# *Antenna Impedance Models – Old and New*

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# Outline

- ☐ ***Electromagnetics and antenna engineering basics***
- ☐ ***Dipole impedance by antenna theory***
  - ➢ Induced EMF method
  - ➢ King-Harrison-Middleton iterative methods (1943-46)
  - ➢ Hill's radiation pattern integration method (1967)
  - ➢ MoM solution of Hallen's or Pocklington's integral equations
- ☐ ***Antenna impedance models***
  - ➢ What are they; what are they good for; why are they needed?
- ☐ ***Kinds of impedance models***
  - ➢ General mathematical approximations
  - ➢ Equivalent circuits

# Outline continued

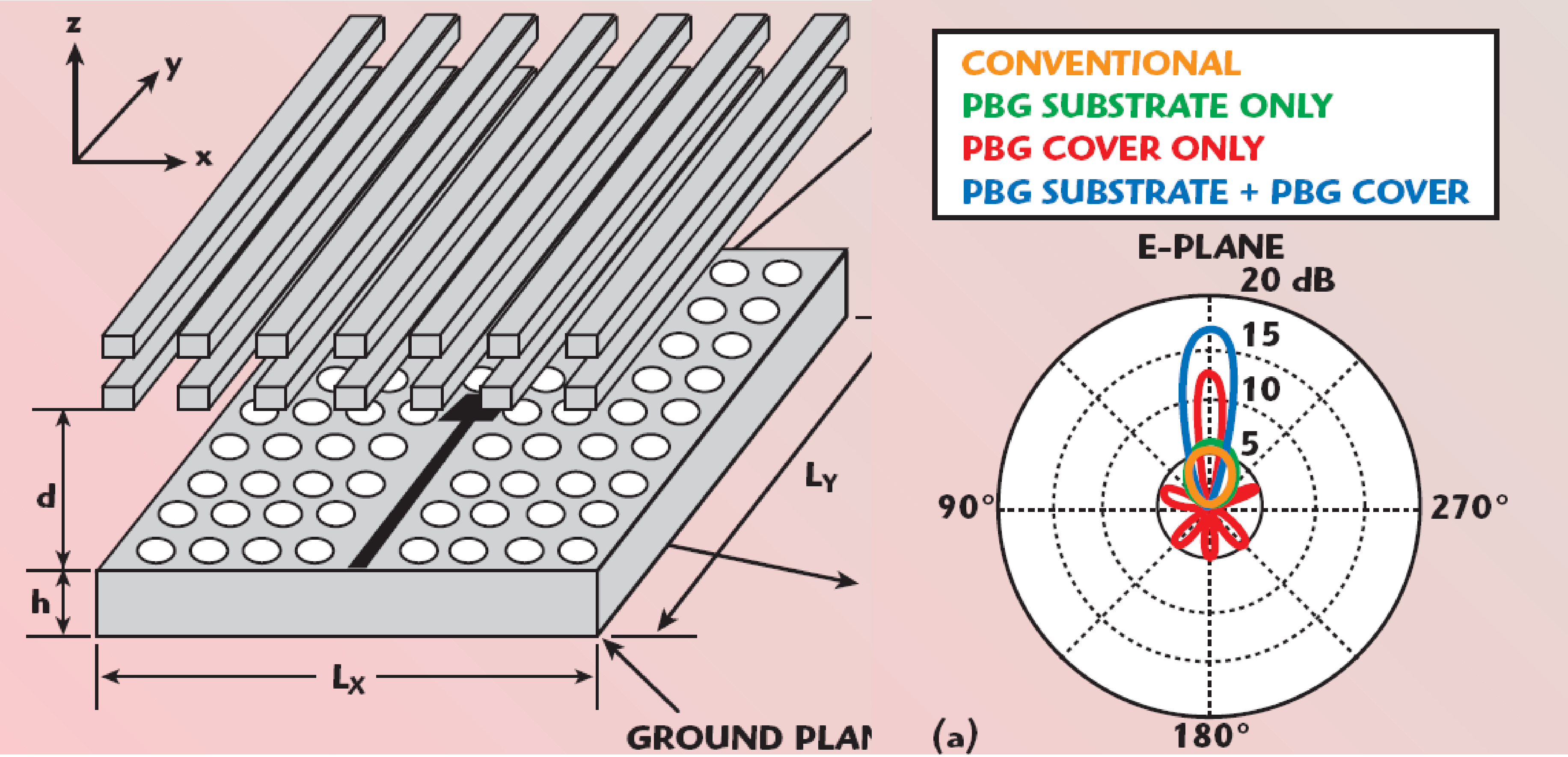
- ☐ ***Previous narrowband impedance models for dipoles at resonance & antiresonance***
  - ➢ Series and parallel RLC equivalent circuit models
  - ➢ Witt’s series stub model (1995)
- ☐ ***New (better) narrowband models***
  - ➢ Immittance functions
  - ➢ Approximating dipole impedance with immittance functions
  - ➢ Converting immittance functions to equivalent circuits
  - ➢ Using EDA software to compare models
- ☐ ***Broadband models that span multiple resonances***
  - ➢ Hamid-Hamid model (1997)
  - ➢ Long-Werner-Werner model (2000)
  - ➢ Streable-Pearson model (1981)

# *Antenna Engineering Basics*

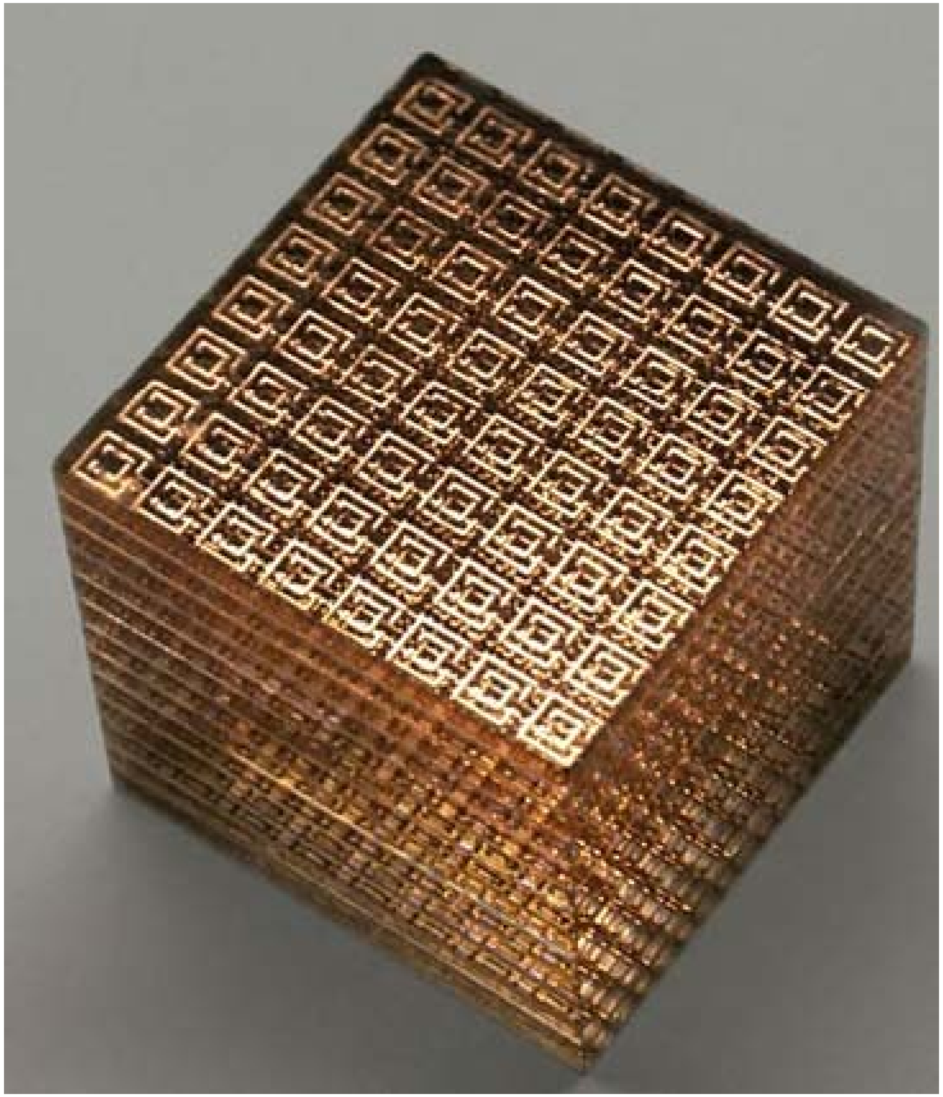
# Hot Topics in Antenna Engineering Today

- ☐ ***Photonic/Electronic band-gap surfaces (PBG/EBG)***
- ☐ ***Engineered “metamaterials”***
- ☐ ***Twisted light***

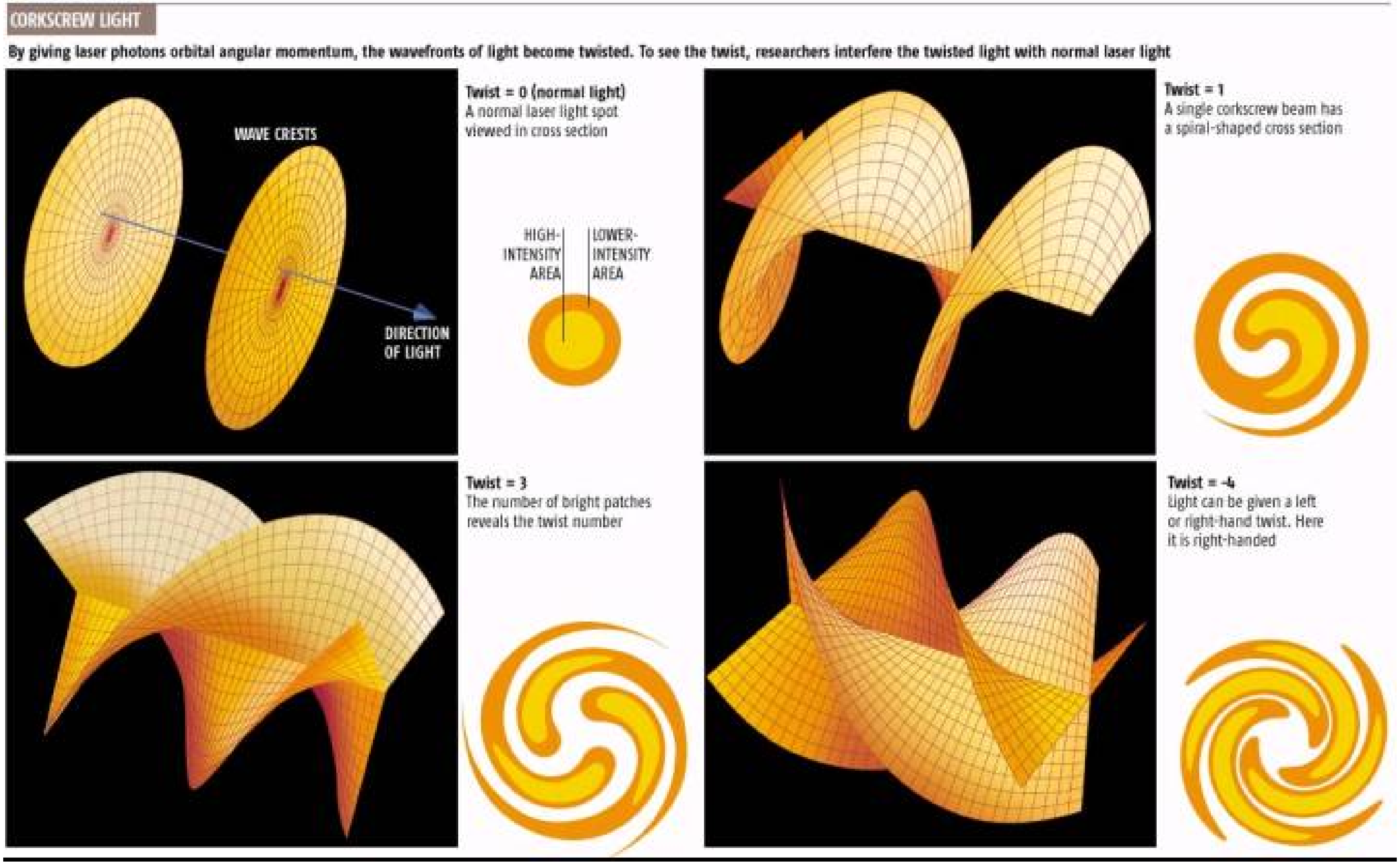
# Antenna Using PBG/EBG



# Metamaterials - The Boeing Cube



# Twisted Light Modes



CORKSCREW LIGHT

By giving laser photons orbital angular momentum, the wavefronts of light become twisted. To see the twist, researchers interfere the twisted light with normal laser light

**Twist = 0 (normal light)**
A normal laser light spot viewed in cross section

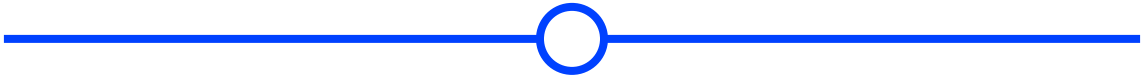
**Twist = 1**
A single corkscrew beam has a spiral-shaped cross section

**Twist = 3**
The number of bright patches reveals the twist number

**Twist = -4**
Light can be given a left or right-hand twist. Here it is right-handed

# *Fact or Myth?*

- ☐ ***A dipole is center fed***

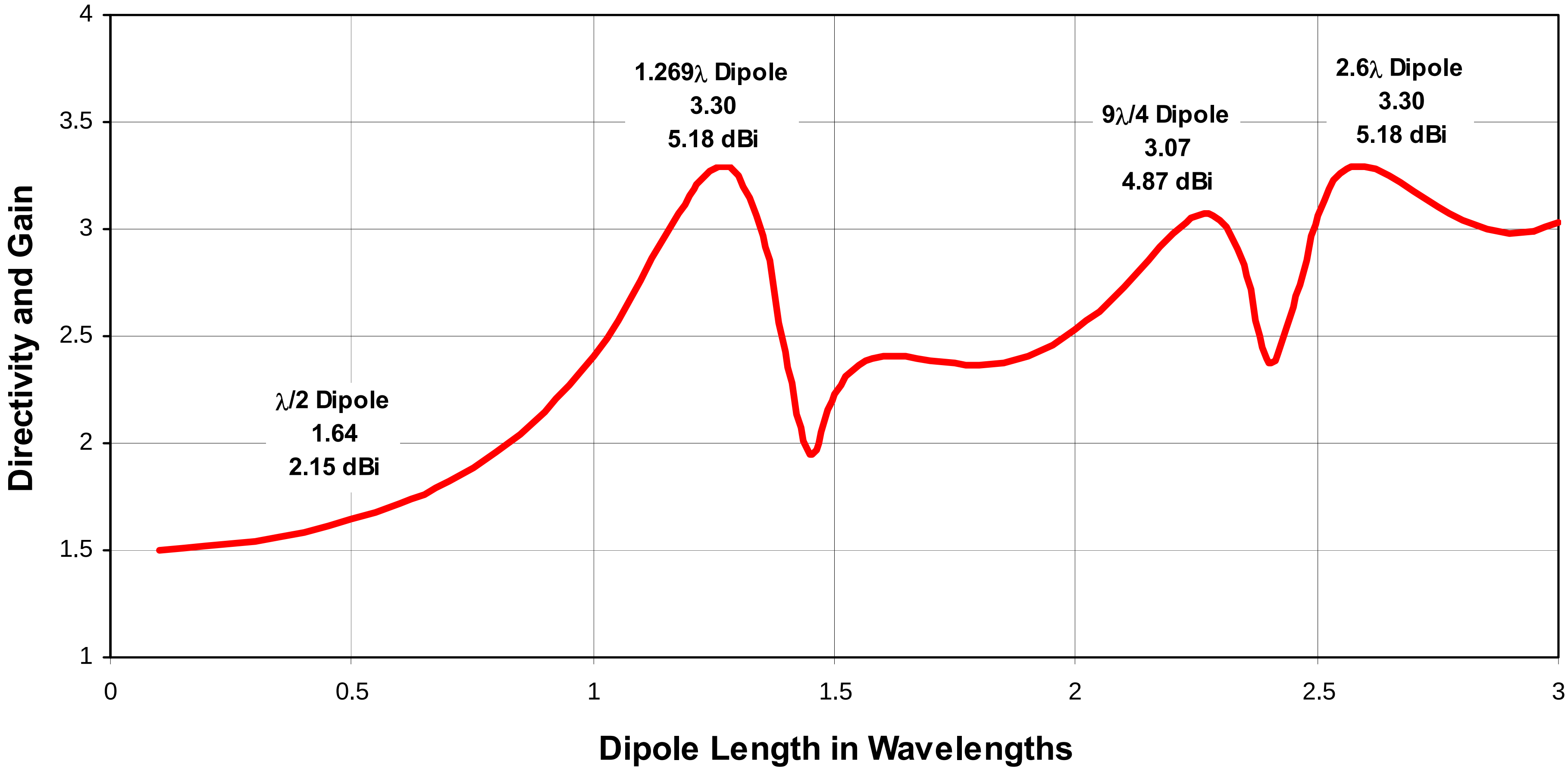


- ☐ ***For lossless antennas, directivity and gain are the same***
- ☐ ***A dipole has maximum gain when it is a half wavelength long***
- ☐ ***An antenna's radiation resistance is not unique. It depends on a reference current or location***
- ☐ ***In the far-field, the electric and magnetic fields have the same waveform as the transmitted signal***
- ☐ ***In free space, a digital data signal transmitted with a dipole and received with a loop will have low bit error rate if the SNR is high enough***

# *Fact or Myth?*

- ☐ ***A dipole is resonant when its length is a half wavelength***
- ☐ ***In free space, a half-wavelength dipole has a real (resistive) feedpoint impedance***
- ☐ ***The feedpoint resistance of a half-wave dipole depends on its diameter***
- ☐ ***The feedpoint reactance of a half-wave dipole depends on its diameter***
- ☐ ***The resonant length of a dipole depends on its diameter***
- ☐ ***Dipoles are resonant at lengths slightly shorter than an odd number of half-wavelengths***
- ☐ ***Dipoles are anti-resonant at lengths slightly longer or shorter (which?) than an even number of half-wavelengths***
- ☐ ***As frequency increases, a dipole's impedance converges to a finite value or diverges to infinity (which?)***
- ☐ ***If a linear wire antenna is resonant, then its feedpoint impedance is real everywhere along its length***

# *Dipole Directivity and Gain versus Length*



# Antenna Impedance Calculation

- ***Getting the current distribution***
  - Induced EMF method
  - Hallen's integral equation (1938)
  - Pocklington's integral equation (1897)
- ***Mathematical solution***
  - Iterative and variational methods
    - Approximation as ratio of infinite series
    - King-Harrison (*Proc IRE*, 1943); Middleton-King (*J Appl Phys*, 1946)
  - Hill's radiation pattern integration method (*Proc IEE*, 1967)
  - Harrington's method of moments (*Proc. IEEE*, 1967)
- ***Numerical solution***
  - Many software programs are available for electromagnetic analysis
  - Finite difference method (FD)
  - Finite element method (FEM)
  - Method of moments (MoM)
  - Geometric theory of diffraction (GTD)

# *Design Software for Antennas and Matching Networks*

- ***Software for antennas and fields***
  - NEC (NEC-2 is public domain, NEC-4 is restricted)
  - WIRA (Dr. Frank Harris's program used at Technology for Communications International)
  - WIPL-D (MoM for wires, plates, and dielectrics; free Lite version)
  - Ansoft HFSS (finite element method, professional, expensive)
  - Zeland IE3D (MoM) and Fidelity (finite difference method)
  - CST Microwave Studio (MWS) (free 30-day trial)
  - Many others …
- ***Electronic Design Automation (EDA) software for rf circuits and networks***
  - SPICE and its variants… (Orcad pSPICE, free Lite version)
  - ARRL Radio Designer (10 variable optimizer, discontinued)
  - Ansoft's Serenade SV (4 variable optimizer, discontinued)
  - Ansoft's Designer SV (no optimizer, free)
  - Agilent's Advanced Design System (ADS)
  - Applied Wave Research's Microwave Office (MWO) (free 30-day trial)

# *Induced EMF Method*

- ***Assumes sinusoidal current distribution***
- ***Method gives pattern, radiation resistance, and reactance***
- ***Accurate for pattern and impedance of dipoles up to half-wavelength and verticals up to quarter-wavelength***
- ***Inaccurate for impedance of dipoles longer than half-wavelength and verticals longer than quarter wavelength***
- ***Used widely for the design of AM broadcast vertical towers***

# *Induced EMF Method continued*

- ***Radiation resistance***

$$R_{in} = \frac{\eta}{2\pi \sin^2\left(\frac{kl}{2}\right)} \left\{ C + \ln(kl) - \mathrm{Ci}(kl) + \frac{1}{2}\sin(kl)\left[\mathrm{Si}(2kl) - 2\mathrm{Si}(kl)\right] + \frac{1}{2}\cos(kl)\left[ C + \ln\left(\frac{kl}{2}\right) + \mathrm{Ci}(2kl) - 2\mathrm{Ci}(kl) \right] \right\}$$

**Terms vanish when *l*/λ is a half integer**

- ***Reactance***

**Wire radius term**

$$X_{in} = \frac{\eta}{2\pi \sin^2\left(\frac{kl}{2}\right)} \left\{ \mathrm{Si}(kl) + \frac{1}{2}\sin(kl)\left[ \mathrm{Ci}(2kl) - 2\mathrm{Ci}(kl) + \mathrm{Ci}\left(\frac{2ka^2}{l}\right) \right] + \frac{1}{2}\cos(kl)\left[\mathrm{Si}(2kl) - 2\mathrm{Si}(kl)\right] \right\}$$

# Method of Moments

- ☐ ***Is a general method for solving integro-differential equations by converting them into matrix equations***
- ☐ ***Introduced to electromagnetics by Roger Harrington in 1967***
- ☐ ***Gives better results with Hallen's integral than Pocklington's***
- ☐ ***Basis functions can be global or local***
- ☐ ***Local basis functions break antenna into small conducting segments or patches***
  - ➢ Expresses current as weighted sum of basis functions
  - ➢ Solves for the coefficients of the basis functions on all segments
  - ➢ Calculates radiation pattern and feedpoint impedance from currents
- ☐ ***Software for antennas made of round wires, no dielectrics***
  - ➢ Numerical Electromagnetic Code (NEC), EZNEC, EZNEC ARRL, and NEC WinPlus
  - ➢ WIRA (proprietary to Technology for Communications International)
- ☐ ***For antennas of round wires, flat plates, and dielectric slabs***
  - ➢ WIPL-D and WIPL-D Lite

# *Limitations of Antenna Modeling by MoM (NEC)*

- ☐ ***NEC is "blind" to current modes – computes total current, not resolved into common and differential current modes***
  - ➢ Current modes are "noumena;" total current is "phenomena"
- ☐ ***Antennas that rely on interacting modes do not scale if $\lambda/\lambda_g$ or $v_p$ changes***
  - ➢ Dielectric insulation on wires affects common and differential current modes differently ⇒ published antenna designs often irreproducible
- ☐ ***Antennas of dielectric covered wire can't be analyzed by NEC***
  - ➢ Twin lead folded dipole
  - ➢ Twin lead J-pole
  - ➢ Butternut radials
- ☐ ***Amateur literature***
  - ➢ "Plastic-insulated wire lowers the resonant frequency of halfwave dipoles by about 3%." (*ARRL Antenna Book*, p. 4-31)
  - ➢ "Plastic-insulated wire increases the antiresonant frequency of 1$\lambda$ dipoles by about 5%." (K6OIK, ARRL Pacificon 2003)

# Example Dipole Used in this Talk

- ☐ ***Freespace***
- ☐ ***Omega***
    - ➢ 20 (exact)

$$\Omega' = 2\ln\left(\frac{l}{a}\right) = 2\ln\left(\frac{2l}{d}\right)$$

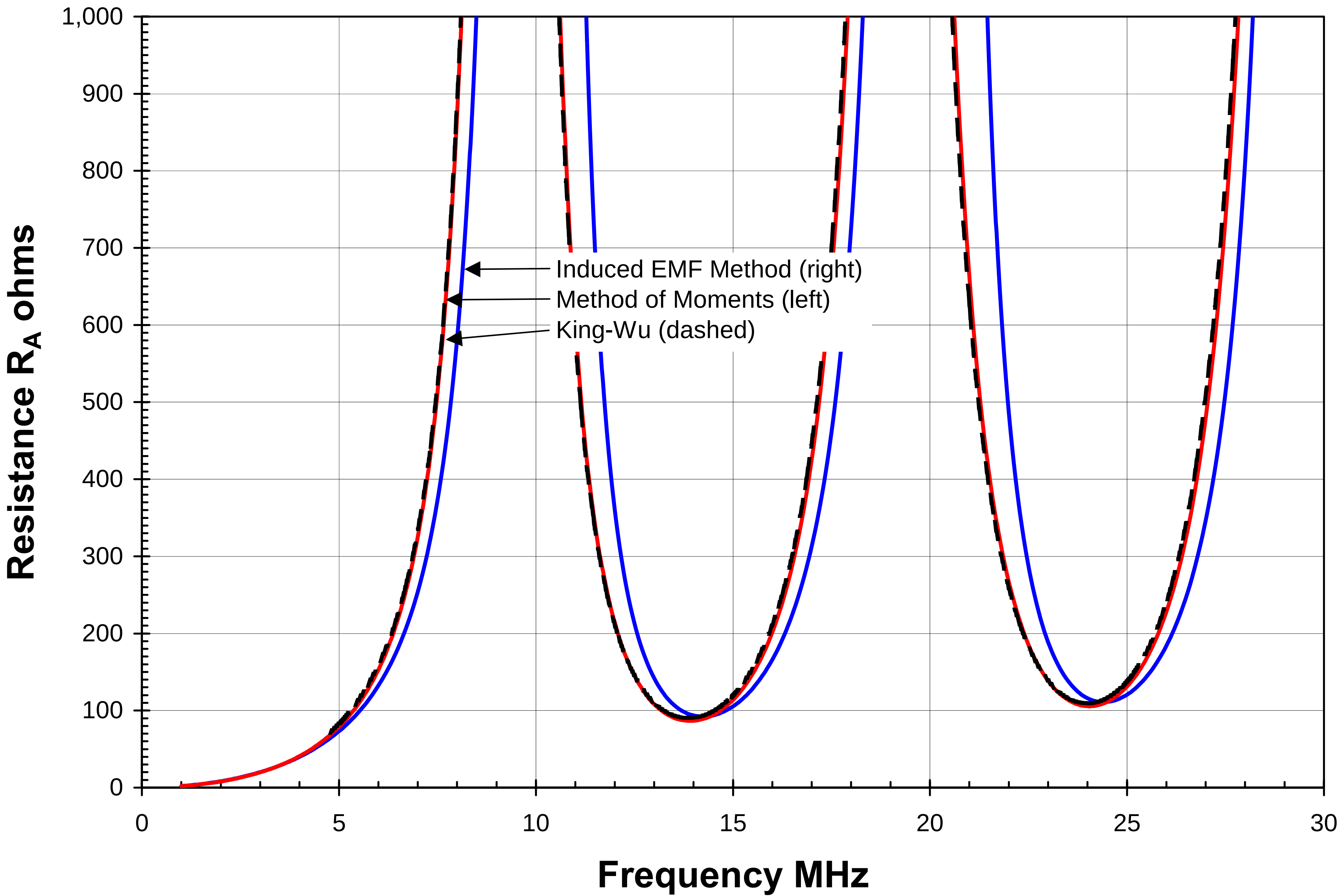
$l$ is total length
$d$ is wire diameter
$a$ is wire radius

- ☐ ***Length: Half wavelength at 5 MHz***
    - ➢ 29.9792458 meters
    - ➢ 98.3571056 feet
- ☐ ***Length-to-diameter ratio***
    - ➢ 11,013
- ☐ ***Diameter***
    - ➢ 0.107170 inches
    - ➢ AWG # 9.56

| ***Resonances*** | ***Antiresonances*** |
|---|---|
| 4.868 MHz<br>72.2 Ω | 9.389 MHz<br>4,970 Ω |
| 14.834 MHz<br>106 Ω | 19.245 MHz<br>3,338 Ω |
| 24.820 MHz<br>122 Ω | 29.158 MHz<br>2,702 Ω |

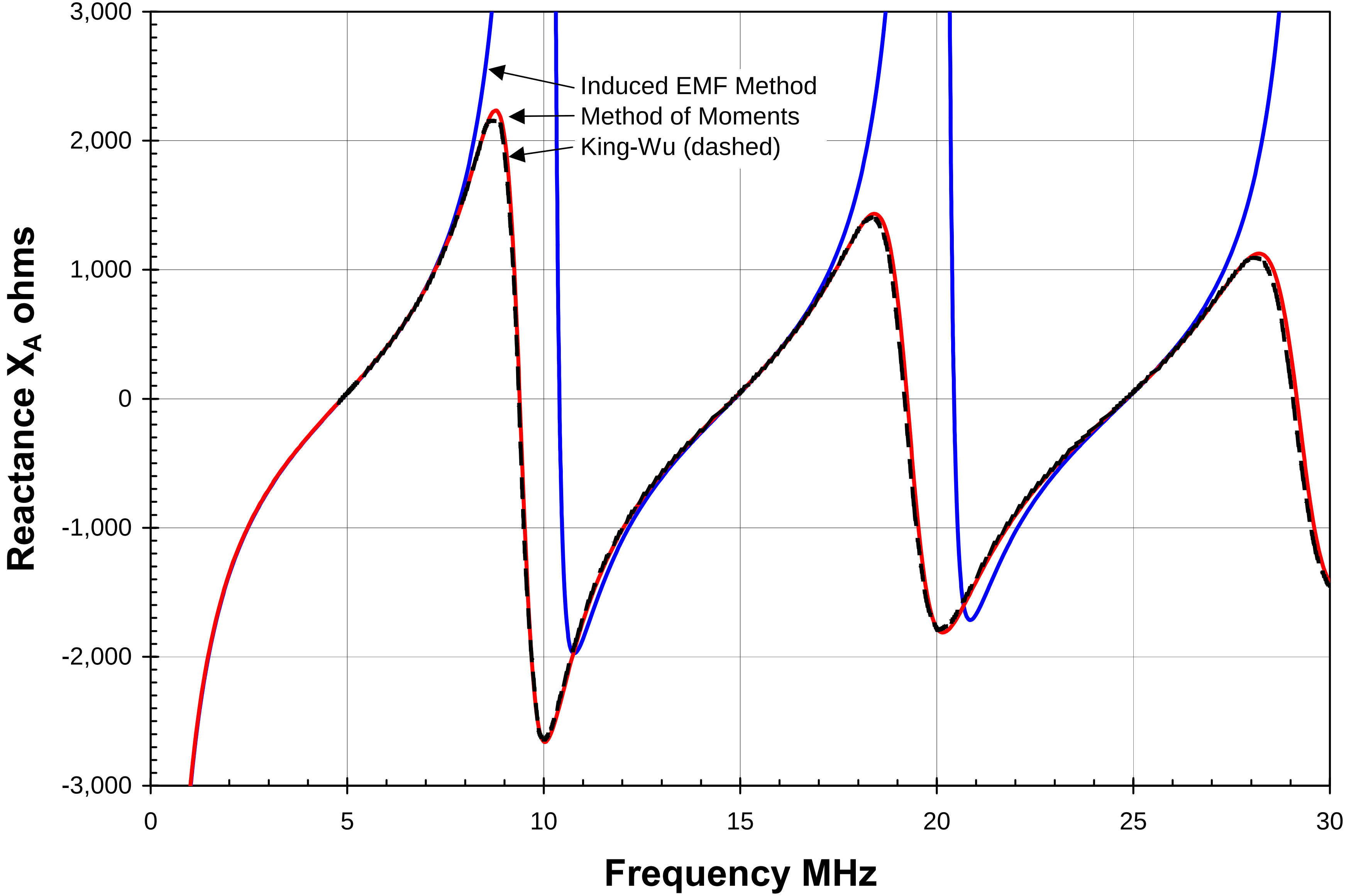
# Feedpoint Resistance

## Induced EMF Method versus MoM



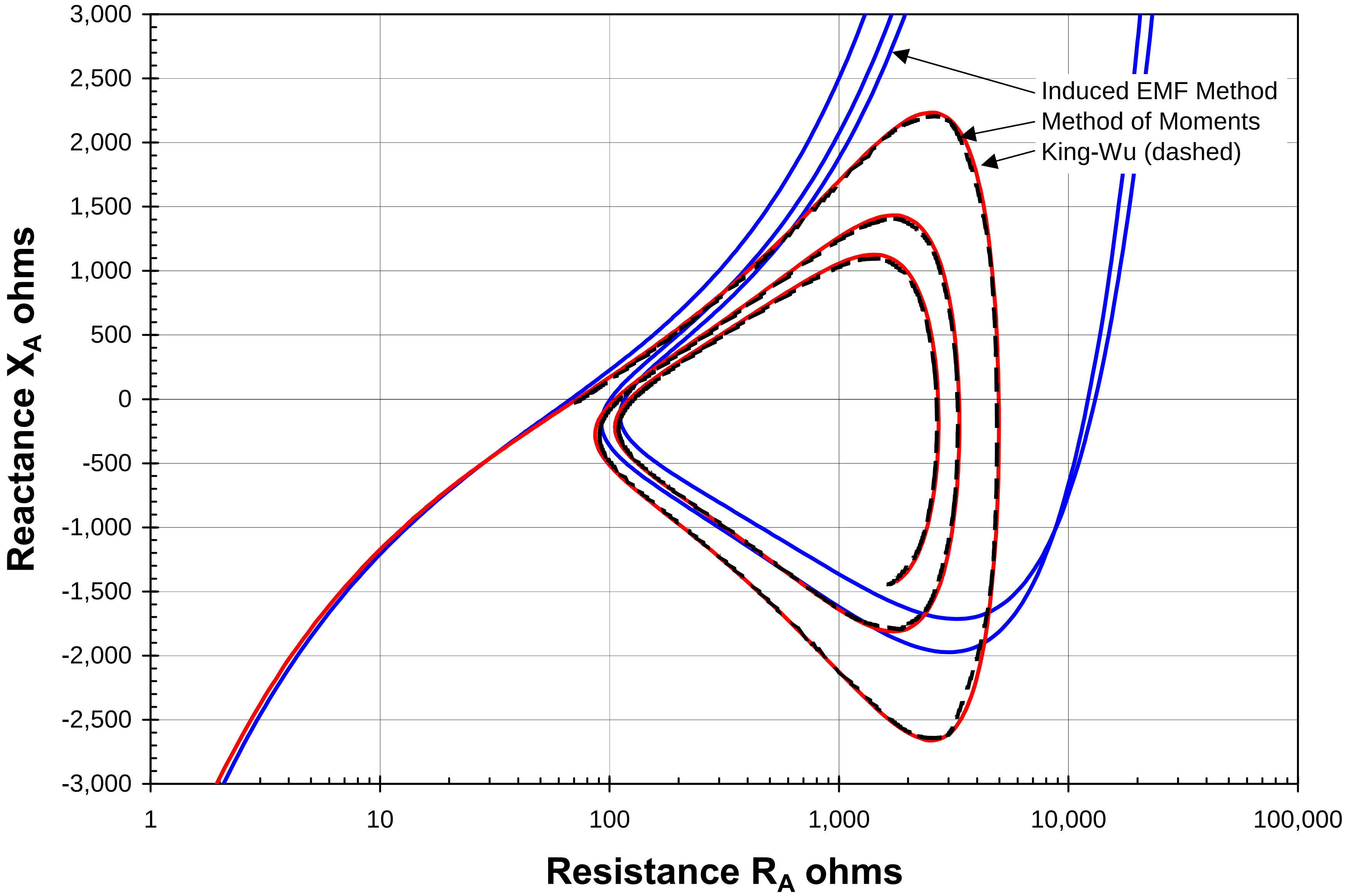
# Feedpoint Reactance

## Induced EMF Method versus MoM

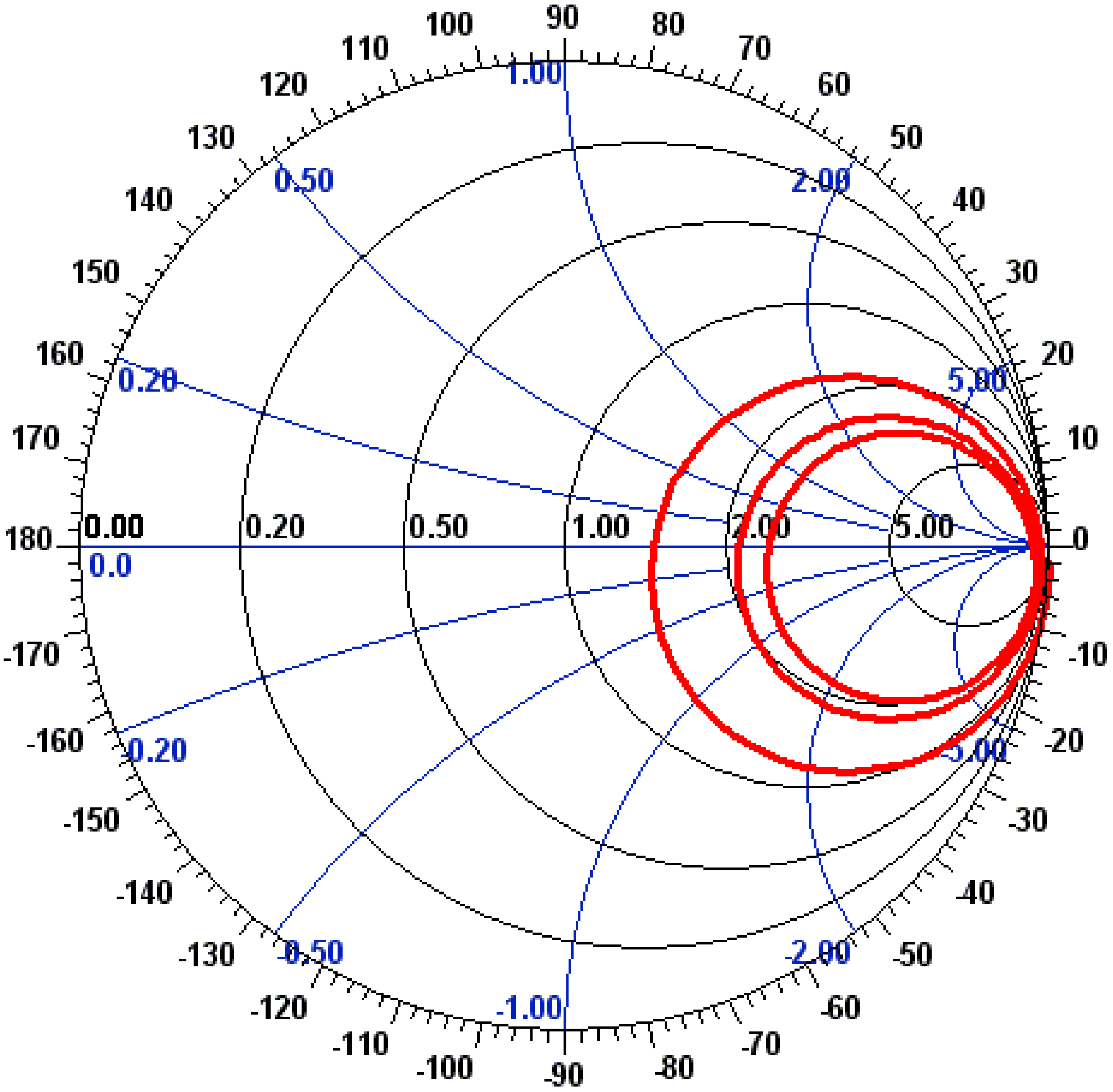


# *Comparison of Induced EMF versus MoM up to 3λ*

**Compare to ARRL Antenna Book, p. 2-4, Figure 3.**



# Dipole Impedance by MoM on the Smith Chart



# *Dipole Impedance Near 1st Resonance*

- ***For exact half-wave dipole, l = $\lambda/2$***

$$Z_A = 73.08 + j41.52$$

Independent of
wire diameter

- ***For resonant dipole, l < $\lambda/2$***

$$Z_A = R_A + j0$$

$$R_A < 73.08$$

Depends on
wire diameter

- ***Dipole thickness***

$$\frac{l}{d} = \frac{l}{2a}$$

$l$ is total length
$d$ is wire diameter
$a$ is wire radius

$$\Omega' = 2\ln\left(\frac{l}{a}\right)$$

# *Favorite Antenna Books*

- ☐ ***Books for antenna engineers and students***
  - ➢ *Antenna Engineering Handbook*, 3rd ed., R. C. Johnson editor, McGraw-Hill, 1993, ISBN 007032381X. First edition published in 1961, Henry Jasik editor.
  - ➢ C. A. Balanis, *Antenna Theory*, 2nd ed., Wiley, 1996, ISBN 0471592684. First edition published in 1982 by Harper & Row.
  - ➢ J. D. Kraus & R. J. Marhefka, *Antennas*, 3rd ed., McGraw-Hill, 2001, ISBN 0072321032. First edition published in 1950; 2nd edition 1988. The 3rd edition added antennas for modern wireless applications.
  - ➢ R. S. Elliott, *Antenna Theory and Design*, revised ed., IEEE Press, 2003, ISBN 0471449962. First published in 1981 by Prentice Hall.
  - ➢ S. J. Orfanidis, *Electromagnetic Waves and Antennas*, draft textbook online at http://www.ece.rutgers.edu/~orfanidi/ewa/
- ☐ ***Books for radio amateurs***
  - ➢ *ARRL Antenna Book*, 20th ed., Dean Straw editor, American Radio Relay League, 2003, ISBN 0872599043.

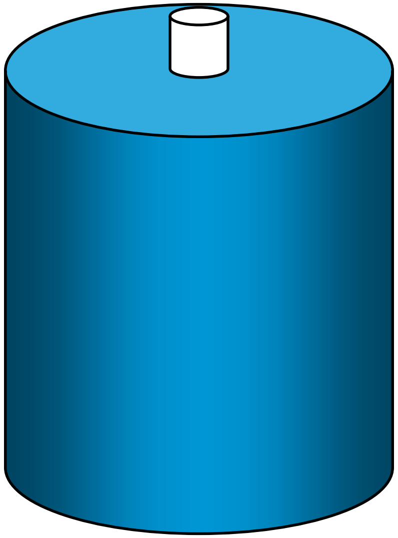
# *Narrowband Models of Dipole Impedance*

## *Near the 1st Resonance*

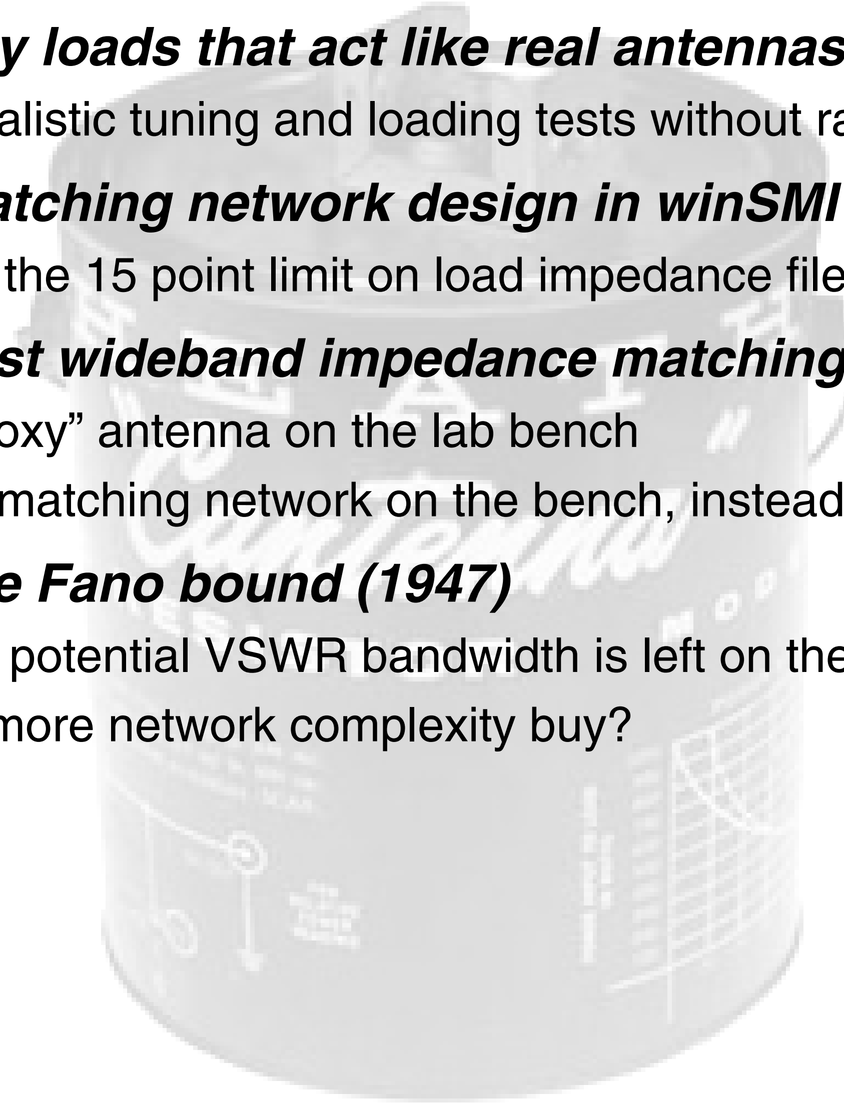
# Blind Observer Problems

- ☐ ***Albert Einstein (1916)***
  - ➢ Blind observer can only measure force
  - ➢ Gravity or acceleration?
  - ➢ Equivalence principle & General theory of relativity
- ☐ ***Alan Turing (1950)***
  - ➢ Blind observer can only send and receive text messages to unknown entity
  - ➢ Man, woman or machine?
  - ➢ Turing test for Artificial Intelligence
- ☐ ***Steve Stearns, K6OIK (2004)***
  - ➢ Blind observer can only measure impedance at any frequency
  - ➢ Antenna or circuit?
  - ➢ ? ? ?

# Introducing the Smart Dummy



# *What Are Equivalent Circuits for Antenna Impedance Good For?*



- ***Build dummy loads that act like real antennas***
  - Perform realistic tuning and loading tests without radiating
- ***Facilitate matching network design in winSMITH***
  - Overcome the 15 point limit on load impedance files
- ***Build and test wideband impedance matching networks***
  - Put the “proxy” antenna on the lab bench
  - Adjust the matching network on the bench, instead of on the tower
- ***Calculate the Fano bound (1947)***
  - How much potential VSWR bandwidth is left on the table?
  - What can more network complexity buy?

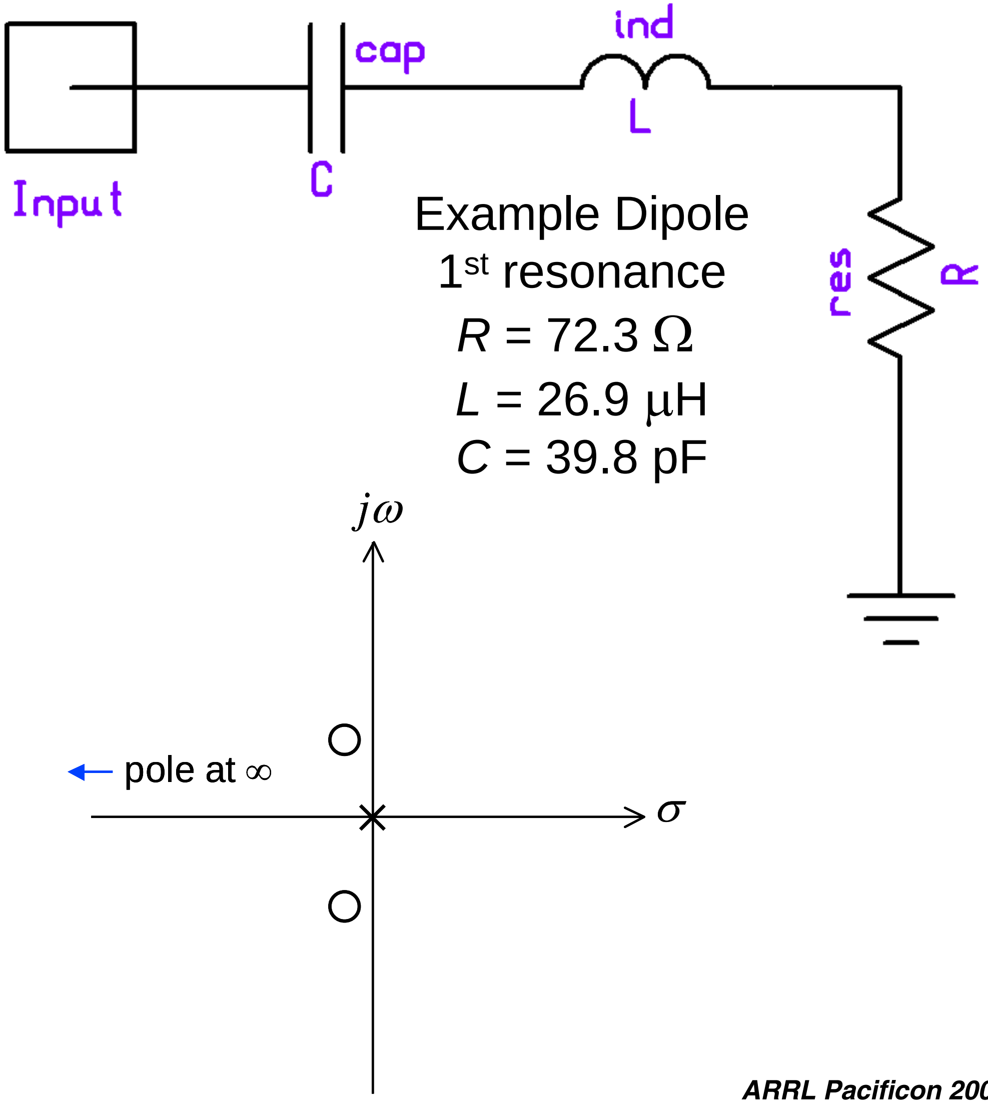
# Series RLC Equivalent Circuit for Dipoles at Resonance

$$Z_{in} = \frac{1}{sC} + sL + R$$

$$= \frac{LCs^2 + RCs + 1}{Cs}$$

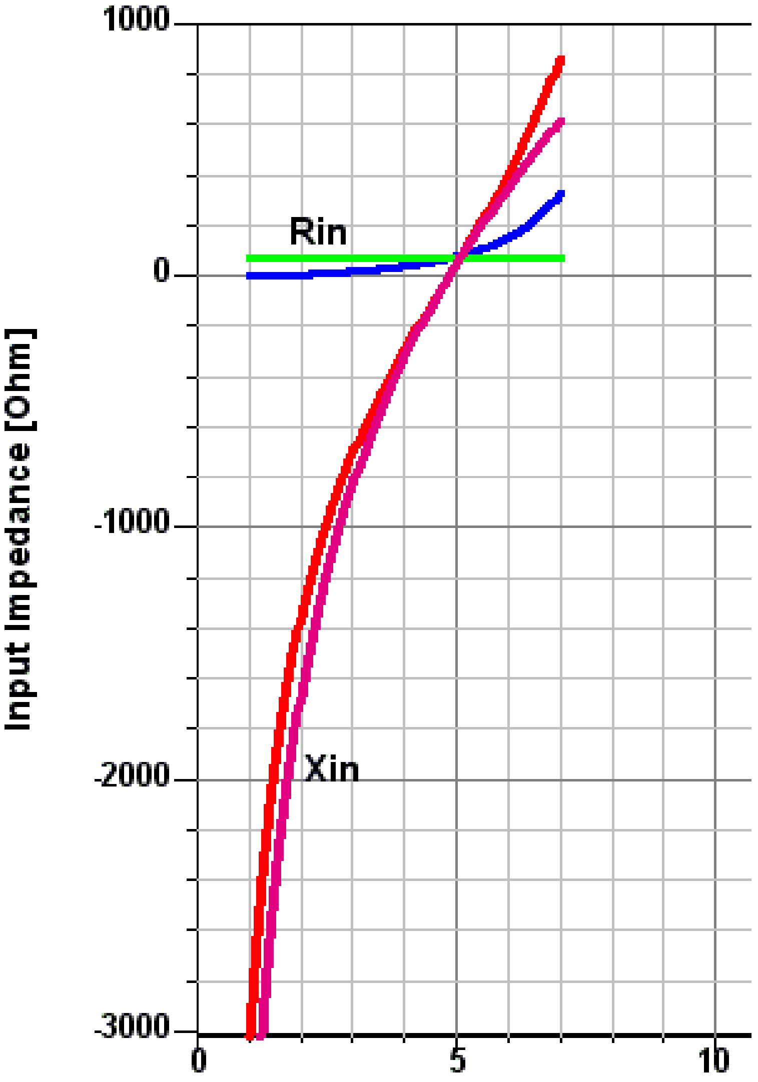
$$= L\frac{s^2 + \frac{R}{L}s + \frac{1}{LC}}{s}$$

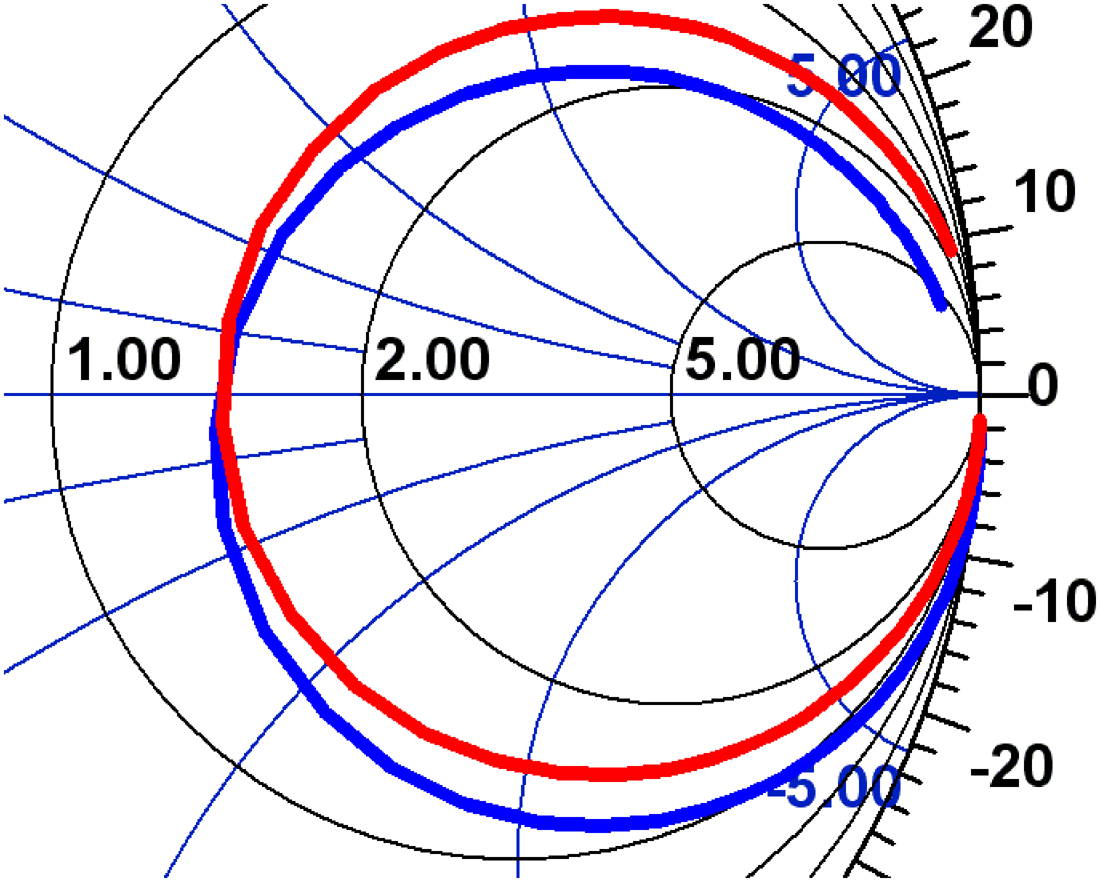
$$= \frac{quadratic}{linear}$$



Example Dipole
1st resonance
$R$ = 72.3 Ω
$L$ = 26.9 µH
$C$ = 39.8 pF

# *Accuracy of Series RLC Model*





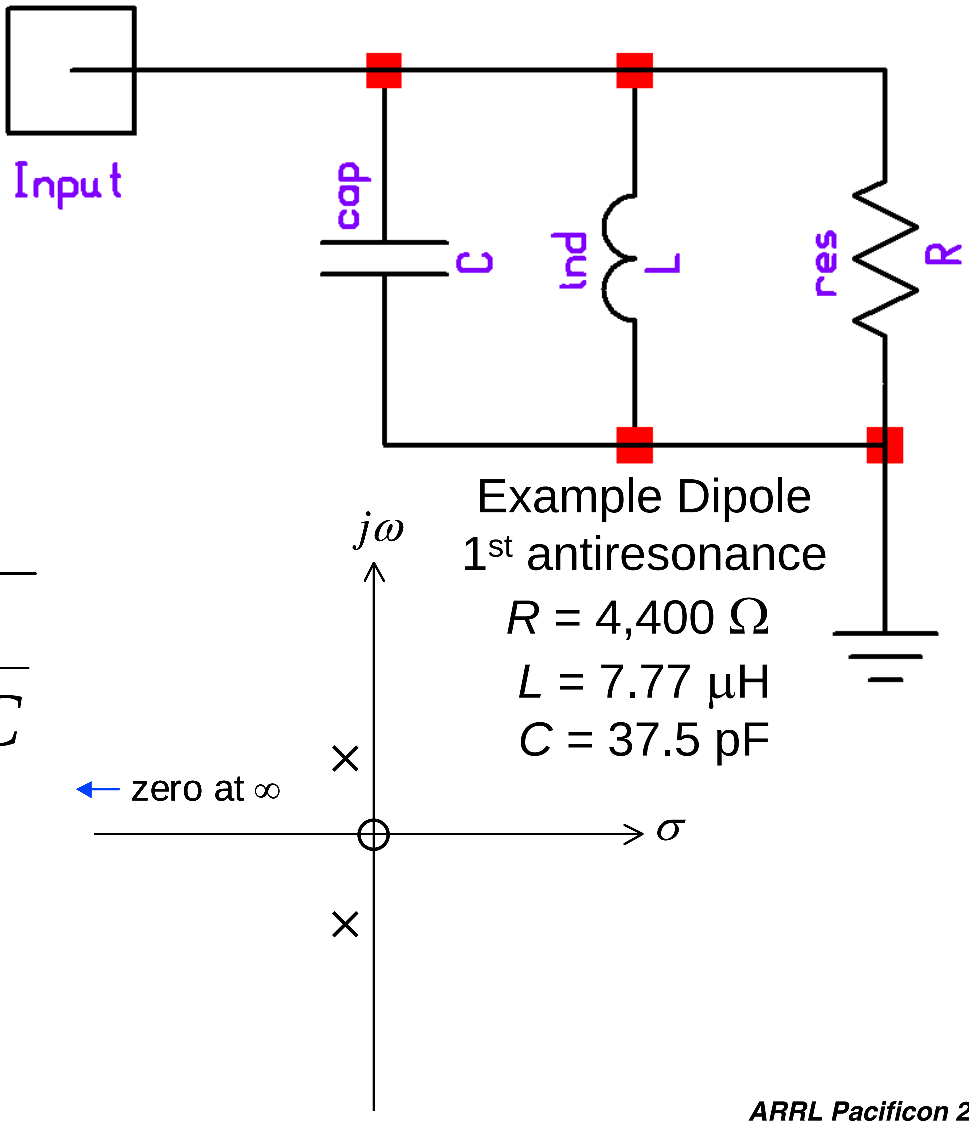
# Parallel RLC Equivalent Circuit for Dipoles at Antiresonance

$$Z_{in} = \frac{1}{sC + \frac{1}{sL} + \frac{1}{R}}$$

$$= \frac{LRs}{LRCs^2 + Ls + R}$$

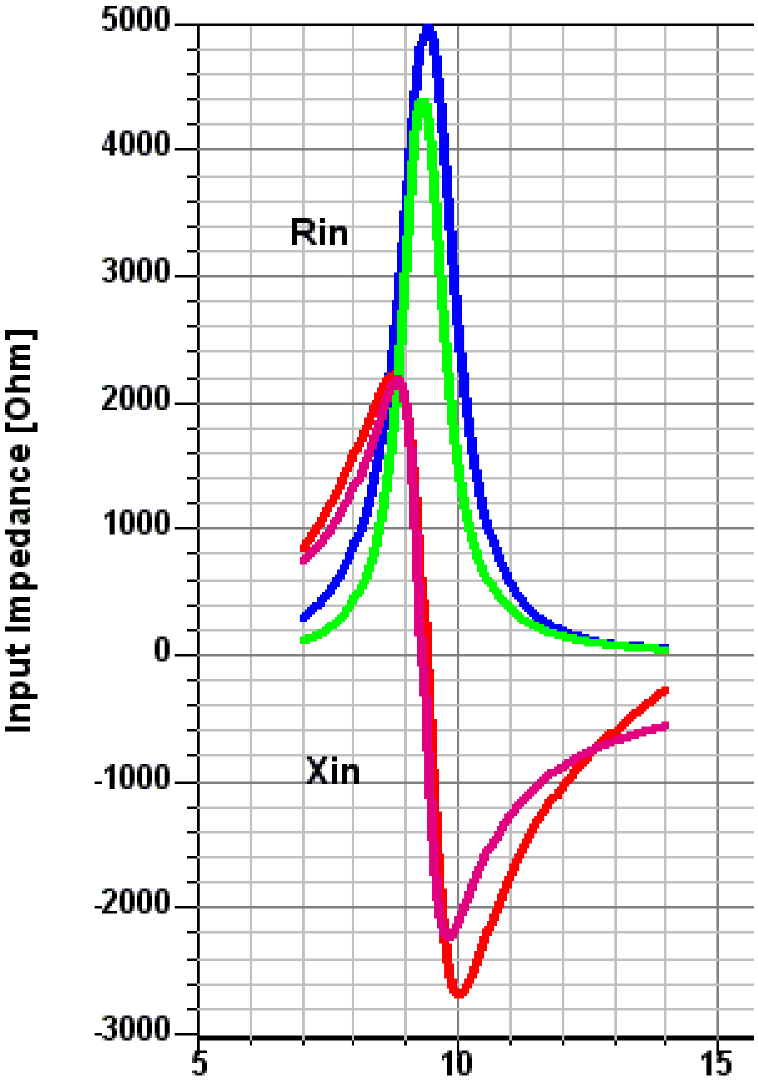
$$= \frac{1}{C}\,\frac{s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

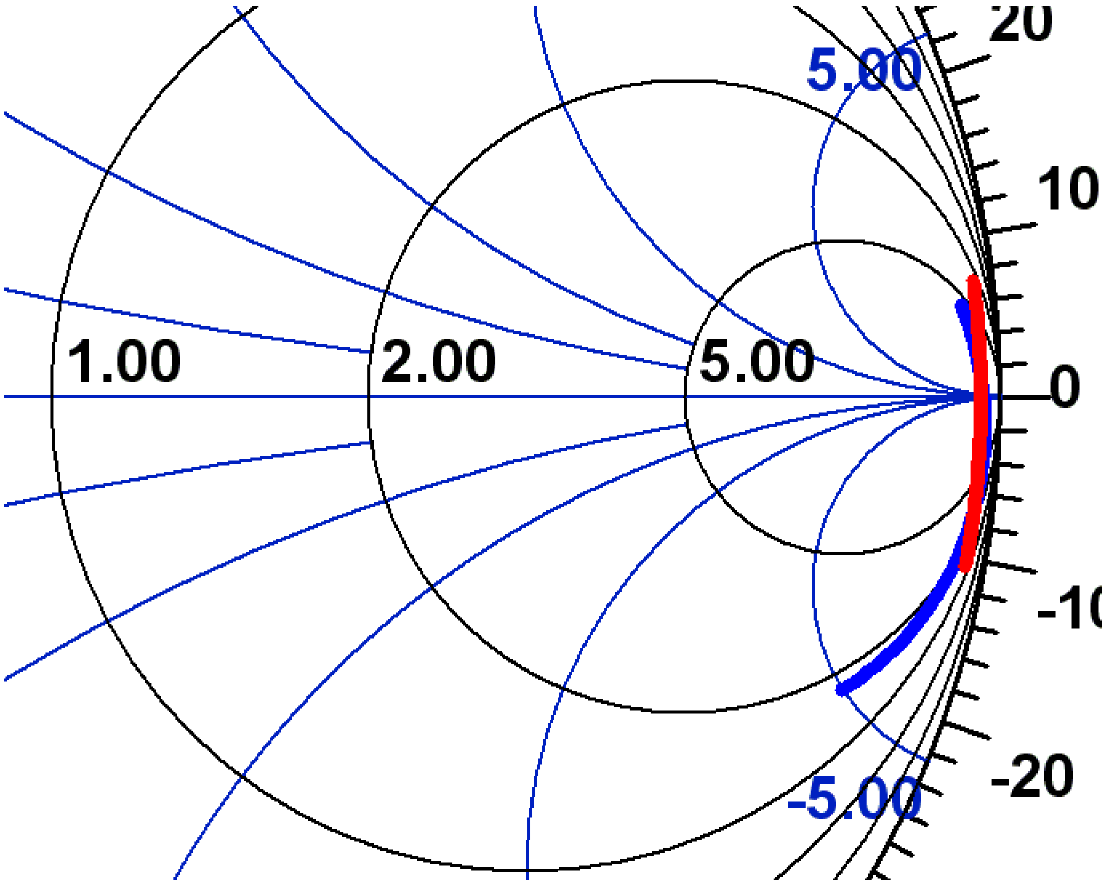
$$= \frac{linear}{quadratic}$$



Example Dipole
1st antiresonance
*R* = 4,400 Ω
*L* = 7.77 μH
*C* = 37.5 pF

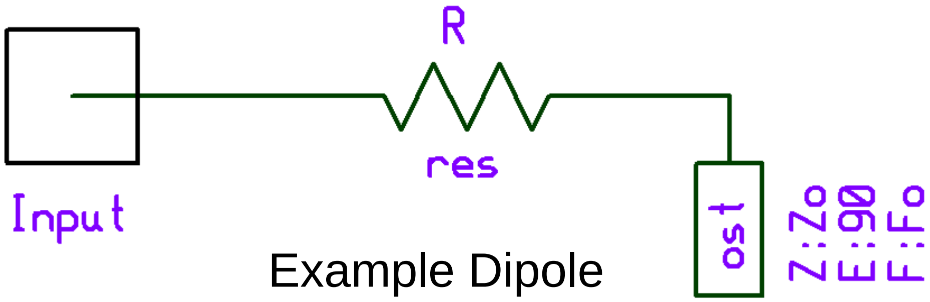
# Accuracy of Parallel RLC Model





# Witt’s Open Circuited Quarter-Wave Stub Model for Dipoles at Resonance

$$Z_{in} = R(f) + jX(f)$$

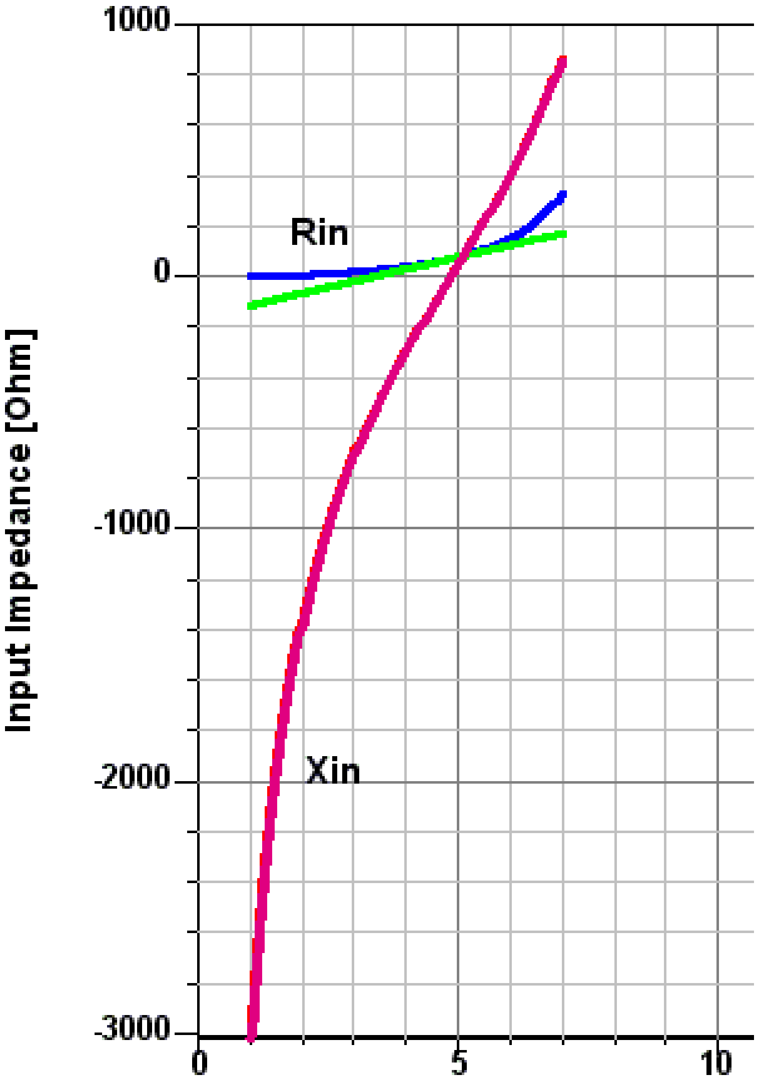


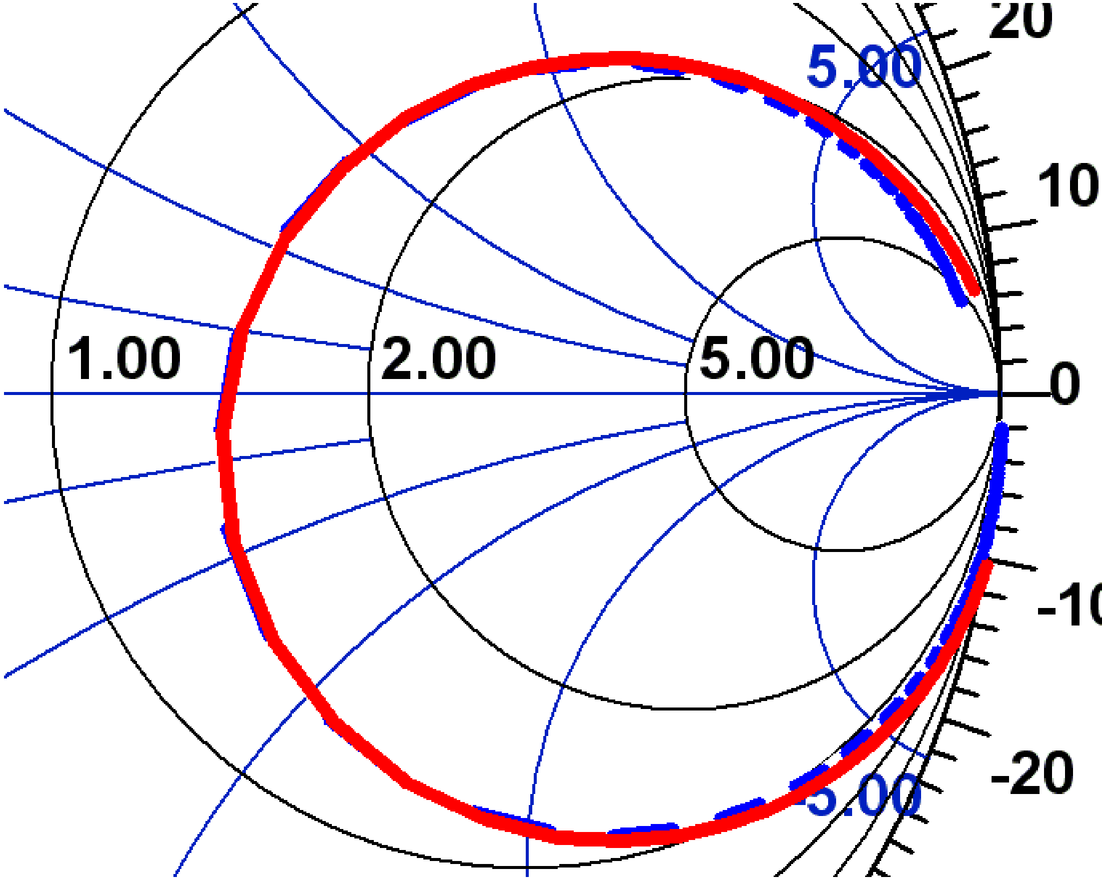
Example Dipole
$R_0$ = 72.6 Ω
$K_R$ = 3.18

$$R(f) = R_0\left[1 + K_R\left(\frac{f}{f_0} - 1\right)\right] \quad \text{where } \ 3 \le K_R \le 3.5$$

$$X(f) = -Z_0 \cot\frac{\pi f}{2 f_0} \quad \text{where } \ Z_0 = \frac{\eta}{\pi}\left[\ln\left(\frac{8110}{d f_0}\right) - 1\right]$$

# *Accuracy of Witt's Open Stub Model*





# ***Better Lumped-Element Equivalent Circuits for Dipoles***

*From DC to Beyond the 1st Resonance*

# Objective and Approach

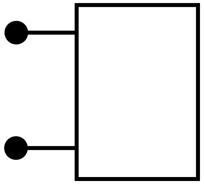
- ***Find simple lumped-element equivalent circuits that approximate the impedance of a resonant dipole better than existing models, by using network synthesis***
- ***Step 1: Obtain reference impedance data for 5 MHz half-wave dipole from 1 MHz to 30 MHz***
  - Run broadband EZNEC sweep, and write to a MicroSmith .gam file
- ***Step 2: Fit the rational function to the dipole's impedance***
  - Order must be at least $\dfrac{quadratic}{linear}$
  - Program a general rational function by using Ansoft Serenade SV's "RJX" element or ARRL Radio Designer's "SRL" element
  - Use optimizer for S matrix goal from 1 MHz to 7 MHz
  - Factor to ensure no poles or zeros in right half plane (RHP)
  - Test to ensure positive real (p.r.)

# *Approach continued*

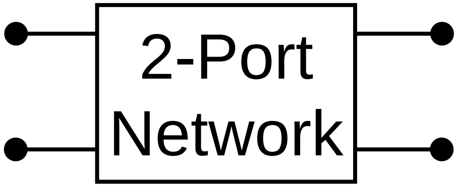
- ☐ ***Step 4: Synthesize equivalent circuit from rational function***
  - Extract lumped-element circuit topology in Darlington form
  - Continued fraction expansion gives ladder network
  - Partial fraction expansion gives series/parallel network
- ☐ ***Step 5: Check the result***
  - Program the circuit into Ansoft Serenade SV or ARRL Radio Designer
  - Compare against original dipole
  - Compare against other approximations

# The Subject of Ports is an Important Subject

Resistors
Capacitors
Inductors
Stubs
Diodes
?Antennas?



1-Port
Network



Filters
Matching networks
Transformers
Transmission lines
Amplifiers
?Antennas?

- ❑ ***N-port networks:***
  - ➢ Terminals are paired
  - ➢ Port voltages defined across terminal pairs
  - ➢ Port currents defined as differential current into/out of terminal pairs
- ❑ ***Laws of physics determine properties and relations among, port impedances***
  - ➢ Conservation of energy
  - ➢ Causality

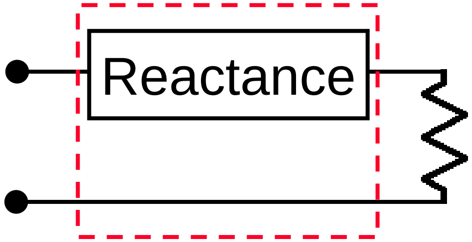
# *Immittance (Impedance & Admittance) Functions*

- ☐ ***Analytic in the RHP, and no poles or zeros***
- ☐ ***Poles and zeros allowed only on j$\omega$ axis and in LHP***
- ☐ ***Input immittances of passive reciprocal networks and devices***
- ☐ ***Real and imaginary parts are related by Poisson integral***
- ☐ ***Every immittance function has a Darlington equivalent circuit,***
- ☐ ***Port immittances of lumped R, L, C networks***
  - ➢ Are rational functions with positive coefficients
  - ➢ Degrees of numerator and denominator polynomials differ by 0 or 1
  - ➢ If the degrees are the same, the network has losses

# *Darlington Forms*

- ***Any one-port immittance function can be realized by a lossless two-port terminated by a resistor***
- ***A resistor in series or shunt with a lossless one-port lacks generality – antennas don't act like this***

**This……**



**Not This!**

- ***Every antenna impedance function has an equivalent circuit in Darlington form***
- ***The Darlington form is the starting point for understanding the Fano bound on impedance matching***

# *Finding a Rational Approximating Function*

- ☐ ***Initial form***

$$Z_A(s) = \frac{cubic}{quadratic}$$

$$= \frac{as^3 + bs^2 + cs + d}{es^2 + s}$$

- ☐ ***Real part***

$$R_A(j\omega) = \frac{(be - a)\omega^2 + (c - de)}{e^2\omega^2 + 1}$$

- ☐ ***Imaginary part***

$$X_A(j\omega) = \frac{ae\omega^4 + (b - ce)\omega^2 - d}{e^2\omega^3 + \omega}$$

# ARRL Radio Designer Optimization Code

```
* This file was generated initially by Serenade Schematic Netlister
* Edited manually for ARRL Radio Designer by K6OIK
 A : 74.3954E-24
 B : 27.5199E-6
 D : 25.3813E9
 E : 4.66048E-9
 C : 72.2976
 w :(2*pi*f)
 r :(((b*e-a)*w^2+(c-d*e))/((e*w)^2+1))
 x :((a*e*w^4+(b-c*e)*w^2-d)/(w*((e*w)^2+1)))
BLK
 srl 122 R=r L=(x/w)
dipole5: 1POR 122
END
FREQ
 Step 1MHz 7MHz 50kHz
END
NOUT
 R1 = 50
END
OPT
dipole5  R1 = 50
 F 1MHz 7MHz S=antdata
END
NOPT
 R1 = 50
END
DATA
antdata: Z RI INTP=CUB
*Impedance of 5-MHz dipole by EZNEC.  Length=98.35710566 ft., Dia=0.1071697366 in., Omega=20
1.00MHz 1.89876587 -3035.57432668
...  [impedance data file continued...]
END
```

# *Coefficients Found By ARD's Optimizer in Four Tries*

- ❑ ***First attempt with no constraints; negative coefficient*** ☹

$$Z_A(s) = \frac{-7.74\times10^{-14}s^3 + 2.70\times10^{-5}s^2 + 1.83\times10^{-5}s + 2.50\times10^{10}}{1.83\times10^{-9}s^2 + s}$$

- ❑ ***Second attempt, forced coefficients > 0; but $R_A < 0$ at low f*** ☹

$$Z_A(s) = \frac{7.44\times10^{-23}s^3 + 2.75\times10^{-5}s^2 + 72.3s + 2.54\times10^{10}}{4.66\times10^{-9}s^2 + s}$$

- ❑ ***Third attempt, constrained c = de, so $R_A(j\omega) \geq 0$ for all $\omega$*** ☺

$$Z_A(s) = \frac{5.36\times10^{-23}s^3 + 2.72\times10^{-5}s^2 + 72.3s + 2.52\times10^{10}}{2.88\times10^{-9}s^2 + s}$$

- ❑ ***Fourth attempt, eliminated negligible cubic term*** ☺

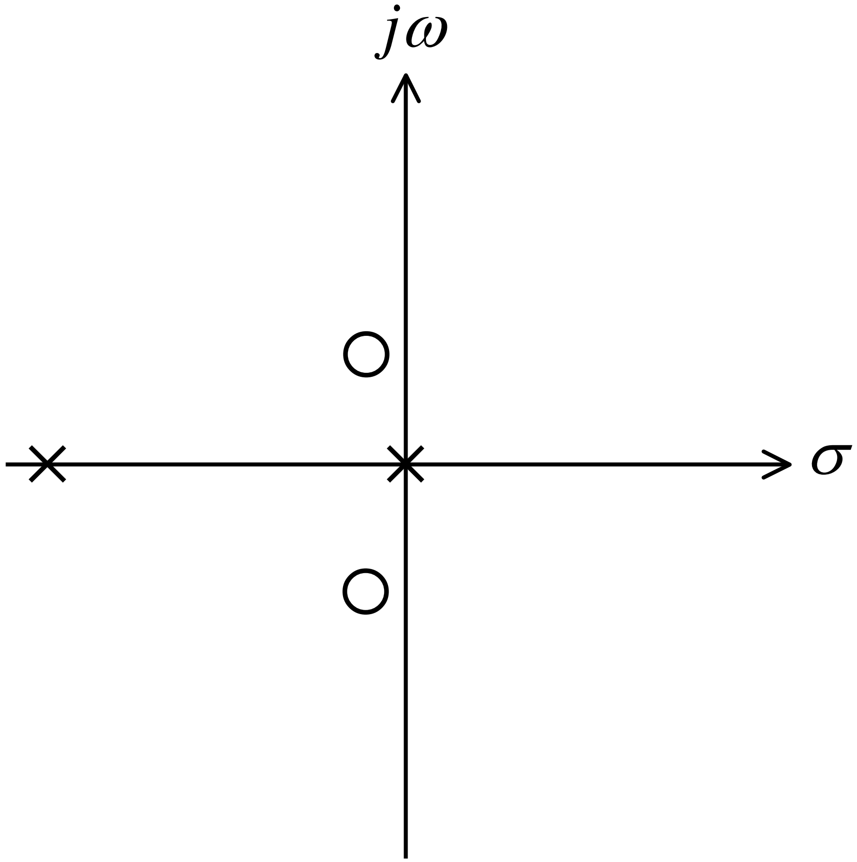
$$Z_A(s) = \frac{2.72\times10^{-5}s^2 + 72.3s + 2.52\times10^{10}}{2.88\times10^{-9}s^2 + s}$$

# Finding a Rational Approximating Function
# Final Solution with Proper Constraints

- ❑ ***Final form***

$$
\begin{aligned}
Z_A(s) &= \frac{quadratic}{quadratic} \\
&= \frac{bs^2 + des + d}{es^2 + s} \\
&= \frac{2.72\times10^{-5}s^2 + 72.3s + 2.52\times10^{10}}{2.88\times10^{-9}s^2 + s} \\
&= 9{,}445\,\frac{\left(s + (0.13 \pm j3.04)\times10^{7}\right)}{s\left(s + 3.48\times10^{8}\right)}
\end{aligned}
$$

# Confirm that Approximation is Positive Real



| | |
|---|---|
| ***$Z_A$ analytic in RHP*** | ☑<br>***pass*** |
| ***$Z_A$ real if s is real*** | ☑<br>***pass*** |
| ***Poles on $j\omega$ axis are simple and have positive real residues*** | ☑<br>***pass*** |
| ***Real part of $Z_A \geq 0$ on $j\omega$ axis*** | ☑<br>***pass*** |

☺

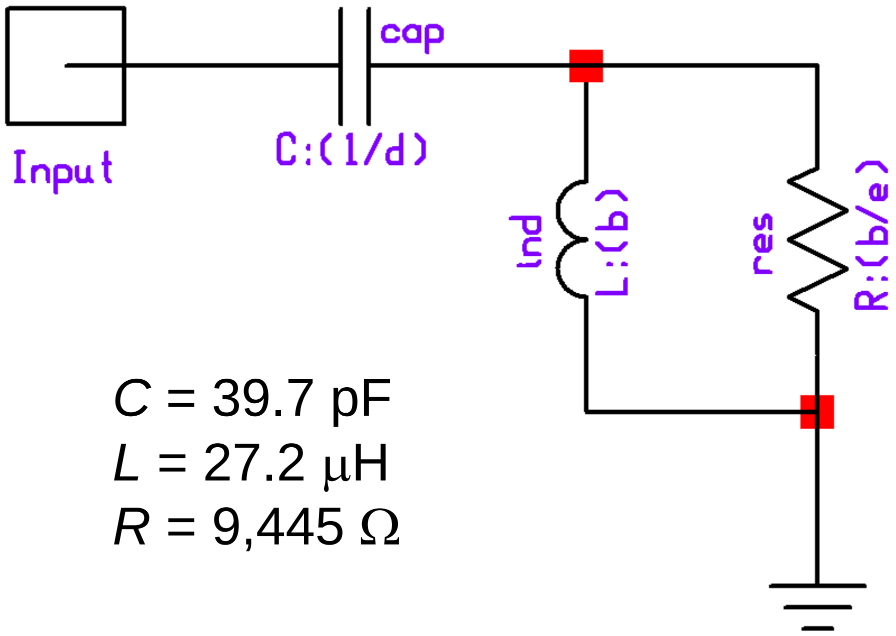
# Network Synthesis

- ❑ ***Divide, and voila !***

$$Z_A(s) = \frac{bs^2 + des + d}{es^2 + s}$$

$$= \frac{d}{s} + \frac{1}{\frac{1}{bs} + \frac{e}{b}}$$

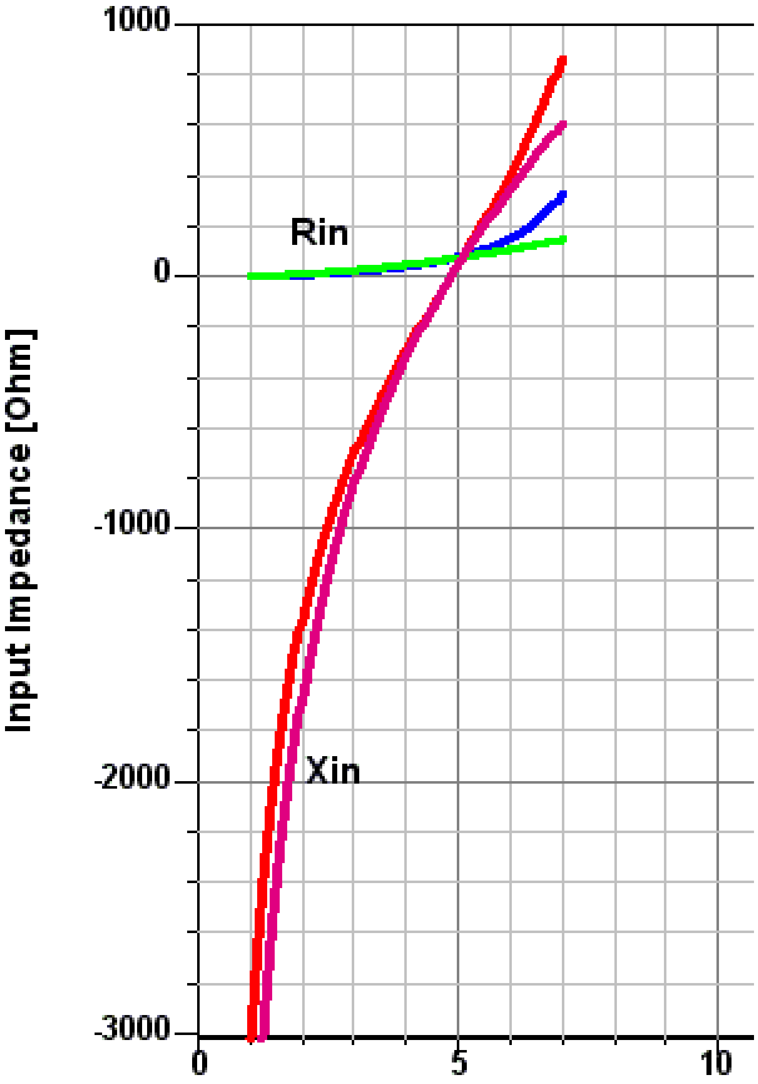
$$= \frac{1}{sC} + \frac{1}{\frac{1}{sL} + \frac{1}{R}}$$

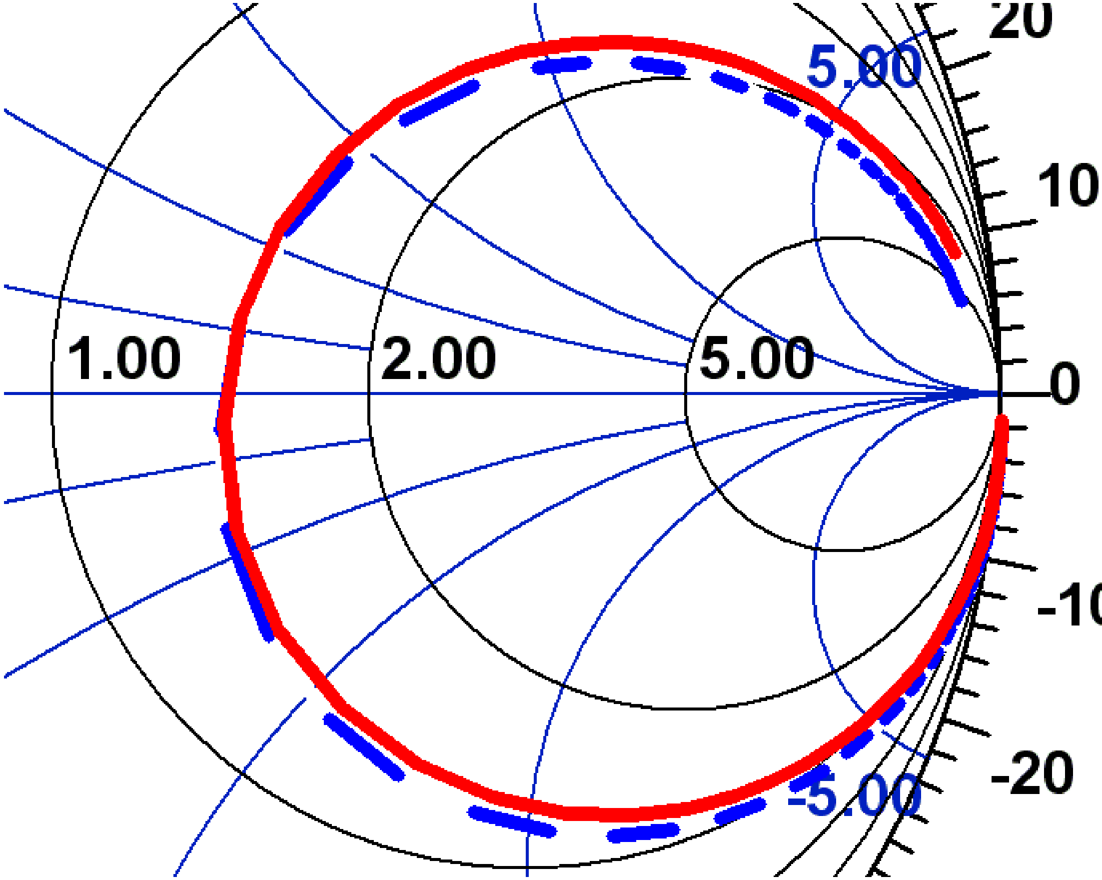


*C* = 39.7 pF
*L* = 27.2 μH
*R* = 9,445 Ω

- ❑ ***A three-element equivalent circuit in Darlington form !***

# *Accuracy of 3-Element Equivalent Circuit*



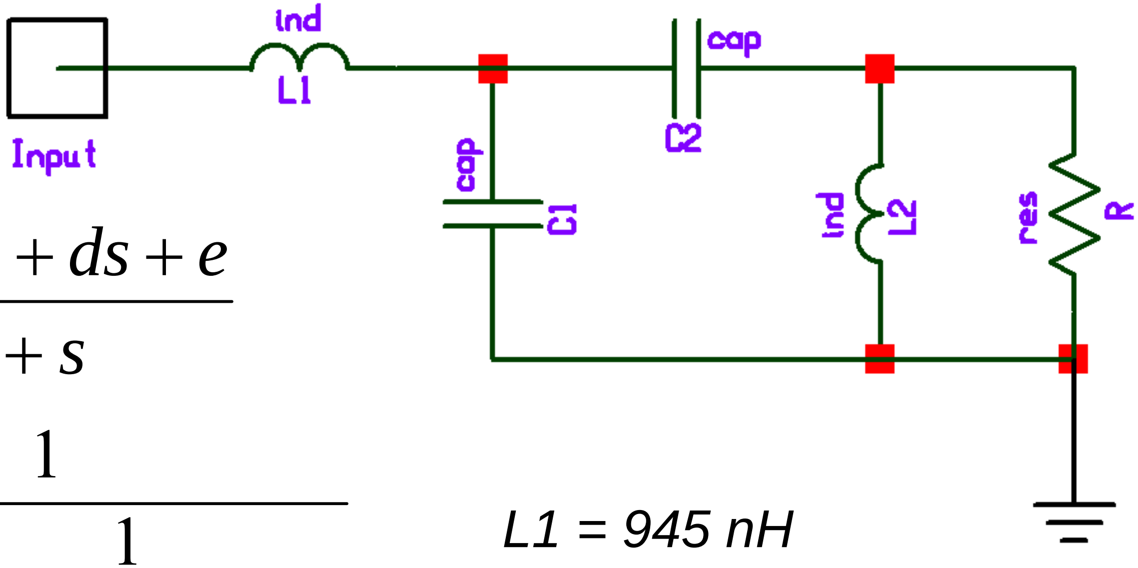


# 5-Element Equivalent Circuit

$$Z_A(s) = \frac{quartic}{cubic}$$

$$= \frac{as^4 + bs^3 + cs^2 + ds + e}{fs^3 + gs^2 + s}$$

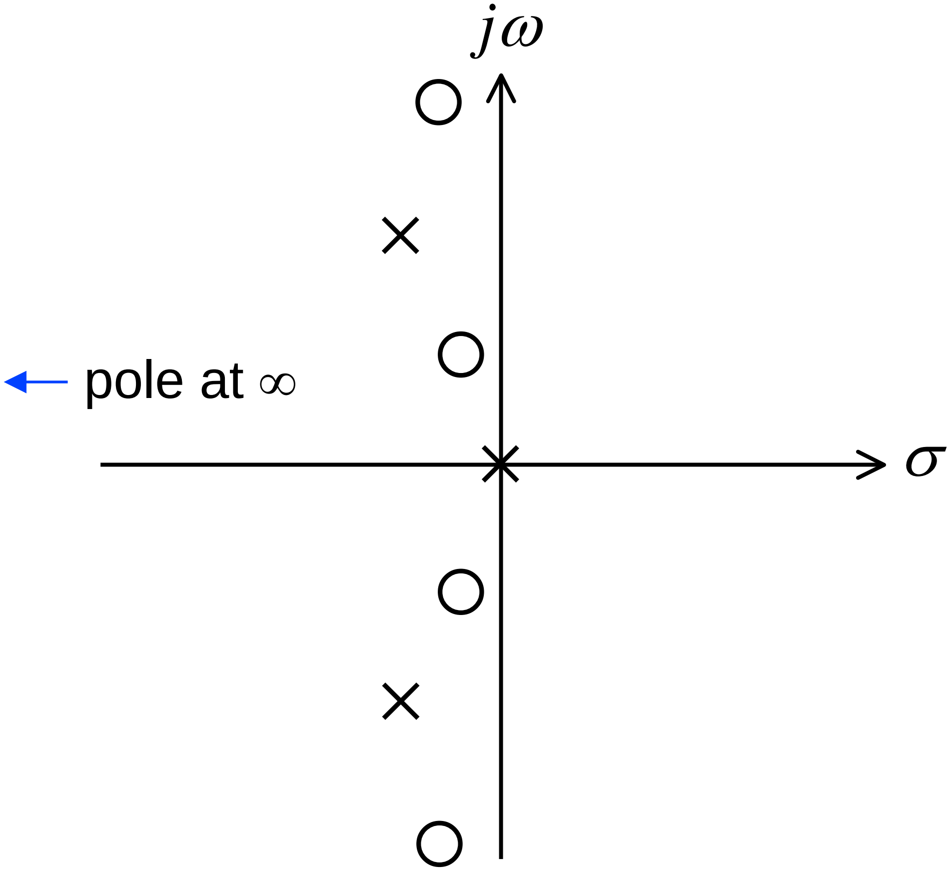
$$= sL_1 + \cfrac{1}{sC_1 + \cfrac{1}{\cfrac{1}{sC_2} + \cfrac{1}{\cfrac{1}{sL_2} + \cfrac{1}{R}}}}$$



*L1 = 945 nH*
*C1 = 12.5 pF*
*C2* = 39.0 pF
*L2* = 26.7 μH
*R* = 8,992 Ω

- ☐ ***A five-element equivalent circuit in Darlington form !***
- ☐ ***1 pole at the origin, 1 pole at infinity, 1 pair conjugate poles, 2 pairs of conjugate zeros***

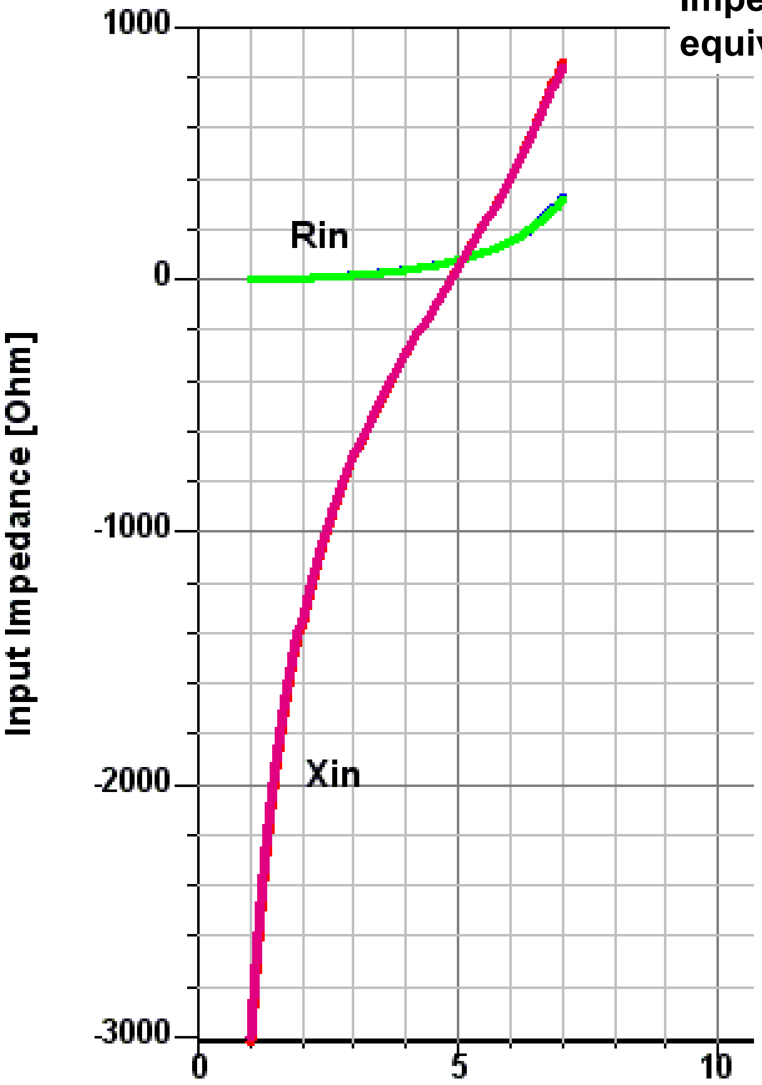
# *Apply Positive Real Tests*

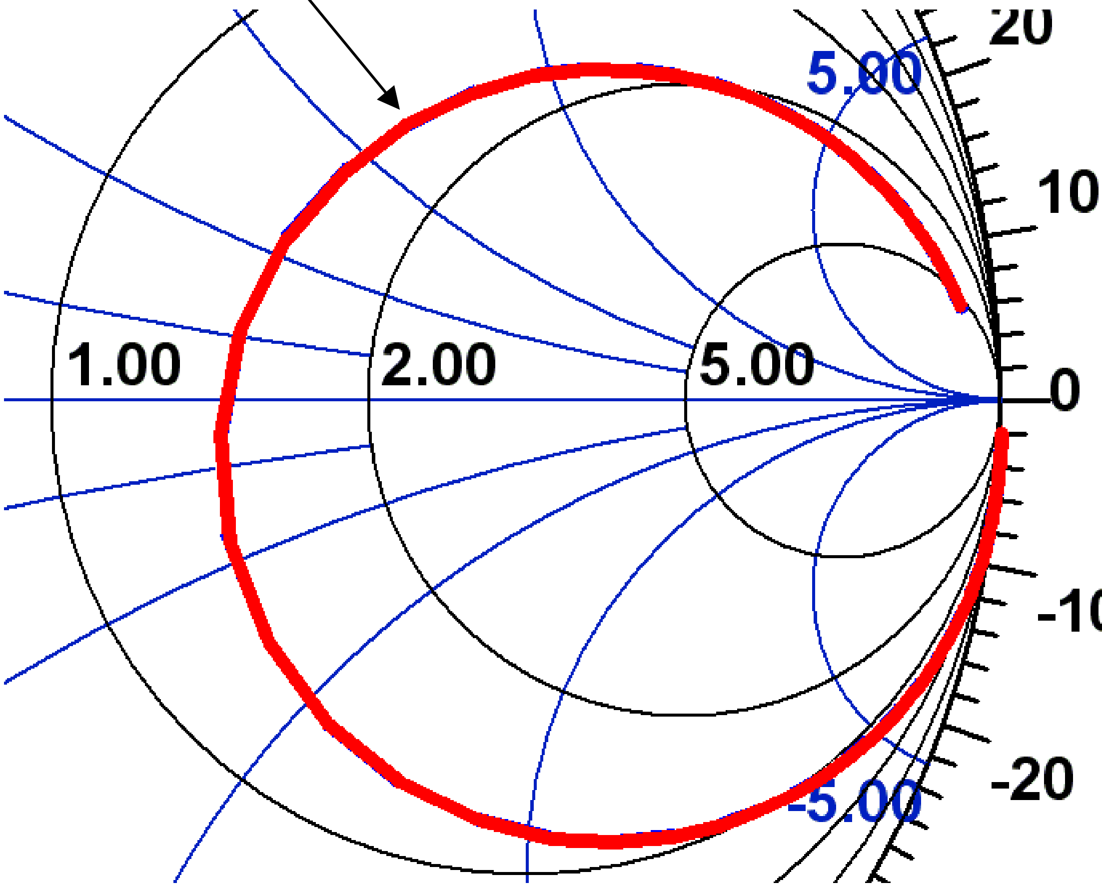


| | |
|---|---|
| ***$Z_A$ analytic in RHP*** | ☑<br>***pass*** |
| ***$Z_A$ real if s is real*** | ☑<br>***pass*** |
| ***Poles on $j\omega$ axis are simple and have positive real residues*** | ☑<br>***pass*** |
| ***Real part of $Z_A \geq 0$ on $j\omega$ axis*** | ☑<br>***pass*** |

☺

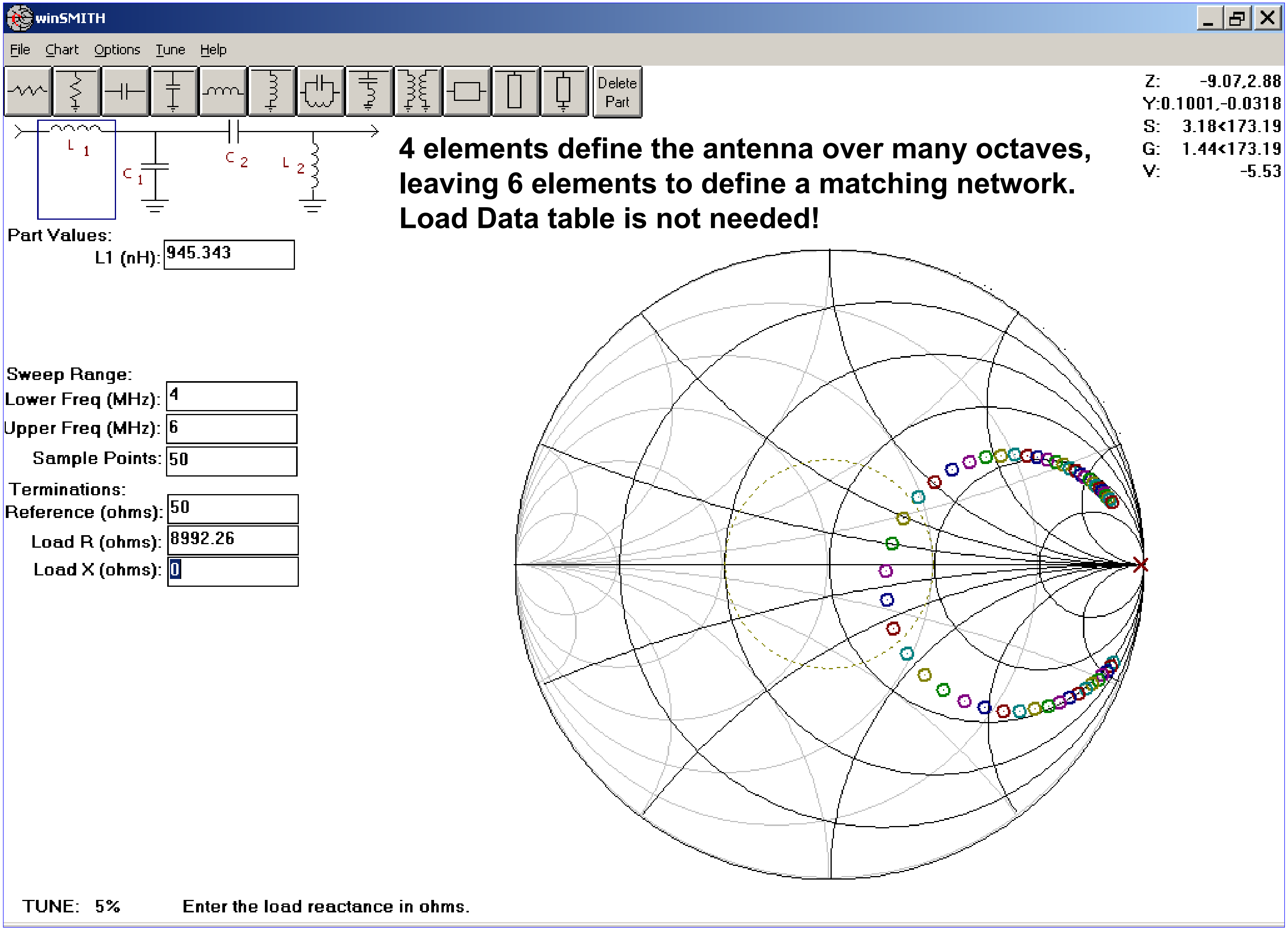
# *Accuracy of 5-Element Equivalent Circuit*





**Impedance of reference dipole and equivalent circuit coincide perfectly**

# Dipole Model in winSMITH



**4 elements define the antenna over many octaves, leaving 6 elements to define a matching network. Load Data table is not needed!**

Z: -9.07,2.88
Y:0.1001,-0.0318
S: 3.18<173.19
G: 1.44<173.19
V: -5.53

Part Values:
L1 (nH): 945.343

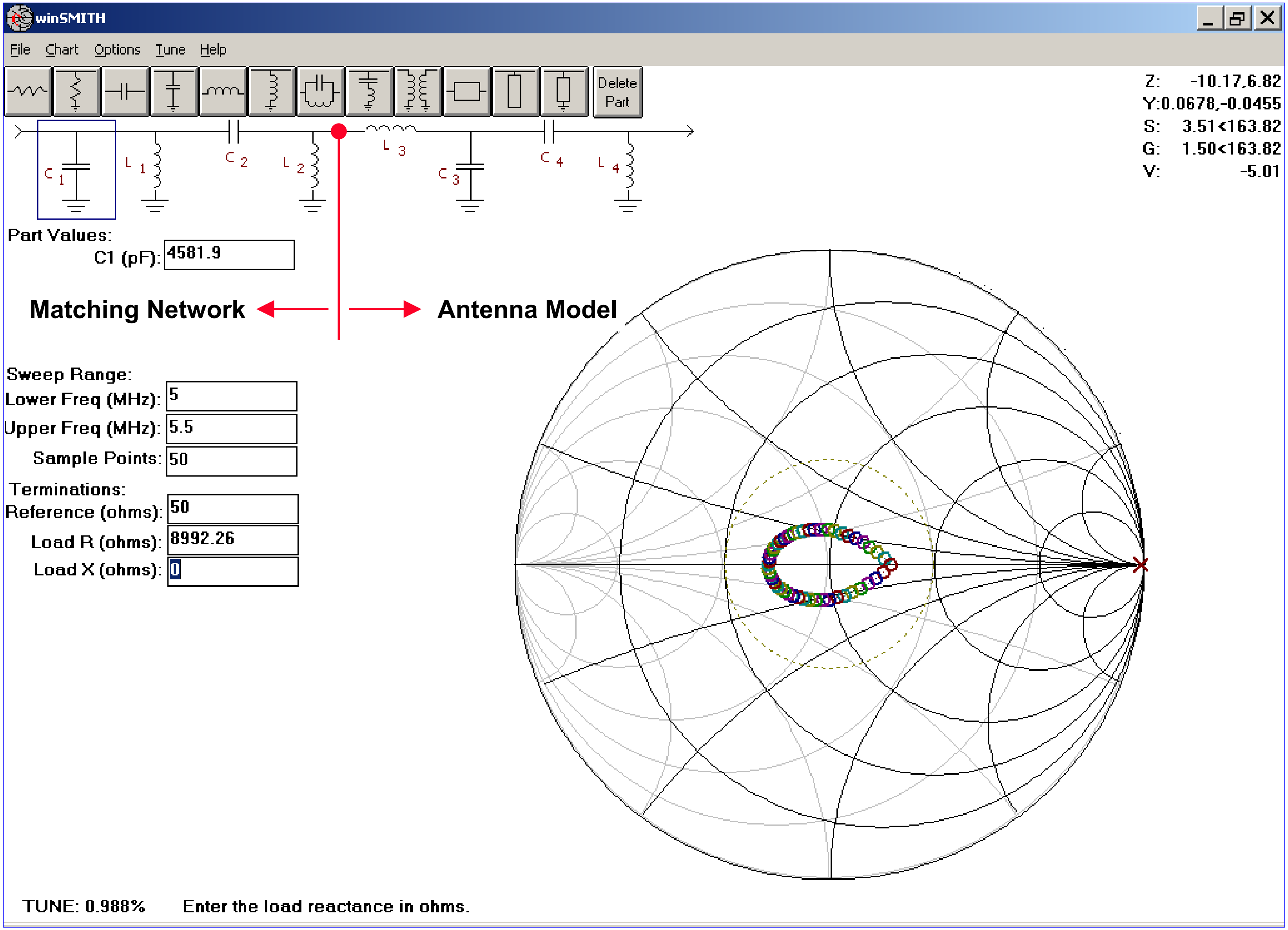
Sweep Range:
Lower Freq (MHz): 4
Upper Freq (MHz): 6
Sample Points: 50

Terminations:
Reference (ohms): 50
Load R (ohms): 8992.26
Load X (ohms): 0

TUNE: 5%

Enter the load reactance in ohms.

# *Matching Network Design in winSMITH*
## *5 MHz to 5.5 MHz, VSWR < 1.48*



Z: -10.17,6.82
Y:0.0678,-0.0455
S: 3.51<163.82
G: 1.50<163.82
V: -5.01

Part Values:
C1 (pF): 4581.9

Sweep Range:
Lower Freq (MHz): 5
Upper Freq (MHz): 5.5
Sample Points: 50

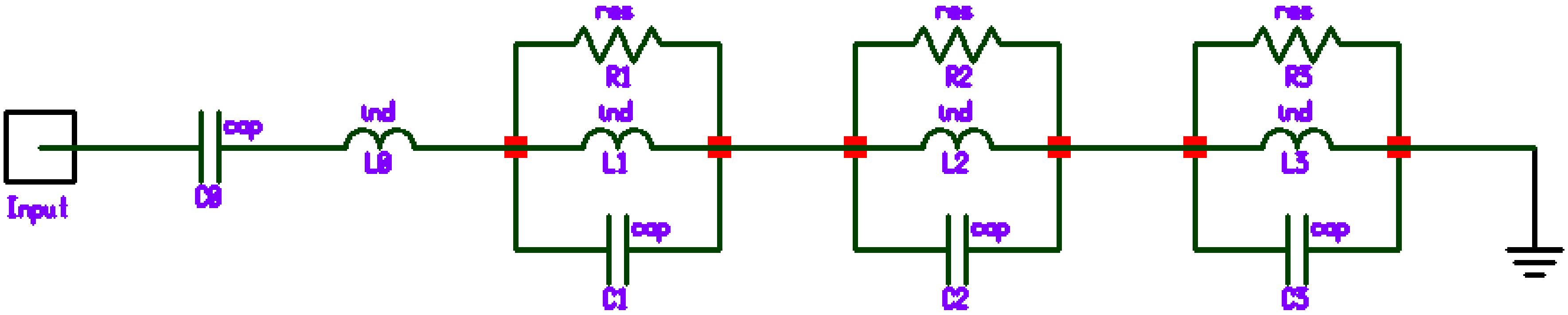
Terminations:
Reference (ohms): 50
Load R (ohms): 8992.26
Load X (ohms): 0

TUNE: 0.988%    Enter the load reactance in ohms.

# ***Broadband Models of Dipole Impedance***

*Spanning Multiple Resonances and Antiresonances*

# *Hamid & Hamid's Broadband Equivalent Circuit (1997)*



Example Dipole:
$C0$ = 43.9 pF
$L\infty$ = 4.49 μH

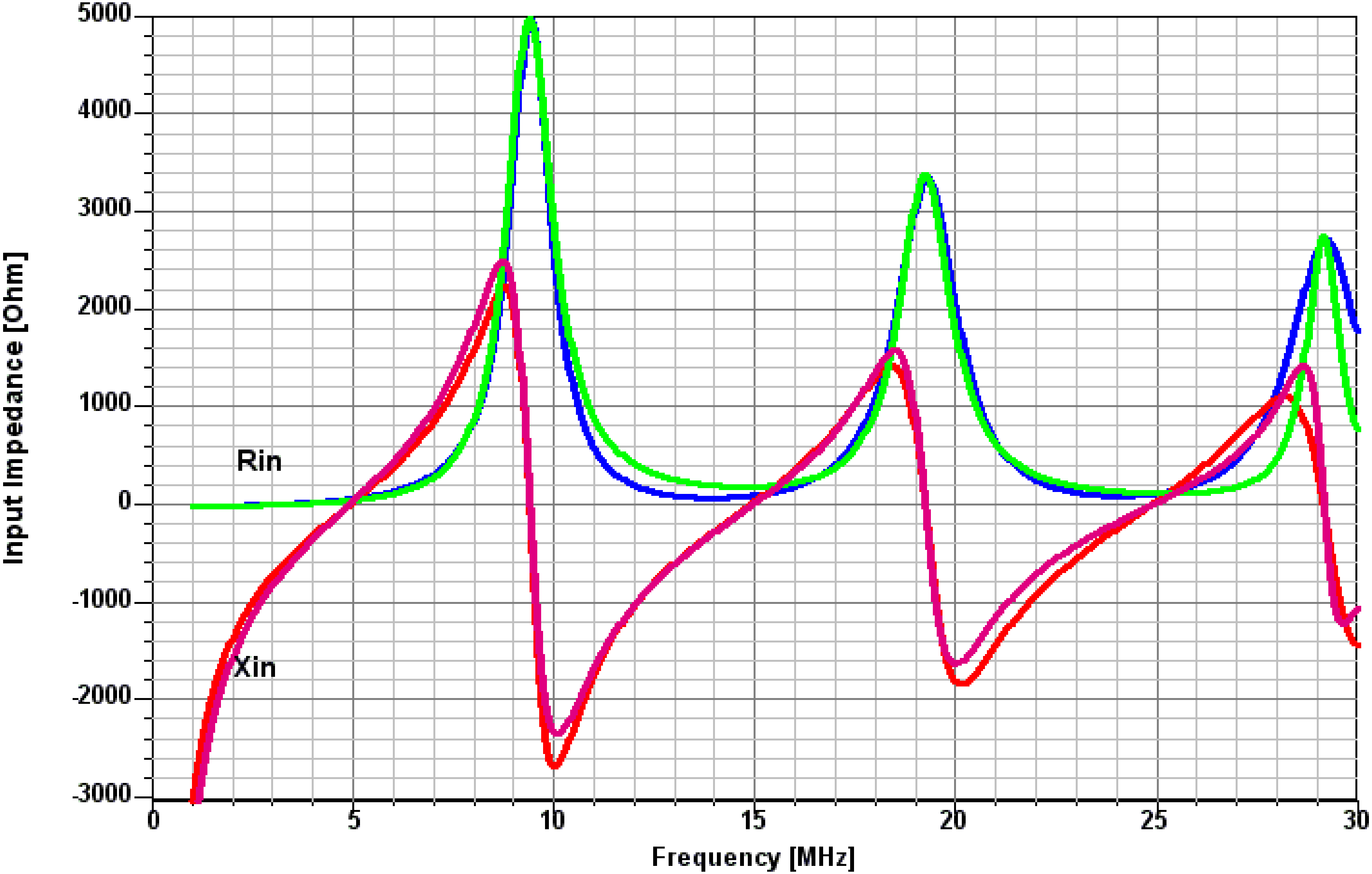
$C1$ = 22.9 pF
$L1$ = 12.5 μH
$R1$ = 4,970 Ω

$C2$ = 30.3 pF
$L2$ = 2.26 μH
$R2$ = 3,338 Ω

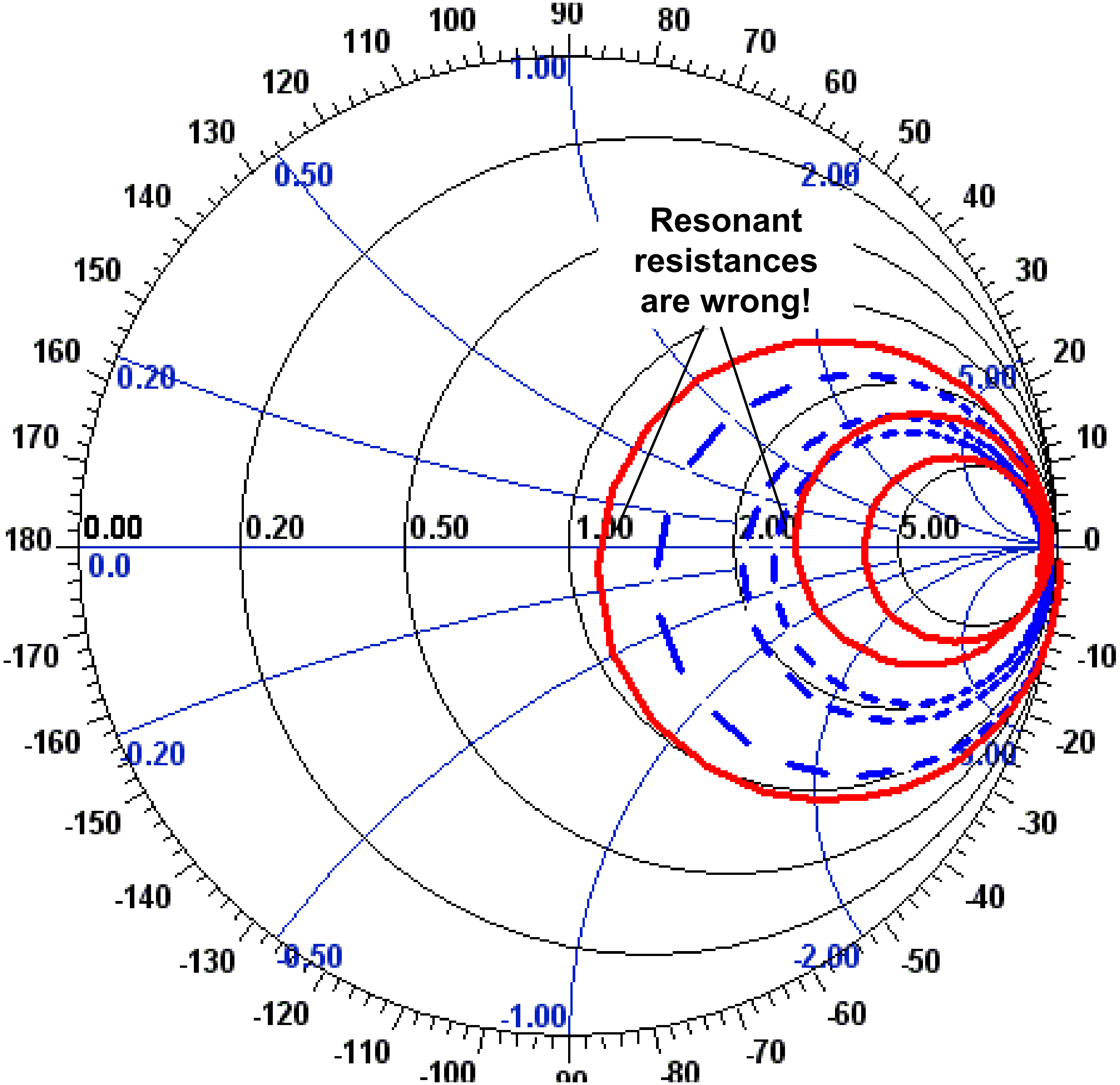
$C3$ = 57.1 pF
$L3$ = 522 nH
$R3$ = 2,702 Ω

- ***Foster's 1st canonical form with small losses added***
- ***Fits dipole impedance best near antiresonances***
- ***Reference: Ramo, Whinnery, and Van Duzer, Fields and Waves in Communication Electronics, Wiley, 1965, Section 11.13***

# *Accuracy of Hamid & Hamid's Equivalent Circuit*

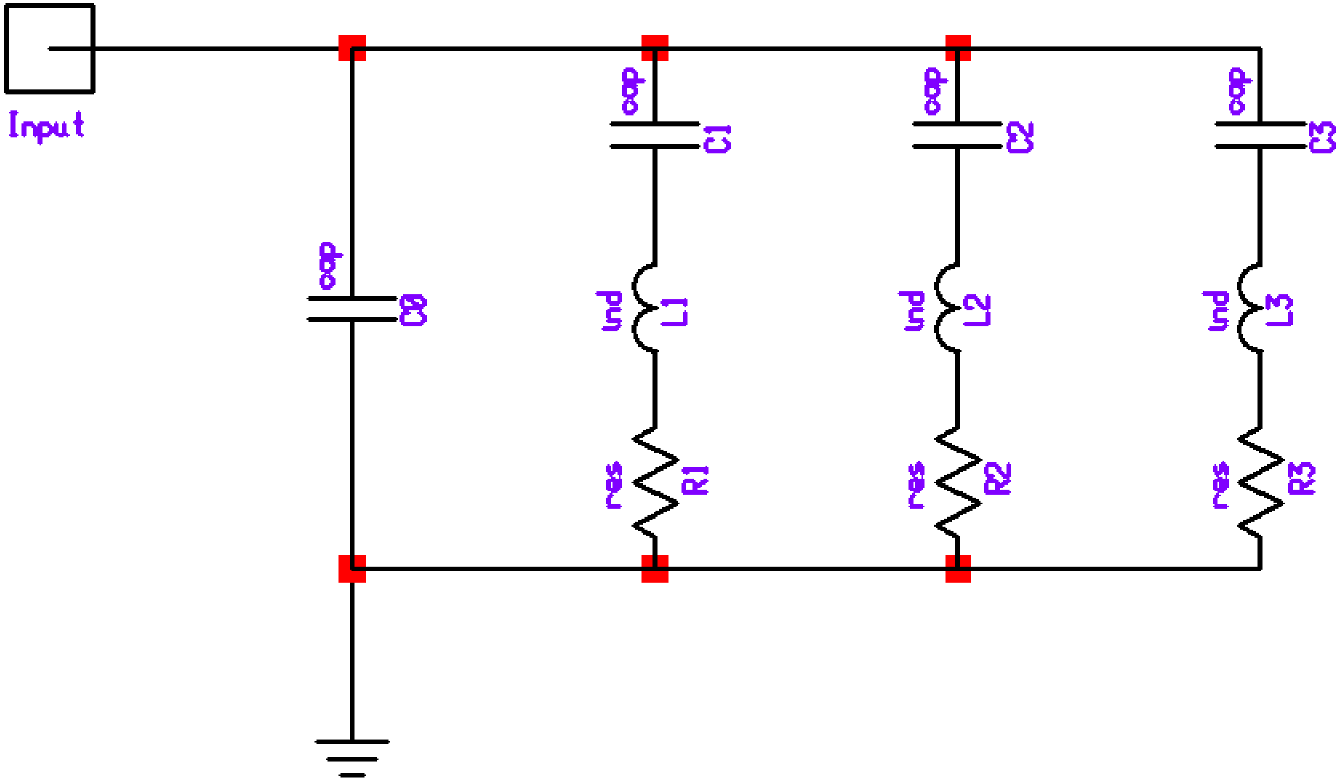


# Accuracy of Hamid & Hamid’s Equivalent Circuit



Resonant resistances are wrong!

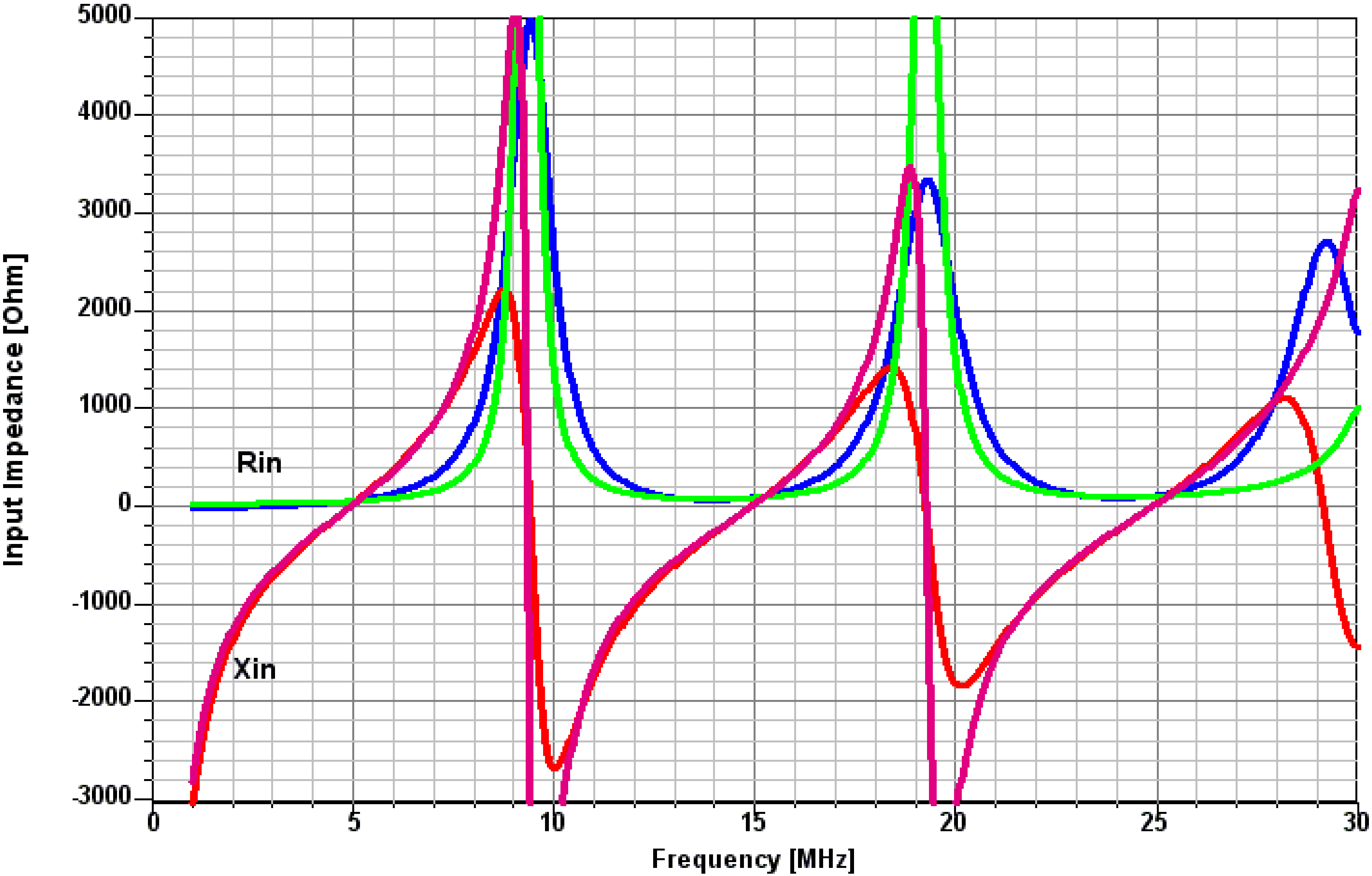
# Foster's 2nd Canonical Form with Small Losses Added



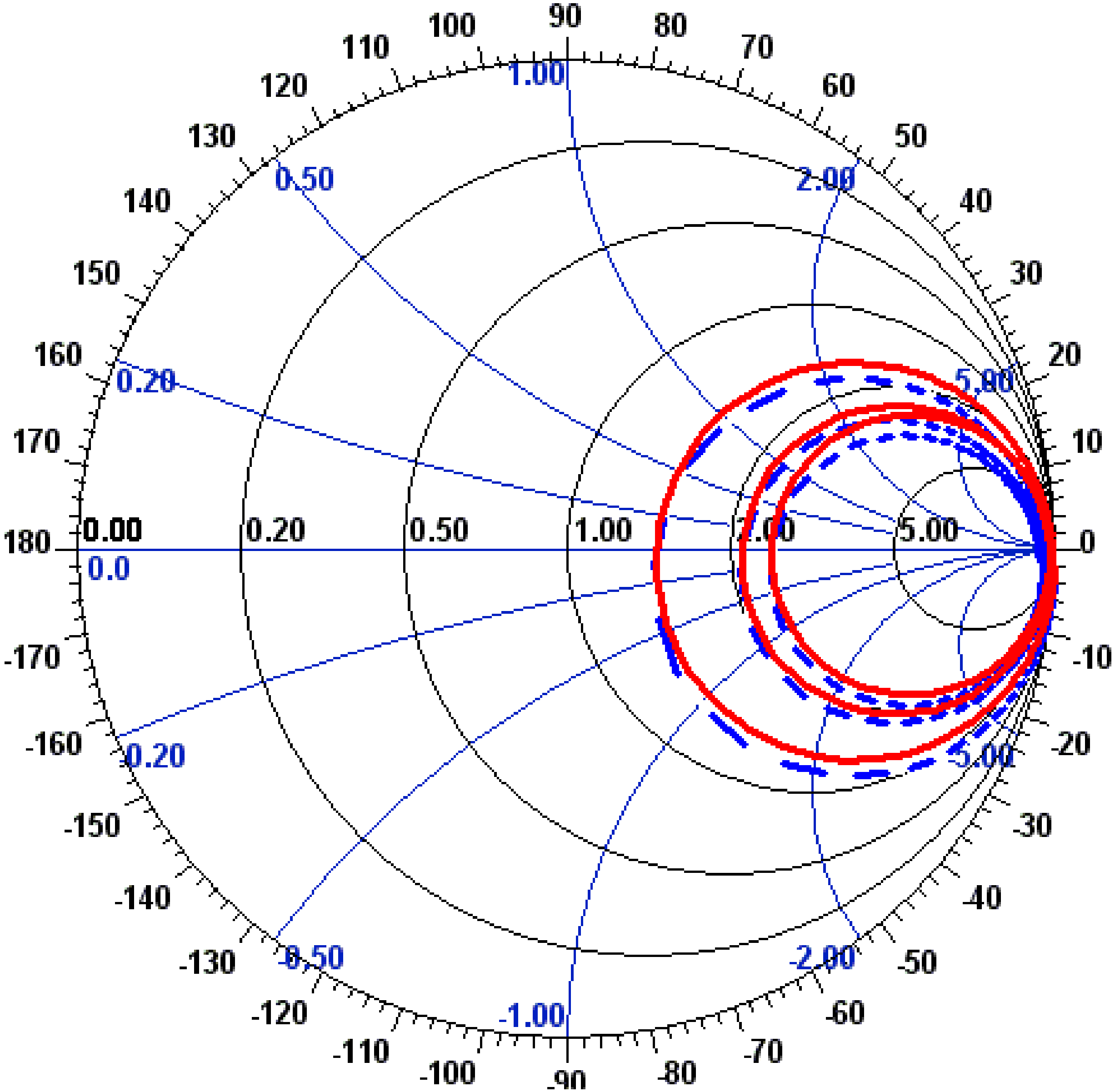
Example Dipole
$C\infty = 5.44$ pF
$C1 = 42.9$ pF
$C2 = 5.05$ pF
$C3 = 1.92$ pF
$L0 = \infty$
$L1 = 24.9\ \mu\mathrm{H}$
$L2 = 22.8\ \mu\mathrm{H}$
$L3 = 21.4\ \mu\mathrm{H}$
$R1 = 72.2\ \Omega$
$R2 = 106\ \Omega$
$R3 = 122\ \Omega$

- ***Fits dipole impedance best near resonances***
- ***Reference: Ramo, Whinnery, and Van Duzer, Fields and Waves in Communication Electronics, Wiley, 1965, Section 11.13***

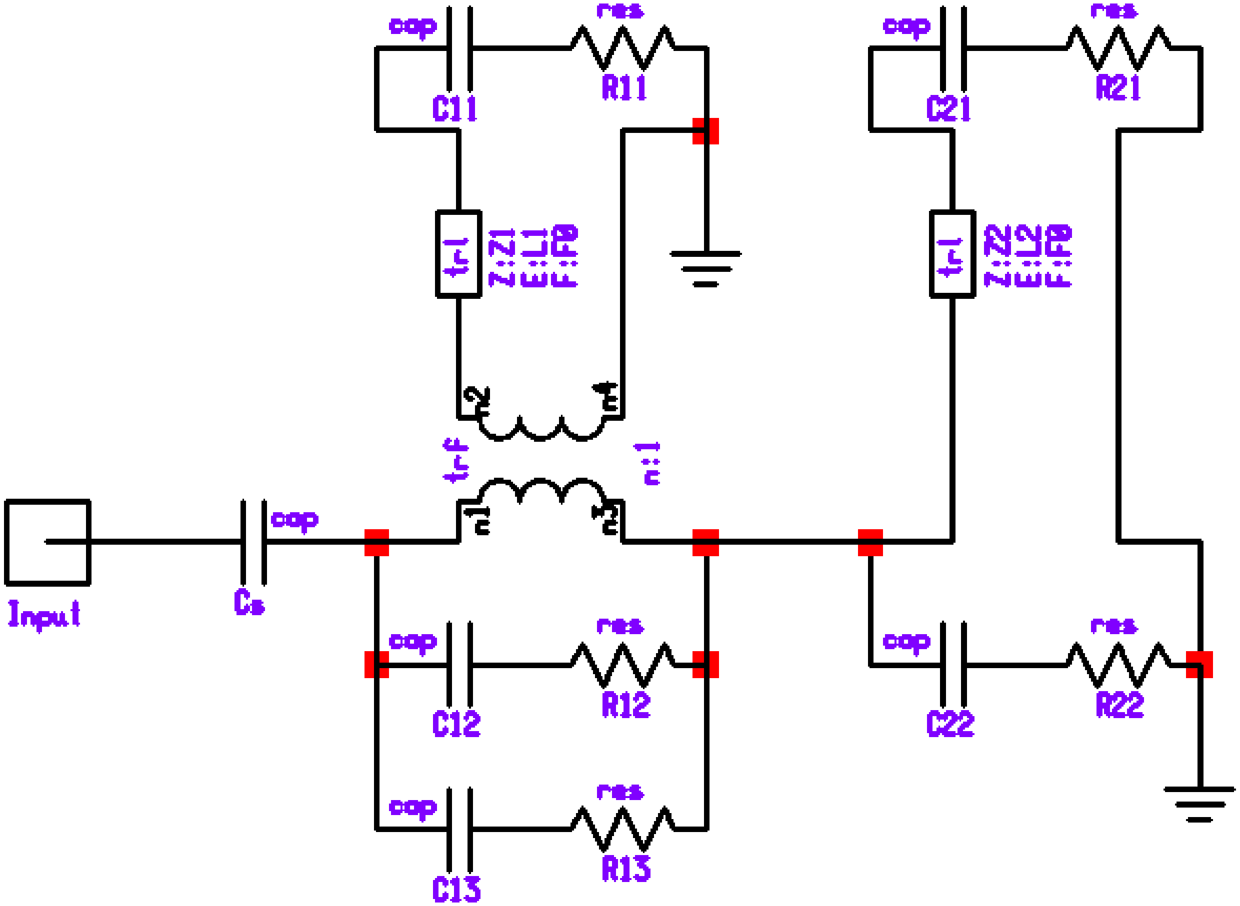
# Accuracy of Foster's 2nd Form With Small Losses



# Accuracy of Foster's 2nd Form With Small Losses



# Long, Werner, & Werner’s Broadband Model (2000)
# Frequency Scaled to $f_0 = 5$ MHz, $\Omega' = 7.8$



*Cs* = 150 pF

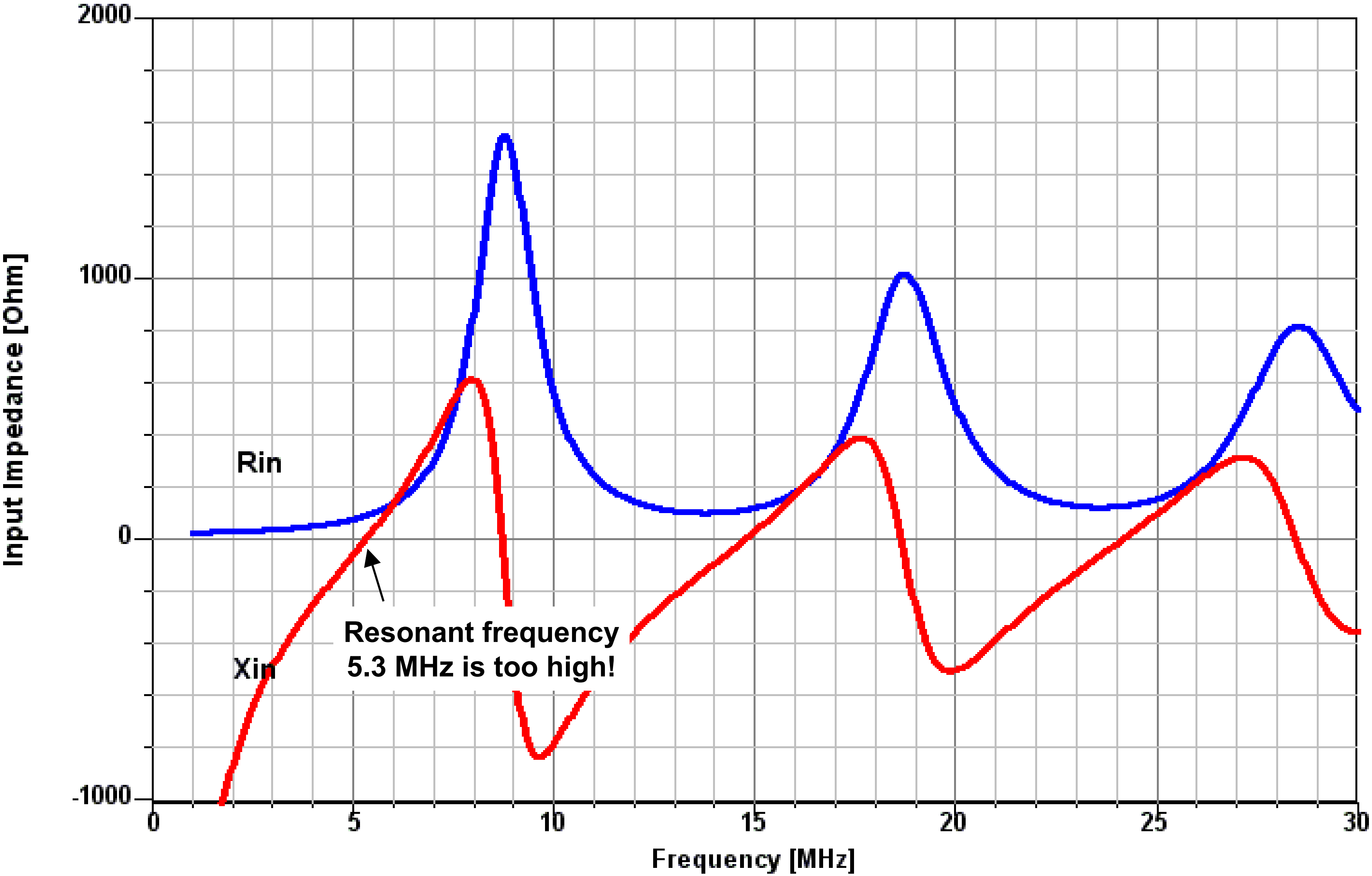
*C*11 = -975 pF
*Z*1 = 215 Ω
*C*12 = 24.0 pF
*C*13 = 8.33 pF

*R*11 = 13.1 Ω
*E*1 = 44.9 deg
*R*12 = 3,600 Ω
*R*13 = 500 Ω

*C*21 = 17.6 pF
*Z*2 = 195 Ω
*C*22 = -3.00 pF

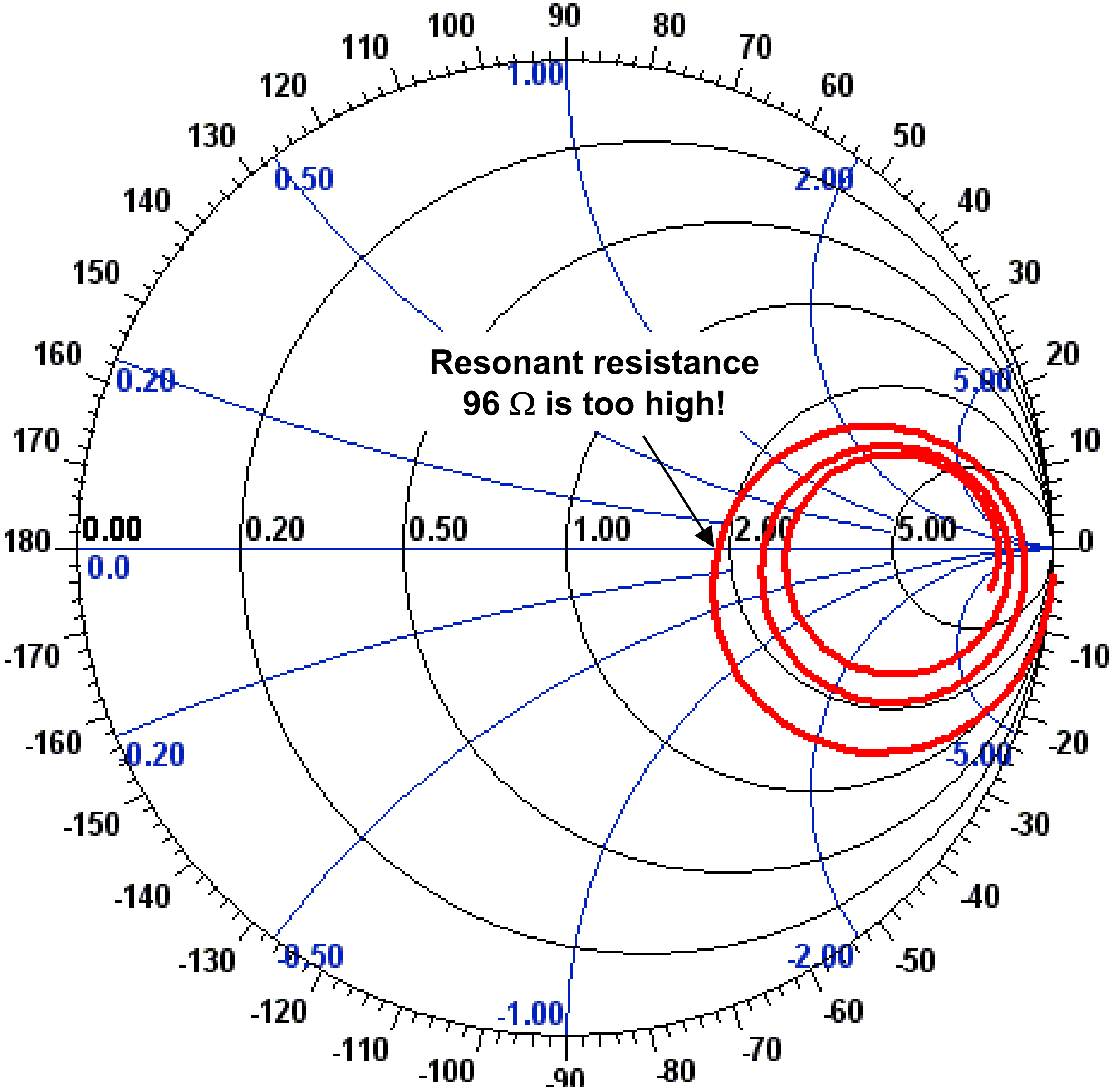
*R*21 = 700 Ω
*E*2 = 46.9 deg
*R*22 = 295 Ω

# *Accuracy of Long, Werner, & Werner's Model*



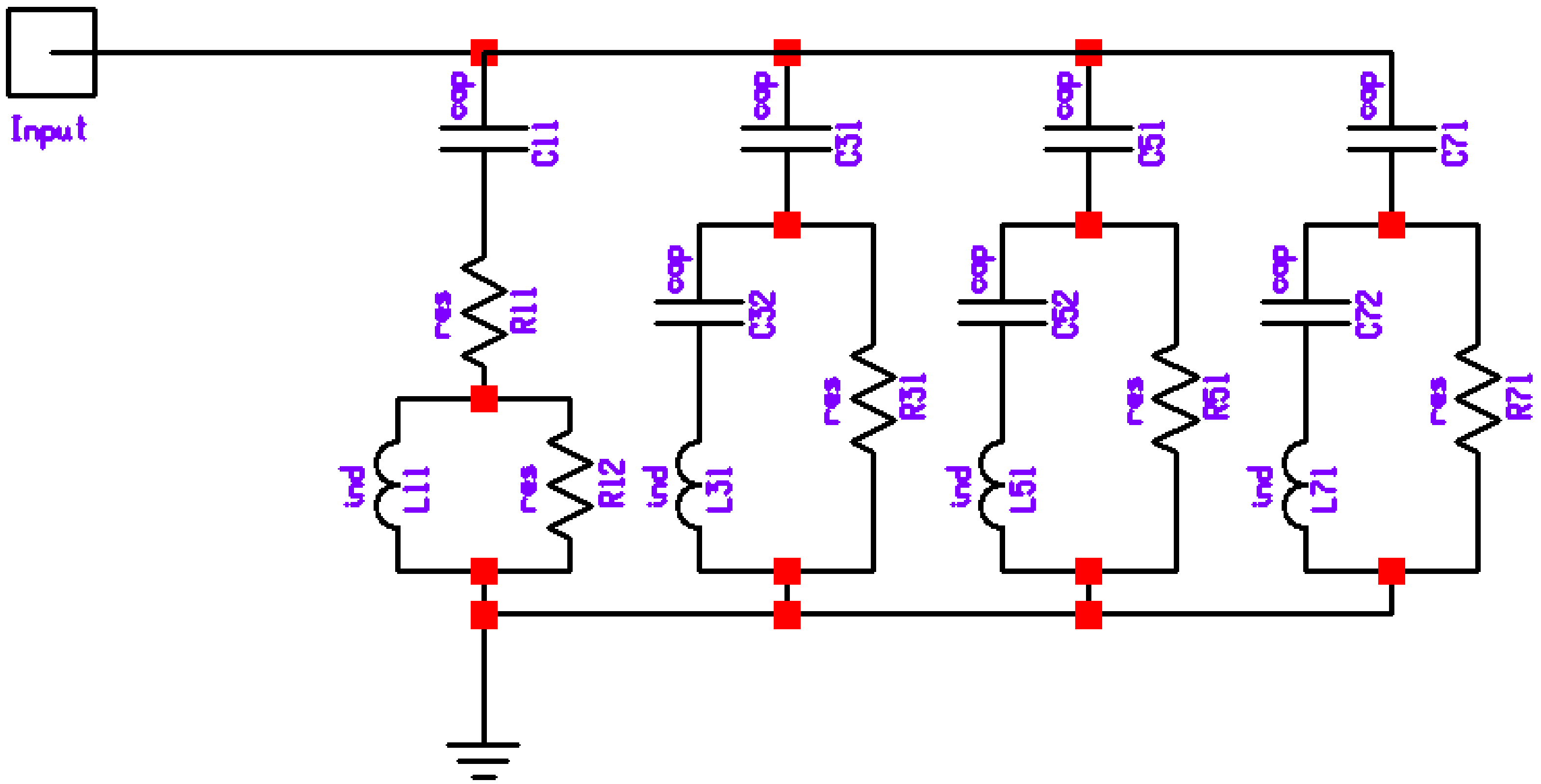
Resonant frequency
5.3 MHz is too high!

# *Accuracy of Long, Werner, & Werner's Model*



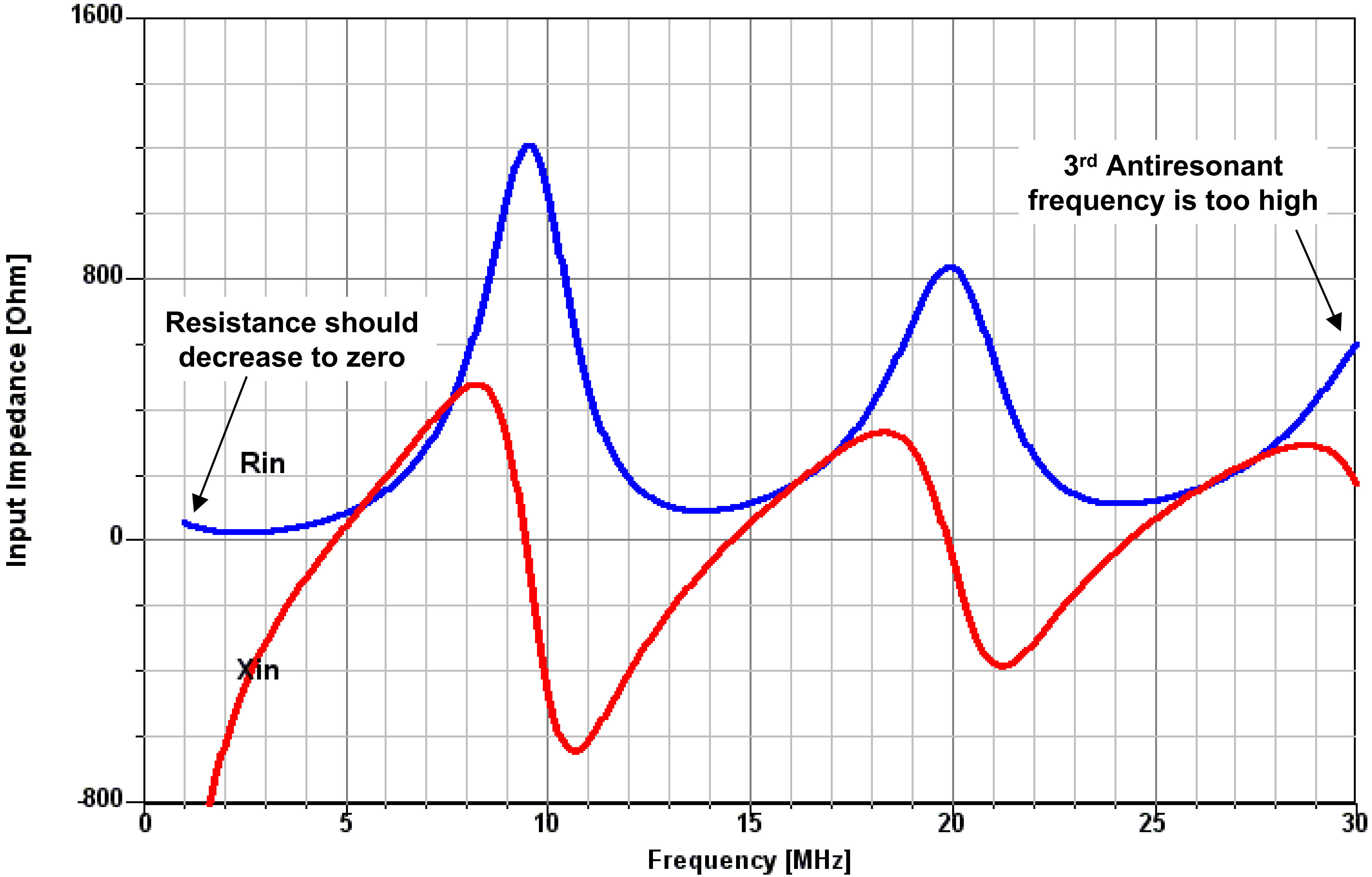
**Resonant resistance**
**96 Ω is too high!**

# *Streable & Pearson's Broadband Equivalent Circuit (1981)*
# *Frequency Scaled to $f_0$ = 5 MHz, $\Omega'$ = 10.6*



| | | | |
|---|---|---|---|
| $C11$ = 86.6 pF | $C31$ = 15.0 pF | $C51$ = 7.17 pF | $C71$ = 4.51 pF |
| $L11$ = 13.8 µH | $C32$ = 33.8 pF | $C52$ = 8.87 pF | $C72$ = 3.98 pF |
| $R11$ = 0.663 Ω | $L31$ = 11.7 µH | $L51$ = 10.9 µH | L71 = 10.3 µH |
| $R12$ = 2,201 Ω | $R31$ = 4,959 Ω | $R51$ = 6,514 Ω | $R71$ = 7,542 Ω |

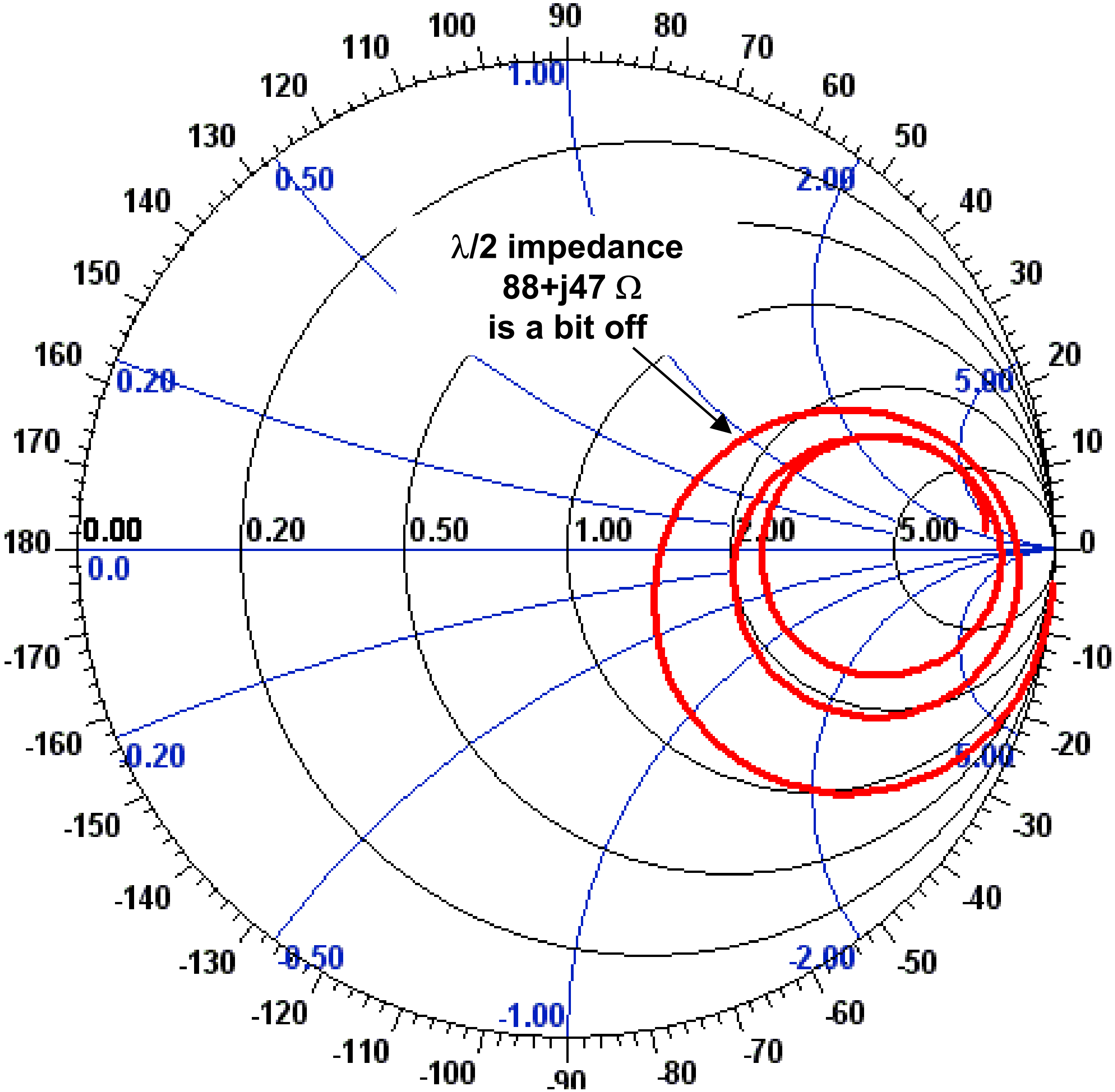
# *Accuracy of Streable & Pearson's Equivalent Circuit*



Resistance should decrease to zero

3rd Antiresonant frequency is too high

# Accuracy of Streable & Pearson’s Equivalent Circuit



λ/2 impedance
88+j47 Ω
is a bit off

# *Comparison of Antenna Impedance Models*

| *Antenna Impedance Model* | *Approximation Accuracy* | *Realizable Equivalent Circuit* | *Darlington Form* | *Element Types* | *Maximum Frequency Range* |
|---|---|---|---|---|---|
| *Series R L C* | *fair* | *yes* | *yes* | *R, L, C* | *0.94 $f_0$ to 1.05 $f_0$* |
| *Witt model* | *good* | *no* | *yes* | *variable resistor, TL stub* | *0.6 $f_0$ to 1.2 $f_0$* |
| *K6OIK 3-Element* | *good* | *yes* | *yes* | *R, L, C* | *0.90 $f_0$ to 1.08 $f_0$* |
| *K6OIK 5-Element* | *excellent* | *yes* | *yes* | *R, L, C* | *DC to 1.4 $f_0$* |
| *Hamid-Hamid* | *poor* | *yes* | *no* | *R, L, C* | *no limit* |
| *Fosters 2nd Form with small losses* | *fair, best near resonances* | *yes* | *no* | *R, L, C* | *no limit* |
| *Long-Werner-Werner* | *fair* | *no* | *no* | *R, C, TL* | *5 octaves* |
| *Streable-Pearson* | *excellent* | *yes* | *no* | *R, L, C* | *no limit* |

# *Ansoft Serenade SV vs ARRL Radio Designer Lessons Learned*

- ☐ ***ARD runs on the netlists generated in Serenade SV with simple modifications to observe ARD restrictions***
  - ➢ ARD restricts names and labels to 8 characters (no spaces)
- ☐ ***Serenade SV's optimizer runs faster than ARD's***
- ☐ ***ARD's optimizer gives better answers than Serenade SV***
  - ➢ ARD 6 digits; Serenade SV 5 digits
- ☐ ***ARD accepts goals on S, Y, or Z matrices, but only one; Serenade SV accepts compound goals***
- ☐ ***Serenade SV accepts data in files or data blocks; ARD uses only data blocks***
- ☐ ***Serenade SV creates the 1st line of a data block of the form***

  Antdata:  IMP  INTP = CUB
- ☐ ***ARD accepts the 1st line of a data block of the form***

  Antdata:  Z  RI  INTP = CUB   (but apparently ignores INTP = CUB)

# *Summary and Conclusions*

- ☐ ***Classical series and parallel RLC approximations of dipoles at resonance and antiresonance are good over very limited bandwidth***
- ☐ ***Approximations of an immittance function can be realizable or not***
- ☐ ***Realizable approximations can be converted to equivalent circuits***
- ☐ ***Two new narrowband approximations for dipole impedance near resonance have been obtained by network synthesis***
  - ➢ Lumped-element RLC networks having 3 and 5 elements
  - ➢ The 5-element network is an extremely accurate fit to the dipole
  - ➢ Darlington form – single resistor terminates lossless 2-port
  - ➢ Stage set for Fano bound analysis
- ☐ ***Broadband, multiple-resonance models were compared***
  - ➢ Streable-Pearson is best equivalent circuit

# References

- ***S. Ramo, J. R. Whinnery, and T. Van Duzer, Fields and Waves in Communication Electronics, Wiley, 1967***
- ***R. F. Harrington, “Matrix Methods for Field Problems,” Proc. IEEE, vol. 55, no. 2, pp. 136-149, Feb. 1967***
- ***G. W. Streable and L. W. Pearson, “A Numerical Study on Realizable Broad-Band and Equivalent Admittances for Dipole and Loop Antennas,” IEEE Trans. AP, vol. 29, no. 5, pp. 707-717, Sept. 1981***
- ***F. Witt, “Broadband Matching with the Transmission Line Resonator” and “Optimizing the 80-Meter Dipole,” ARRL Antenna Compendium, Vol. 4, pp. 30-48, American Radio Relay League, 1995***
- ***M. Hamid and R. Hamid, “Equivalent Circuit of Dipole Antenna of Arbitrary Length,” IEEE Trans. AP, vol. 45, no. 11, pp. 1695-1696, Nov. 1997***
- ***B. Long, P. Werner, and D. Werner, “ A Simple Broadband Dipole Equivalent Circuit Model,” Proc. IEEE Int’l Symp. Antennas and Propagation, vol. 2, pp. 1046-1049, Salt Lake City, July 16-21, 2000***

# *Tomorrow's Presentation*

- ☐ ***Hot topics in antenna engineering today***
  - ➢ PBG/EBG, metamaterials, and twisted light
- ☐ ***Design of impedance matching networks for arbitrary antenna impedance functions***
  - ➢ Perfect matching is always possible at any number of discrete frequencies
  - ➢ Networks for single-frequency matching
  - ➢ Networks for multiple-frequency matching
- ☐ ***The theoretical (Fano) limit on matching a series RLC antenna impedance model***
  - ➢ Perfect matching is impossible over a continuous band of frequencies, even with networks of infinite complexity!
  - ➢ How close can simple networks get to the limit?
- ☐ ***Design software demo***
  - ➢ Network design procedures
- ☐ ***Lots of examples***

# The End