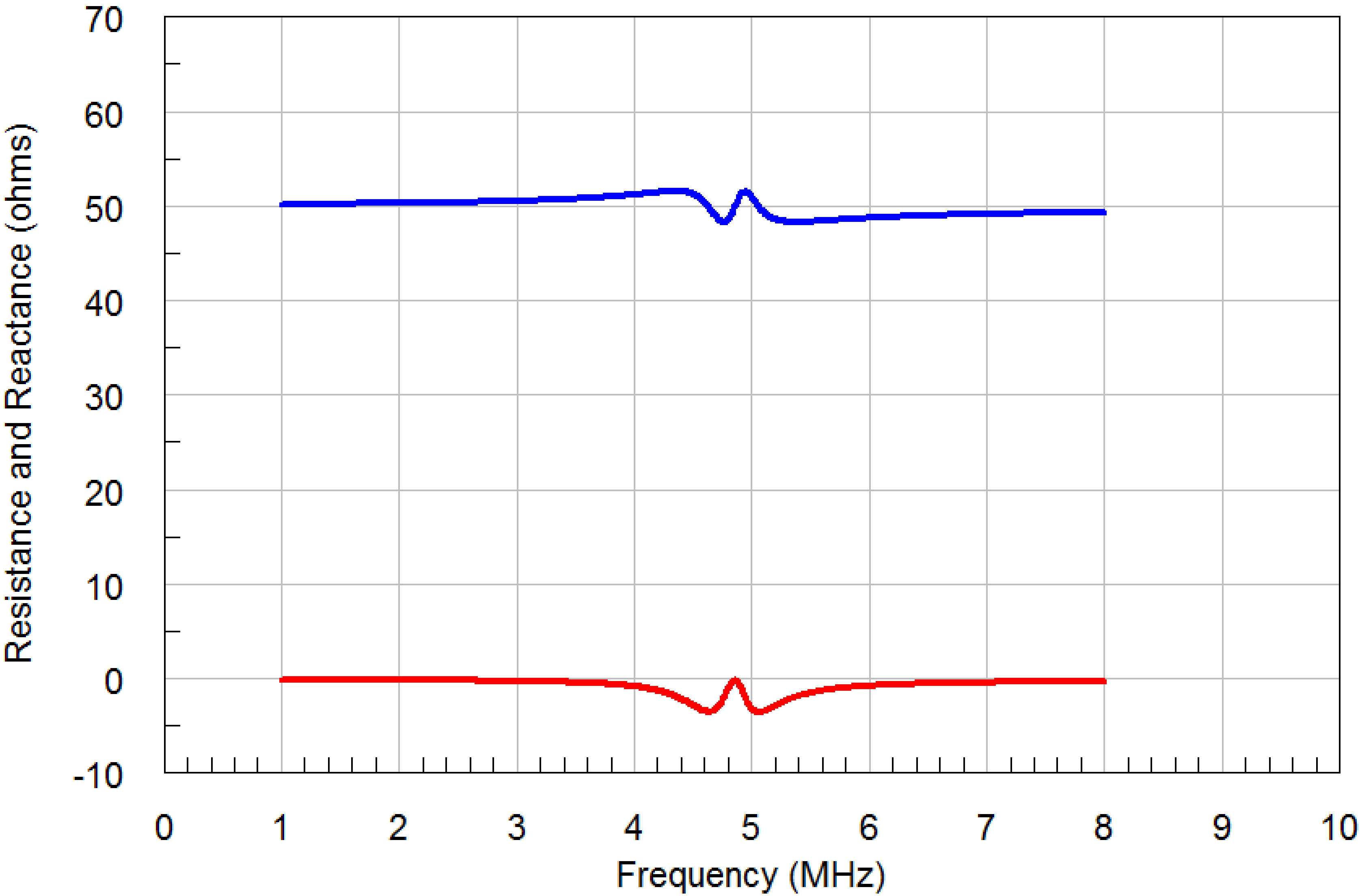
Multi-Octave Match from 1 MHz to 8 MHz



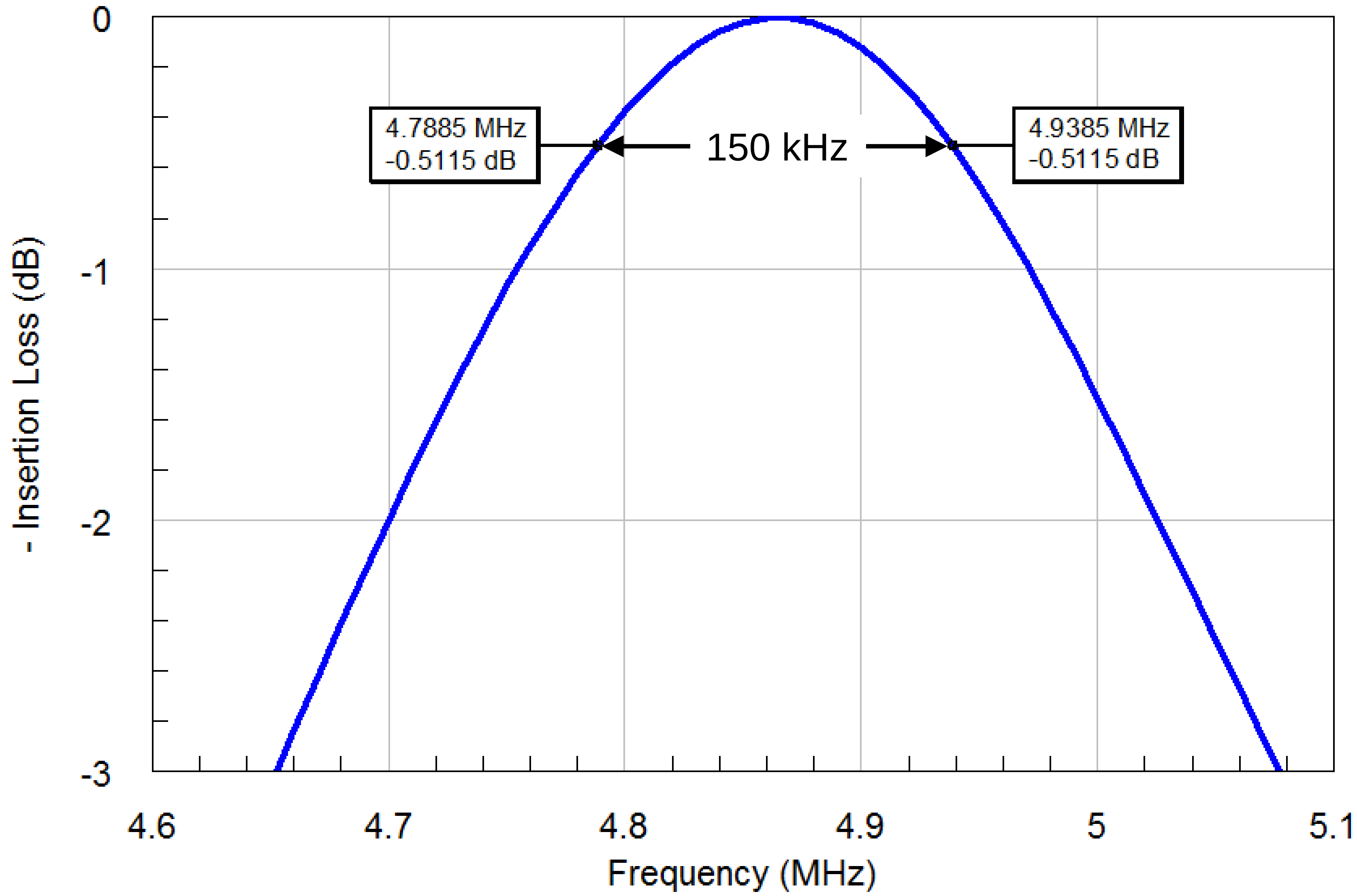
SWR



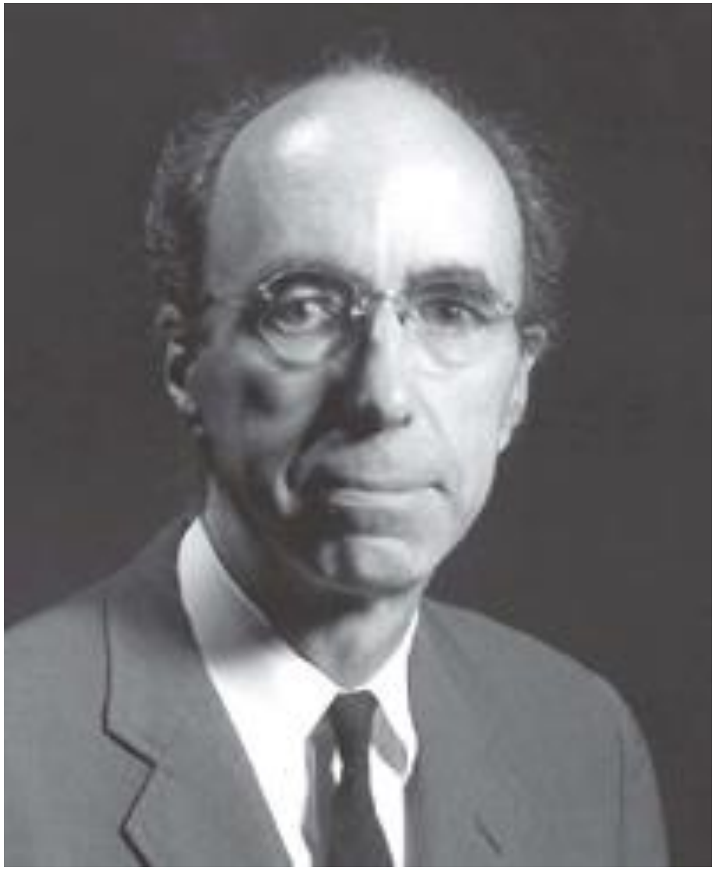
Resistance and Reactance



– Insertion Loss



Sidney Darlington, 1906-1997



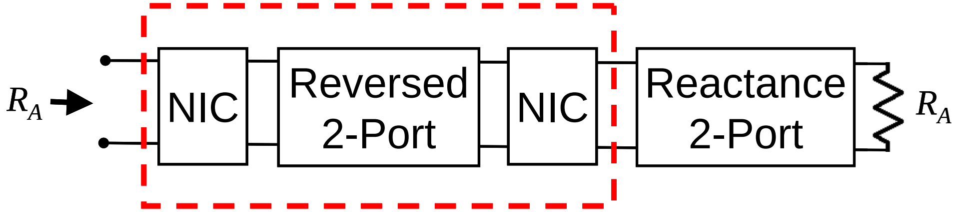
Darlington Forms (1939)

- Every positive-real, rational immittance function can be realized as a lossless lumped 2-port terminated by a resistor

- For antennas, a meromorphic antenna impedance function can be represented by a convergent sequence of equivalent circuits in Darlington form
- The Darlington form is therefore the starting point for a theory of non-Foster matching of antennas

Cascade Inversions Using NICs and NIVs

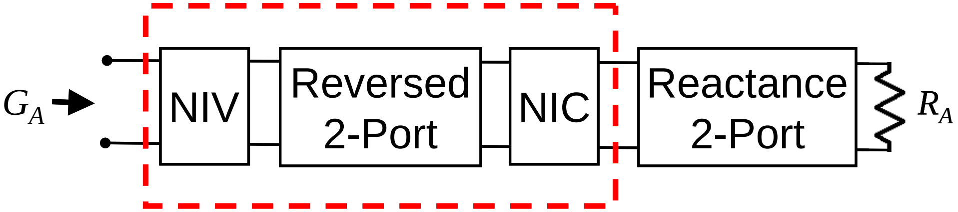
1st Canonical Form



Non-Foster Reactance 2-Port

Darlington Antenna Model

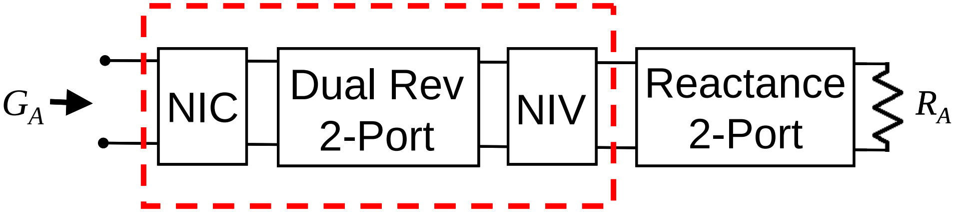
2nd Canonical Form



Non-Foster Reactance 2-Port

Darlington Antenna Model

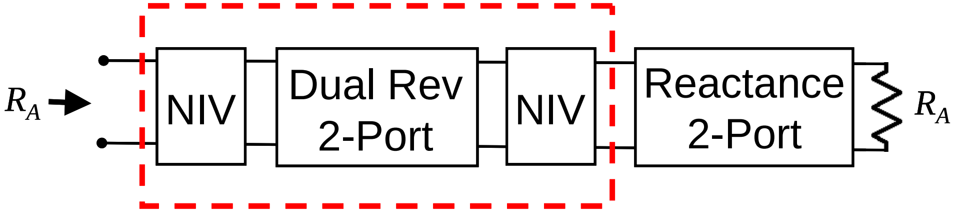
3rd Canonical Form



Non-Foster Reactance 2-Port

Darlington Antenna Model

4th Canonical Form



Non-Foster Reactance 2-Port

Darlington Antenna Model

Formal inverse match networks achieve perfect impedance matching and infinite match bandwidth.

Simple but Incorrect Formulas for Q in Terms of Feedpoint Impedance

- **Series RLC equivalent circuit**

$$Q(f) = \frac{|X(f)|}{R(f)}$$

- **Geyi (2000, 2003)**

$$Q(f) = \frac{f}{2R(f)}\left[\frac{dX(f)}{df} \pm \frac{X(f)}{f}\right]$$

- **Yaghjian and Best (2003, 2005)**

$$Q(f) = \frac{f}{2R(f)}\sqrt{\left(\frac{dR(f)}{df}\right)^2 + \left(\frac{dX(f)}{df} + \frac{|X(f)|}{f}\right)^2}$$

- **Hansen (2007)**

$$Q(f) = \frac{f}{2R(f)}\left|\frac{dX(f)}{df}\right|$$

If feedpoint loading or matching is allowed, Q cannot be computed from Z. Q must be computed directly from field formulas!

Antenna Q Formulas Are Wrong !

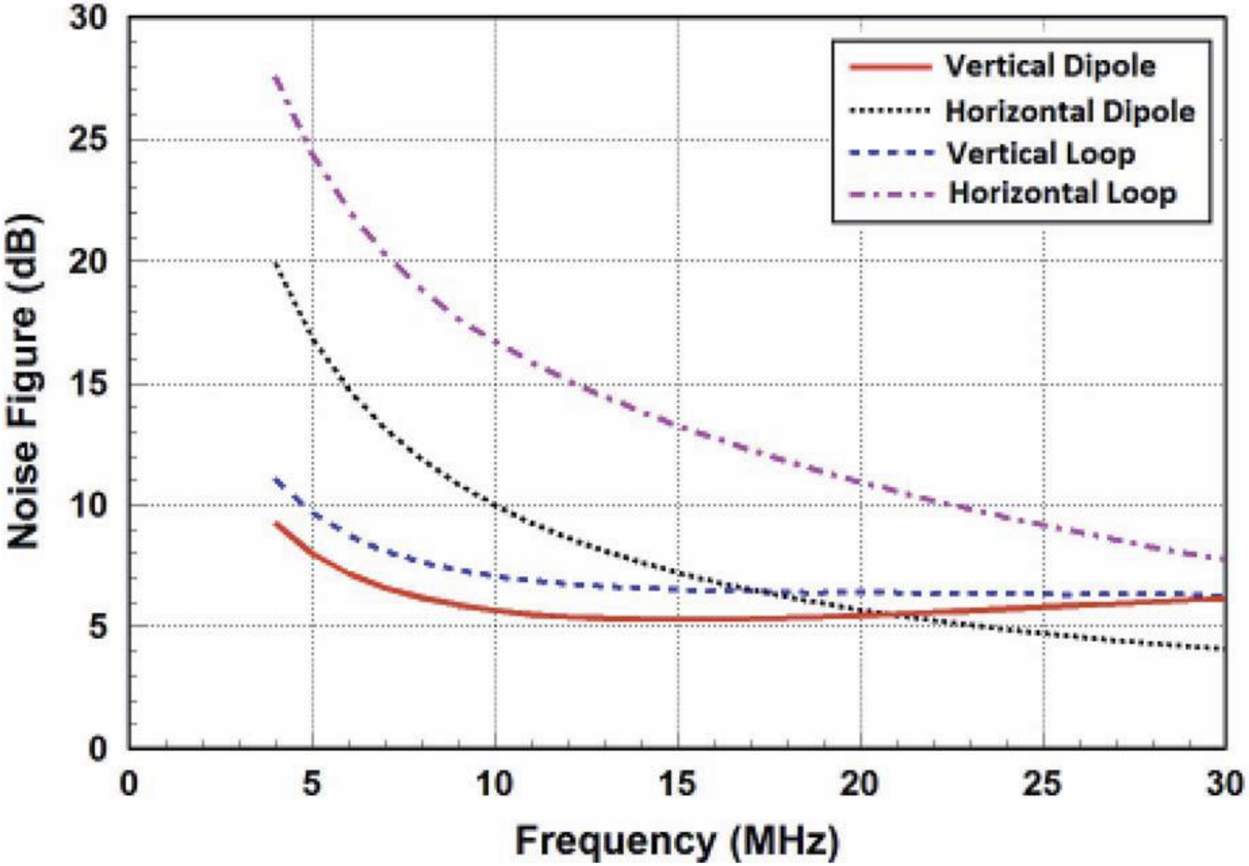
- **Put a reflectionless match network at an antenna's feedpoint; network being**
  - Part of the antenna
  - Nonradiating
- **The feedpoint impedance is a constant real number independent of frequency**

$$Z_A = R_A + j0 \qquad \frac{\partial R_A}{\partial \omega} = 0 \qquad X_A = 0 \qquad \frac{\partial X_A}{\partial \omega} = 0$$

- ***Q* formulas based on feedpoint *Z* yield zero**
  - *Q* by formula is zero
  - Antenna *Q* is not zero

No general formula for computing *Q* from *Z* exists.
Antenna *Q* must be computed from field formulas.

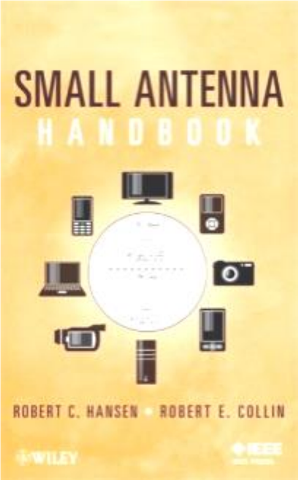
Noise Figures of Four Antennas with Ground Loss



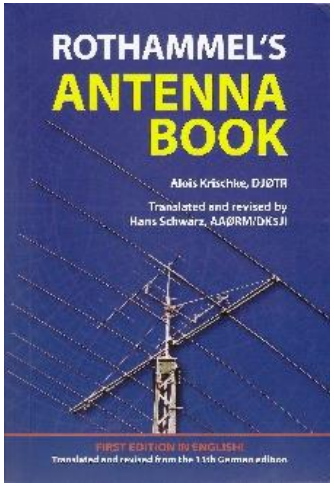
- Noise figure comparison of a 2-meter long dipoles and 1-meter diameter loops located 2 meters above average soil. The external noise is assumed to be galactic.
- Low frequency ranked order: V dipole, V loop, H dipole, H loop
- High frequency ranked order: H dipole, V dipole, V loop, H loop

S.R. Best, “Optimizing the Receiving Properties of Electrically Small HF Antennas,” *Radio Science Bulletin*, Dec. 2016.

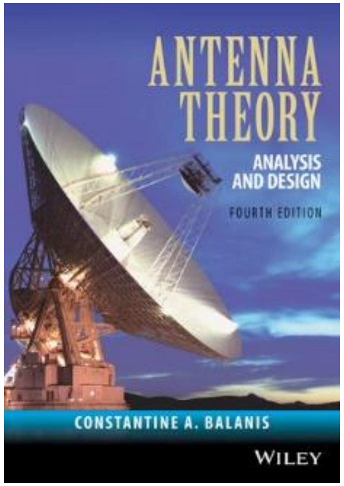
Some Recommended Antenna Books



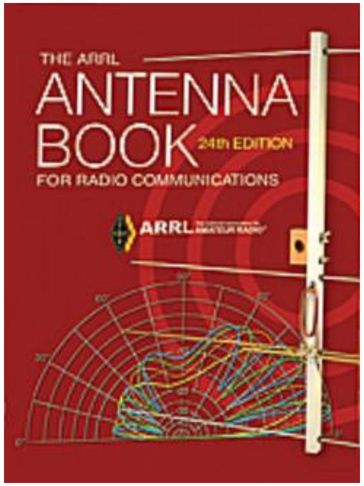
R.C. Hansen and R.E. Collin, *Small Antenna Handbook*, Wiley, 2011



A. Krischke, DJ0TR, ed., *Rothammel's Antenna Book*, 13e, English transl., DARC, 2019



C.A. Balanis, *Antenna Theory: Analysis and Design*, 4e, Wiley, 2016



***H.W. Silver, N0AX, ed., ARRL Antenna Book*, 24e, ARRL, 2019**

The End