A Transmission Line Power Paradox and Its Resolution

Steve Stearns, K6OIK

Technical Fellow
Northrop Grumman
Electromagnetic Systems Laboratory
San Jose, California
stearns@ieee.org
k6oik@arrl.net

ARRL Pacificon Presentations by K6OIK

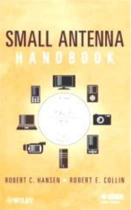
Archived at
http://www.fars.k6ya.org

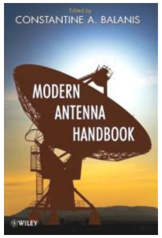
1999	Mysteries of the Smith Chart	
2000	Jam-Resistant Repeater Technology: Signal Separation, Identification, Routing, Control, and Automatic Logging	
2001	Mysteries of the Smith Chart	✓
2002	How-to-Make Better RFI Filters Using Stubs	
2003	Twin-Lead J-Pole Design	
2004	Antenna Impedance Models – Old and New	✓
2005	Novel and Strange Ideas in Antennas and Impedance Matching	
2006	Novel and Strange Ideas in Antennas and Impedance Matching 2	✓
2007	New Results on Antenna Impedance Models and Matching	✓
2008	Antenna Modeling for Radio Amateurs	✓
2009	(convention held in Reno)	
2010	Facts About SWR, Reflected Power, and Power Transfer on Real Transmission Lines with Loss	✓
2011	Conjugate Match Myths	✓
2012	Transmission Line Filters Beyond Stubs and Traps	✓
2013	Bode, Chu, Fano, Wheeler: Antenna Q and Match Bandwidth	✓
2014	A Transmission Line Power Paradox and Its Resolution	✓

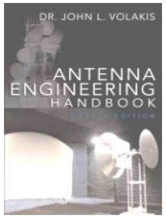
Outline

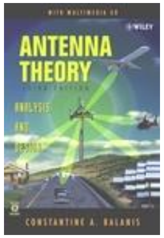
- **Antenna books**
- **Software**
  - Antenna modeling
  - Circuit design
  - Smith chart
- **Transmission line theory – a brief review**
  - Distributed parameters
  - Complex characteristic impedance and propagation constant
  - Attenuation constant and phase constant
  - Published formula for line loss with mismatched load
- **The Loss Paradox**
- **The Resolution**
  - Power delivery myth
  - Improved formulas for line loss with mismatched load
- **References**

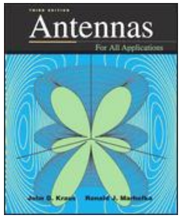
Favorite Antenna Books





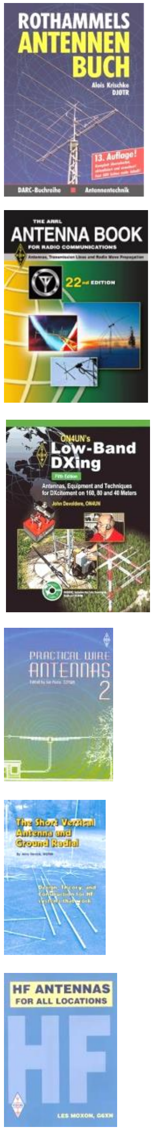




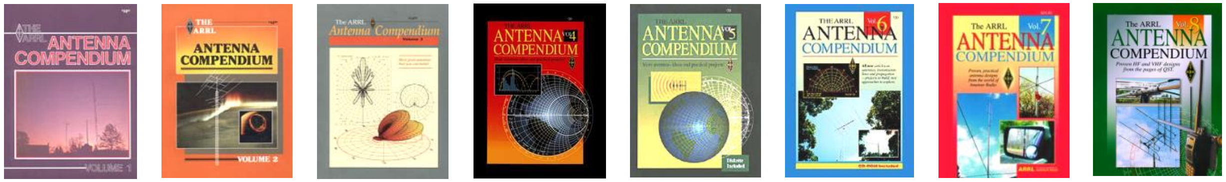


- **Books for antenna engineers and students**
  - R.C. Hansen and R.E. Collin, *Small Antenna Handbook*, Wiley, 2011, ISBN 0470890835
  - *Modern Antenna Handbook*, C.A. Balanis editor, Wiley, 2008, ISBN 0470036346
  - *Antenna Engineering Handbook*, 4th ed., J.L. Volakis editor, McGraw-Hill, 2007, ISBN 0071475745. First published in 1961, Henry Jasik editor
  - C.A. Balanis, *Antenna Theory*, 3rd ed., Wiley, 2005, ISBN 047166782X. First published in 1982 by Harper & Row
  - J.D. Kraus and R.J. Marhefka, *Antennas*, 3rd ed., McGraw-Hill, 2001, ISBN 0072321032. First published in 1950
  - S.J. Orfanidis, *Electromagnetic Waves and Antennas*, draft textbook online at http://www.ece.rutgers.edu/~orfanidi/ewa/
  - E.A. Laport, *Radio Antenna Engineering*, McGraw-Hill, 1952 http://snulbug.mtview.ca.us/books/RadioAntennaEngineering
  - G.V. Ayzenberg, *Shortwave Antennas*, 1962, transl. from Russian, DTIC AD0706545. http://www.dtic.mil/dtic/tr/fulltext/u2/706545.pdf
- **Antenna research papers**
  - IEEE AP-S Digital Archive, 2001-2009 (1 DVD), JD0307
  - IEEE AP-S Digital Archive, 2001-2006 (1 DVD), JD0304
  - IEEE AP-S Digital Archive, 2001-2003 (1 DVD), JD0301
  - IEEE AP-S Digital Archive, 1952-2000 (2 DVDs), JD0351

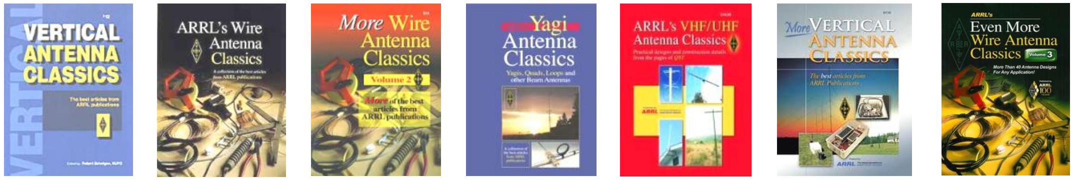
Favorite Antenna Books continued



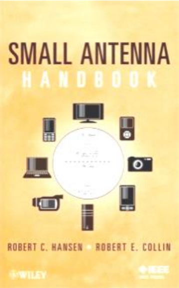
- **Books for Radio Amateurs**
  - *Rothammel's Antennenbuch*, 13th ed., A. Krischke (DJ0TR) editor, DARC Verlag, 2013
  - *ARRL Antenna Book*, 22nd ed., H.W. Silver (N0AX) editor, American Radio Relay League, 2011, ISBN 087259694X
  - J. Devoldere (ON4UN), *ON4UN's Low-Band Dxing*, 5th ed., American Radio Relay League, 2011, ISBN 087259856X
  - *Practical Wire Antennas 2*, I. Poole (G3YWX) editor, Radio Society of Great Britain, 2005, ISBN 1905086040
  - J. Sevick (W2FMI), *The Short Vertical Antenna and Ground Radial*, CQ Communications, 2003, ISBN 0943016223
  - L. Moxon (G6XN), *HF Antennas for All Locations*, 2nd ed., Radio Society of Great Britain, 1983, ISBN 1872309151
- **ARRL Antenna Compendium series – eight volumes**



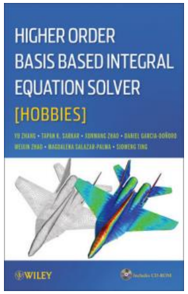
- **ARRL Antenna Classics series – seven titles**



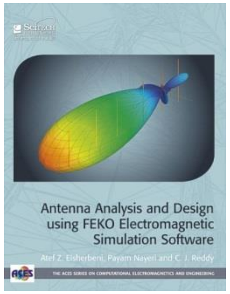
Recent Antenna Books of Interest



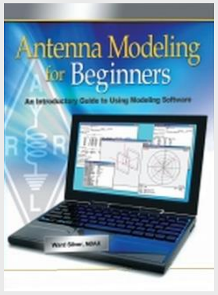
R.C. Hansen and R.E. Collin, *Small Antenna Handbook*, Wiley, 2011



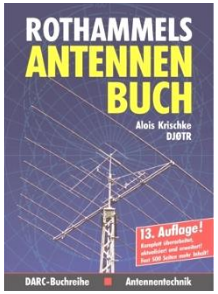
Y. Zhang et al., *Higher Order Basis Based Integral Equation Solver* (*HOBBIES*), Wiley, 2012



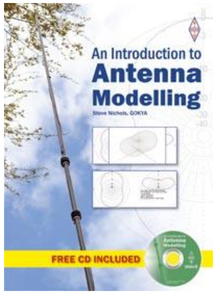
Elsherbeni et al., *Antenna Analysis and Design using FEKO Electromagnetic Simulation Software*, SciTech, 2014



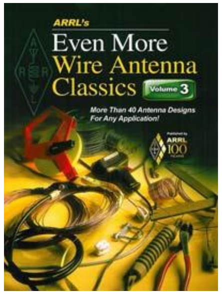
H.W. Silver, N0AX, *Antenna Modeling for Beginners*, ARRL, 2012



A. Krischke, DJ0TR, ed., *Rothammels Antennen Buch*, 13th ed, DARC, 2013



S. Nichols G0KYA, *An Introduction to Antenna Modelling*, RSGB, 2014



Even More Wire Antenna Classics vol. 3, ARRL, 2014

Software

Antenna Modeling Programs for Radio Amateurs

- ***EZNEC*** **http://www.eznec.com**
  - EZNEC v.5 demo program Free
  - EZNEC-ARRL v.3 & v.4 $50 (on ARRL Antenna Book CD-ROM)
  - EZNEC v.5 $90
  - EZNEC+ v.5 $140
  - EZNEC Pro/2 v.5 $500
  - EZNEC Pro/4 v.5 $650 (sold only to NEC-4 licensees)
- ***4nec2*** **http://home.ict.nl/~arivoors/**
  - Free, 11,000 segments, two optimizers
- ***MMANA-GAL*** **http://hamsoft.ca/pages/mmana-gal.php**
  - Free Basic version 8,192 segs. Pro version 32,000 segs, $130
- ***NEC-4*** **https://ipo.llnl.gov/data/assets/docs/nec.pdf**
  - Noncommercial user license $300
- ***FEKO Lite*** **http://www.feko.info**
  - Free LITE version, no time limit
- ***WIPL-D Lite*** **http://www.wipl-d.com**
  - Free 30-day full demo, or Lite version $449 (Artech House)
- ***HOBBIES*** **http://em-hobbies.com**
  - Similar to WIPL-D Lite but more capability, $149 to $217 online

Accessory Software for EZNEC

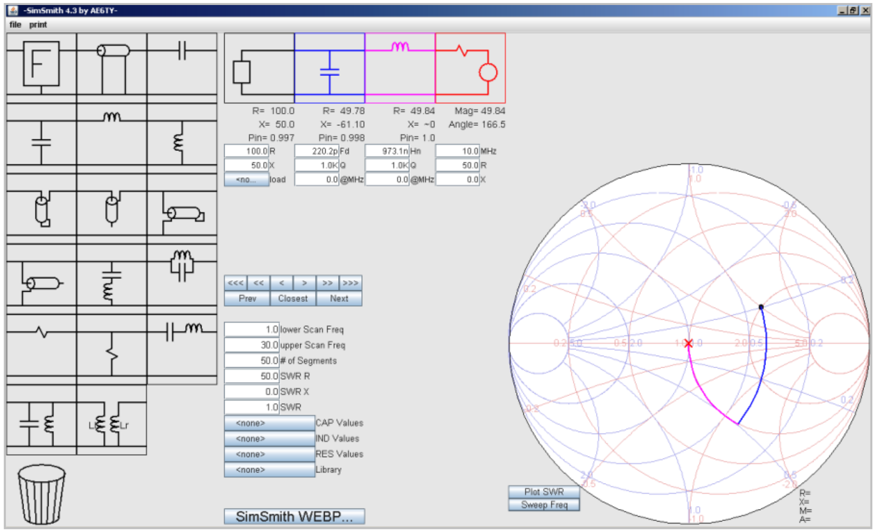
- ***AutoEZ* 2.0.8 by Dan Maguire, AC6LA, http://www.ac6la.com**
  - Visual Basic program $79, or free demo version (30 segment limit)
  - Controls EZNEC to make multiple runs
    - A GUI for a GUI for NEC
  - Requires Excel and EZNEC installed on computer
  - Has an optimizer – Nelder-Mead algorithm
  - Can read NEC, AO, and MMANA-GAL files
  - Doesn’t work with EZNEC-ARRL
  - Replaces MultiNEC which is no longer available

Software for Smith Charting and Network Design

- ***SimSmith*** **9.9 by Ward Harriman AE6TY**
  - Free download from http://ae6ty.com/Smith_Charts.html
- ***Smith Chart Calculator*** **2.1 by Gorik Stevens**
  - Free download from http://sourceforge.net/projects/gnssmithchart/
- ***Smith*** **3.10 by Fritz Dellsperger HB9AJY**
  - Free download from http://fritz.dellsperger.net
- ***QuickSmith*** **4.5 by Nathan Iyer KJ6FOJ**
  - Free download from http://www.nathaniyer.com/qsdw.htm
- ***JJSmith*** **2.10 by J. Bromley K7JEB and J. Tonne W4ENE (SK)**
  - Free download from http://tonnesoftware.com/jjsmith.html
- ***linSmith*** **by James Coppens ON6JC/LW3HAZ**
  - Free download from http://jcoppens.com/soft/linsmith/index.en.php
- ***XLZIZL*** **by Dan Maguire AC6LA, 2005. No longer available.**
- ***WinSMITH*** **2.0, Noble Publishing, 1995. No longer available.**
- ***MicroSmith*** **2.3, ARRL, 1992. No longer available.**

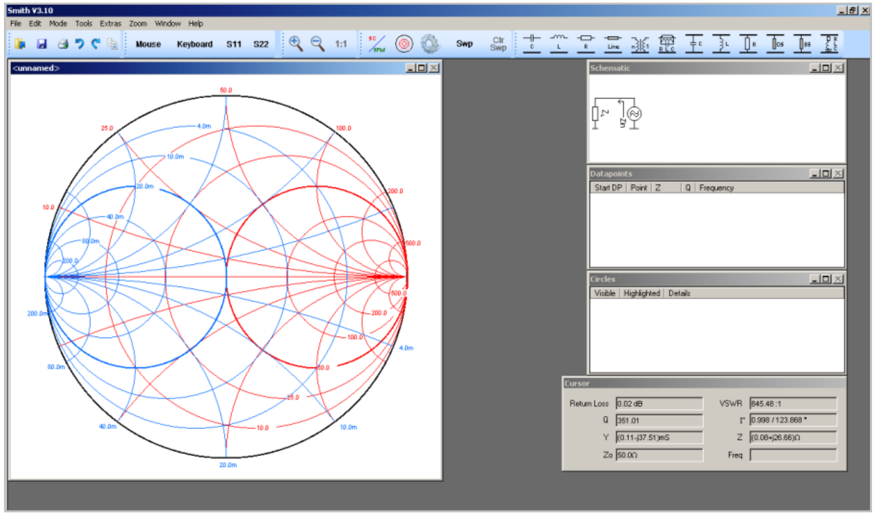
Smith Chart Programs for Ladder Network Design

SimSmith 9.9



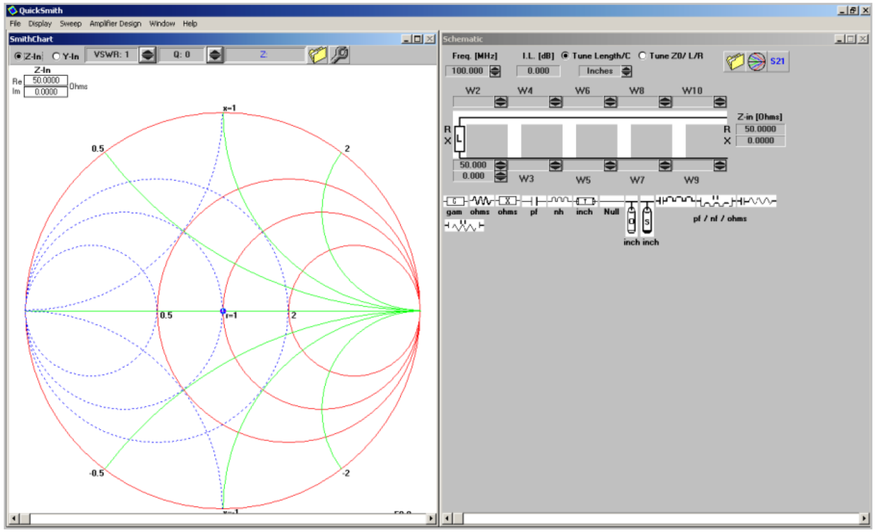
R= 100.0 X= 50.0 Pin= 0.997
R= 49.78 X= -61.10 Pin= 0.998
R= 49.84 X= ~0 Pin= 1.0
Mag= 49.84 Angle= 166.5
100.0 R
50.0 X
<no... load
220.2p Fd
1.0K Q
0.0 @MHz
973.1n Hn
1.0K Q
0.0 @MHz
10.0 MHz
50.0 R
0.0 X
1.0 lower Scan Freq
30.0 upper Scan Freq
50.0 # of Segments
50.0 SWR R
0.0 SWR X
1.0 SWR
<none> CAP Values
<none> IND Values
<none> RES Values
<none> Library

Smith 3.10



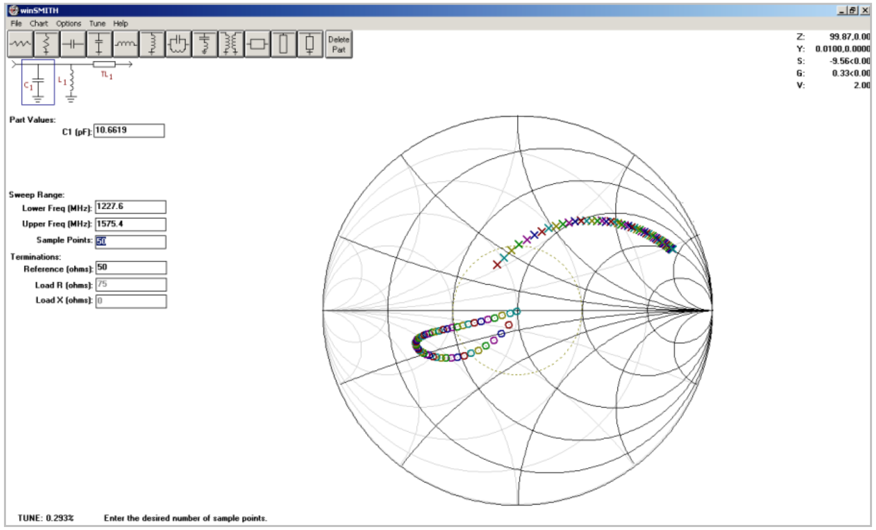
Return Loss 0.02 dB
VSWR 845.48 :1
Q 351.01
Γ 0.998 / 123.868°
Y (0.11-j37.51)mS
Z (0.08+j26.66)Ω
Zo 50.0Ω
Freq

QuickSmith 4.5



● Z-In ○ Y-In VSWR: 1 Q: 0
Z-In Re 50.0000 Im 0.0000 Ohms
Freq. [MHz] 100.000
I.L. [dB] 0.000
● Tune Length/C ○ Tune Z0/ L/R
Inches
Z-in [Ohms] R 50.0000 X 0.0000
50.000
0.000

winSMITH 2.0



Z: 99.87,0.00
Y: 0.0100,0.0000
S: -9.56<0.00
G: 0.33<0.00
V: 2.00

Part Values:
C1 (pF): 10.6619

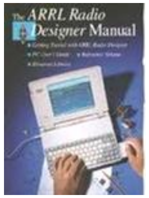
Sweep Range:
Lower Freq (MHz): 1227.6
Upper Freq (MHz): 1575.4
Sample Points: 50

Terminations:
Reference (ohms): 50
Load R (ohms): 75
Load X (ohms): 0

TUNE: 0.293% Enter the desired number of sample points.

General RF Circuit Design, Analysis, and Optimization

- **Software for Radio Amateurs**
  - *Quite Universal Circuit Simulator* (QUCS) 0.0.18, 2014
    - Free download from http://qucs.sourceforge.net
  - *Ansoft Serenade SV* 8.5 (student version), Ansoft, 2000. No longer available. See article by David Newkirk W9VES in *QST* Jan. 2001.
  - *ARRL Radio Designer* 1.5, ARRL, 1995. No longer available.



The ARRL Radio Designer Manual

- **Professional electronic design automation (EDA) software**
  - Agilent *Advanced Design System* (ADS)
  - Applied Wave Research *Microwave Office* (MWO)
  - ANSYS *Designer*

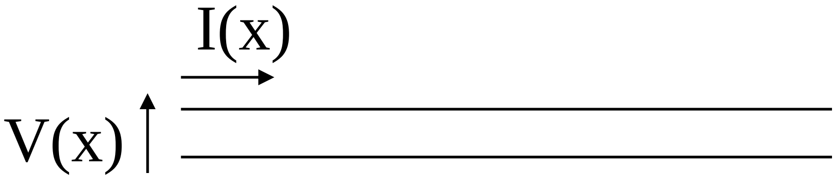
Transmission Line Theory

A Brief Review

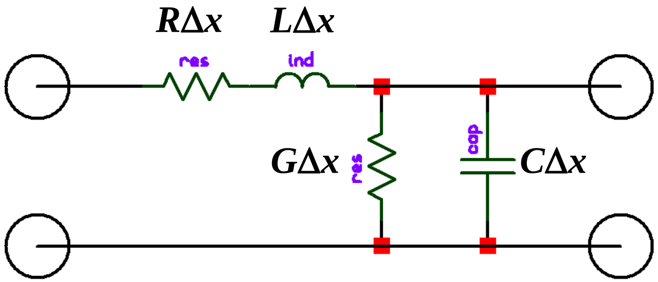
Oliver Heaviside, 1850-1925

Heaviside Telegrapher's Equations

Uniform transmission line



Infinitesimal segment



$$\left.\begin{aligned}\frac{dV}{dx} &= -(R+j\omega L)\,I(x)\\[2ex] \frac{dI}{dx} &= -(G+j\omega C)V(x)\end{aligned}\right\} \Rightarrow \left\{\begin{aligned}\frac{d^2V}{dx^2} &= (R+j\omega L)(G+j\omega C)V(x)\\[2ex] \frac{d^2I}{dx^2} &= (R+j\omega L)(G+j\omega C)\,I(x)\end{aligned}\right.$$

Transmission Line Solution: Waves

- **Two waves traveling in opposite directions**

$$V(x) = V_0^+ e^{\gamma x} \quad + \quad V_0^- e^{-\gamma x}$$

$$I(x) = \frac{V_0^+}{Z_0} e^{\gamma x} \quad - \quad \frac{V_0^-}{Z_0} e^{-\gamma x}$$

- **Propagation constant**

Attenuation per unit length

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

Phase per unit length

- **Characteristic impedance**

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 + jX_0$$

Distortionless Line (Oliver Heaviside, 1887)

- A line is *distortionless* if *R*, *L*, *G*, and *C* are constants independent of frequency and

$$\frac{R}{L}=\frac{G}{C} \quad \text{or} \quad RC=GL \quad \text{or} \quad \frac{R}{G}=\frac{L}{C}$$

- The first and second equations make clear that *dissipationless* or *lossless* line ($R = G = 0$) is also distortionless
- In most practical lines the insulation is so good that

$$\frac{R}{L}>\frac{G}{C} \quad \text{or} \quad RC>GL \quad \text{or} \quad \frac{R}{G}>\frac{L}{C}$$

Oliver Heaviside, “Simple Properties of the Ideally Perfect Telegraph Circuit,” Section XLI of the series “Electromagnetic Induction and Its Propagation,” *The Electrician*, p. 124, June 16, 1887

Seven Properties of Distortionless Line

- Characteristic impedance is purely real

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}} = R_0 + j0$$

- Characteristic impedance is independent of frequency
- Attenuation is independent of frequency
- Attenuation is minimum

$$\frac{\partial \alpha}{\partial L} = \frac{\partial \alpha}{\partial C} = 0$$

- Velocity of propagation is independent of frequency
- Velocity of propagation is maximum

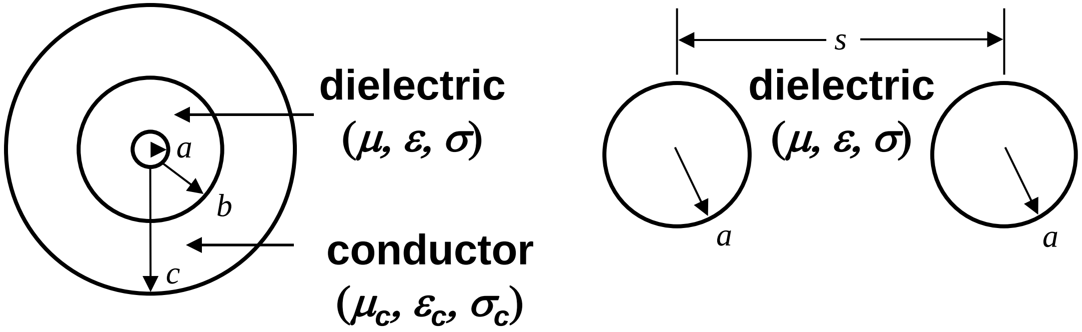
$$\frac{\partial \beta}{\partial R} = \frac{\partial \beta}{\partial L} = \frac{\partial \beta}{\partial G} = \frac{\partial \beta}{\partial C} = 0$$

☞

- Traveling waves account for power delivery along the line

$$P_{Delivered} = P_{Forward} - P_{Reverse}$$

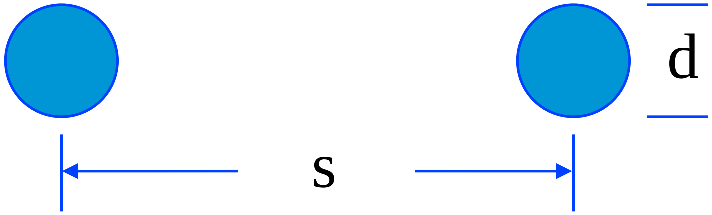
Transmission Line Distributed Parameters from Physical Dimensions and Material Properties



- **Parameter**

R Ω/m	$\frac{1}{2\pi\delta\sigma_c}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{1}{\pi a\delta\sigma_c}$
L H/m	$\frac{\mu}{2\pi}\left[\ln\frac{b}{a}+\frac{\delta}{2}\left(\frac{1}{a}+\frac{1}{b}\right)\right]$	$\frac{\mu}{\pi}\left[\frac{\delta}{2a}+\cosh^{-1}\frac{s}{2a}\right]$
G S/m	$\frac{2\pi\sigma}{\ln\frac{b}{a}}$	$\frac{\pi\sigma}{\cosh^{-1}\frac{s}{2a}}$
C F/m	$\frac{2\pi\varepsilon}{\ln\frac{b}{a}}$	$\frac{\pi\varepsilon}{\cosh^{-1}\frac{s}{2a}}$
δ m	$\frac{1}{\sqrt{\pi f\mu_c\sigma_c}}$	$\frac{1}{\sqrt{\pi f\mu_c\sigma_c}}$

Round Open-Wire Transmission Line (PEC in Air)



- **Accurate formula**
  - For high frequencies (assumes that skin depth is small $\delta << a$)

$$Z_0 = 119.917 \cosh^{-1}\left(\frac{s}{d}\right)$$

- **Approximate formula**
  - Assumes, in addition, large spacings: $s/d > 3$ or large impedances: Z_0 > several hundred

$$Z_0 = 120 \ln\left(\frac{2s}{d}\right) = 276 \log_{10}\left(\frac{2s}{d}\right)$$

Characteristic Impedance

- **Exact formula**

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}} \times \sqrt{\frac{1 + \frac{R}{j\omega L}}{1 + \frac{G}{j\omega C}}}$$

- **High-frequency, low-loss approximation**

$$Z_0 \approx \sqrt{\frac{L}{C}}\, e^{\frac{-j}{2}\left(\frac{R}{\omega L} - \frac{G}{\omega C}\right)}$$

$$|Z_0| \approx \sqrt{\frac{L}{C}}$$

$$R_0 \approx \sqrt{\frac{L}{C}} \cos\left(\frac{R}{2\omega L} - \frac{G}{2\omega C}\right)$$

$$X_0 \approx -\sqrt{\frac{L}{C}} \sin\left(\frac{R}{2\omega L} - \frac{G}{2\omega C}\right)$$

E.W. Kimbark, *Electrical Transmission of Power and Signals*, Wiley, 1949

Propagation Constant

- **Exact formula**

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega\, C)} = j\omega\sqrt{LC} \times \sqrt{1 + \frac{R}{j\omega L}} \times \sqrt{1 + \frac{G}{j\omega\, C}}$$

- **High-frequency, low-loss approximation**

$$\gamma \approx \omega\sqrt{LC}\; e^{j\left[\frac{\pi}{2} - \left(\frac{R}{2\omega L} + \frac{G}{2\omega C}\right)\right]}$$

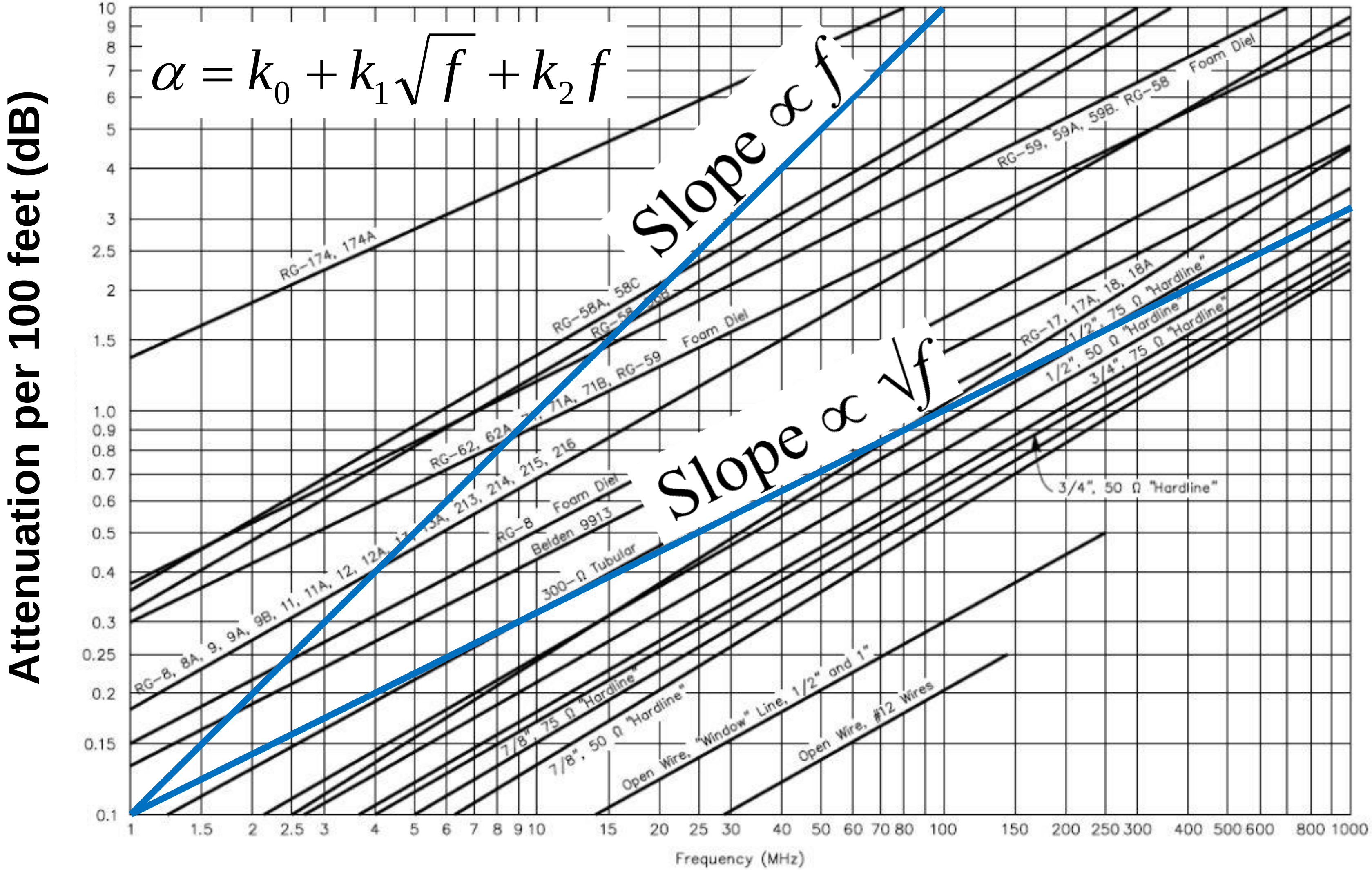
$$\alpha \approx \omega\sqrt{LC}\,\sin\left(\frac{R}{2\omega L} + \frac{G}{2\omega\, C}\right) \approx \frac{1}{2}\left(\frac{R}{|Z_0|} + G\,|Z_0|\right)$$

$$\beta \approx \omega\sqrt{LC}$$

$$v_p = \frac{\omega}{\beta} \approx \frac{1}{\sqrt{LC}}$$

E.W. Kimbark, *Electrical Transmission of Power and Signals*, Wiley, 1949

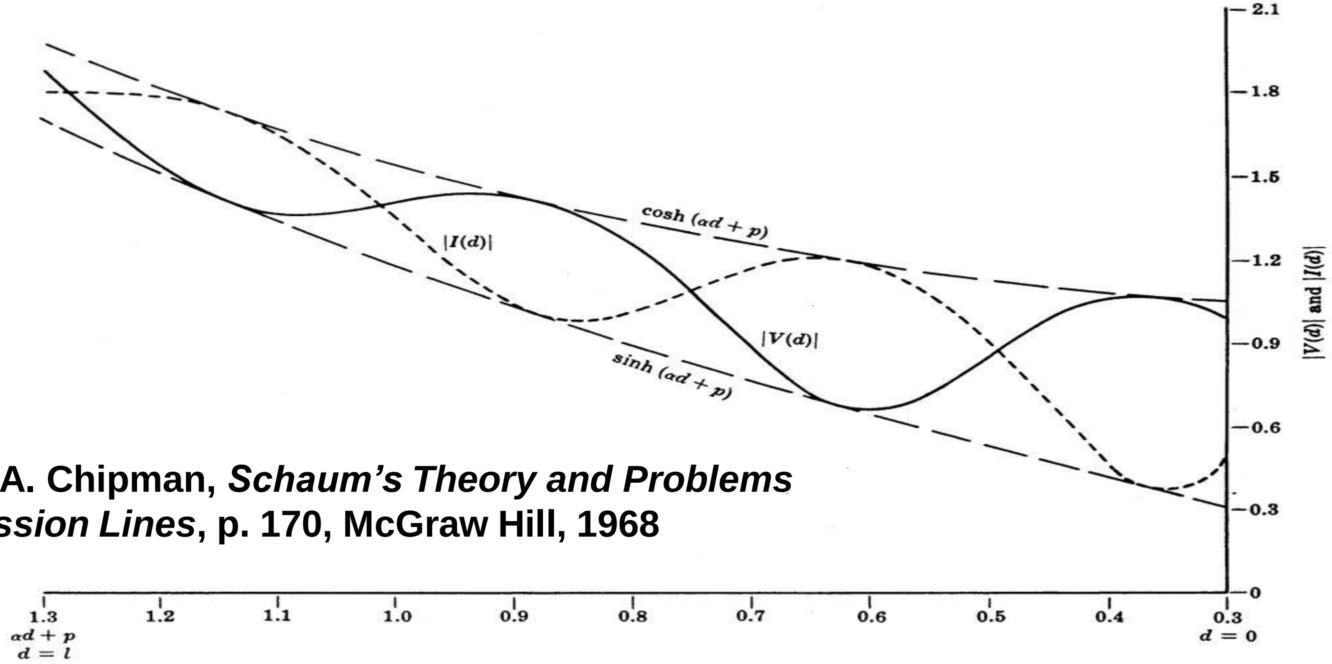
Attenuation Constant of Common Transmission Lines



$$\alpha = k_0 + k_1\sqrt{f} + k_2 f$$

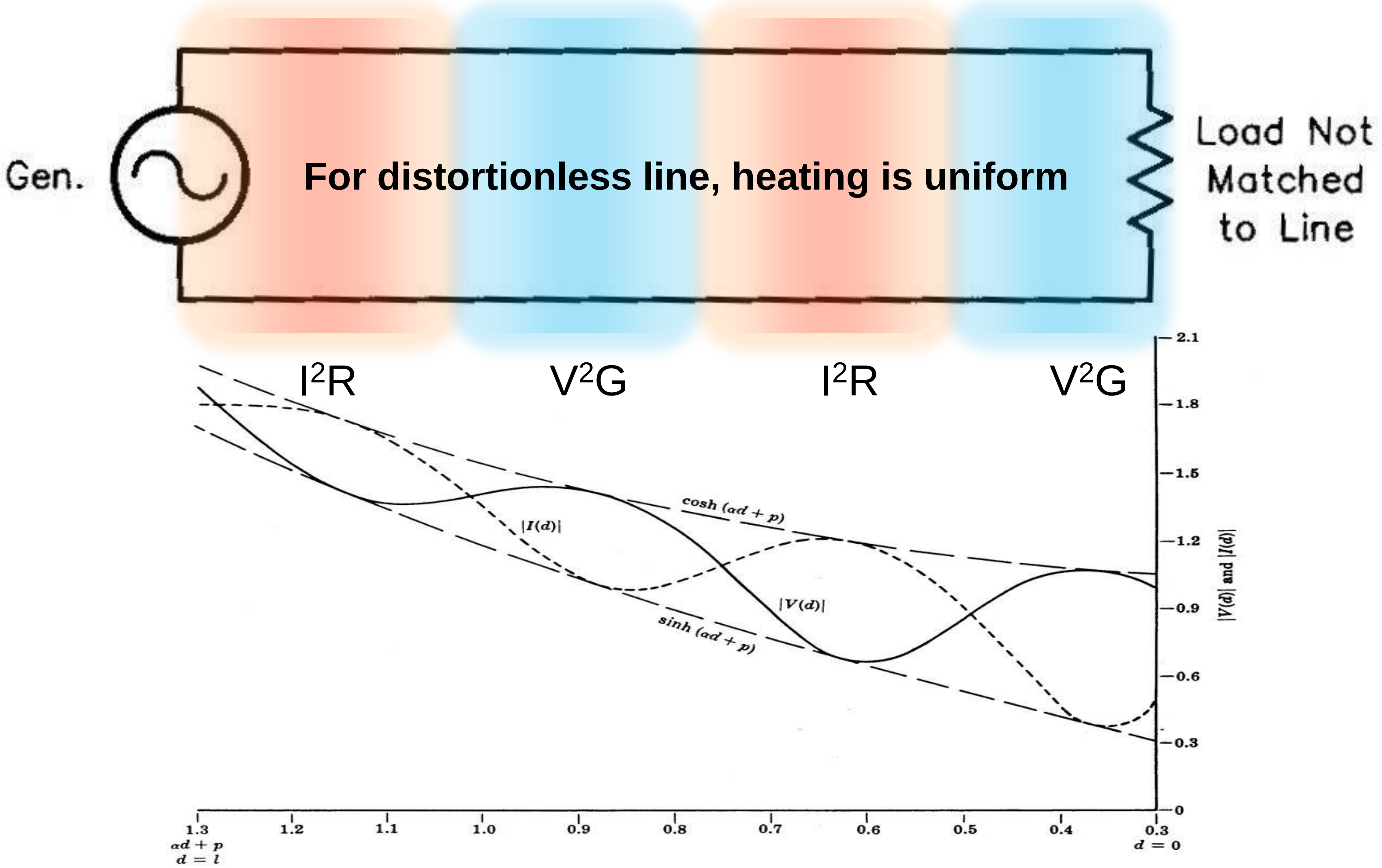
Source: *ARRL Antenna Book*, 21st ed., p. 24-20, 2007

Voltage and Current Standing Waves



Source: R.A. Chipman, *Schaum's Theory and Problems of Transmission Lines*, p. 170, McGraw Hill, 1968

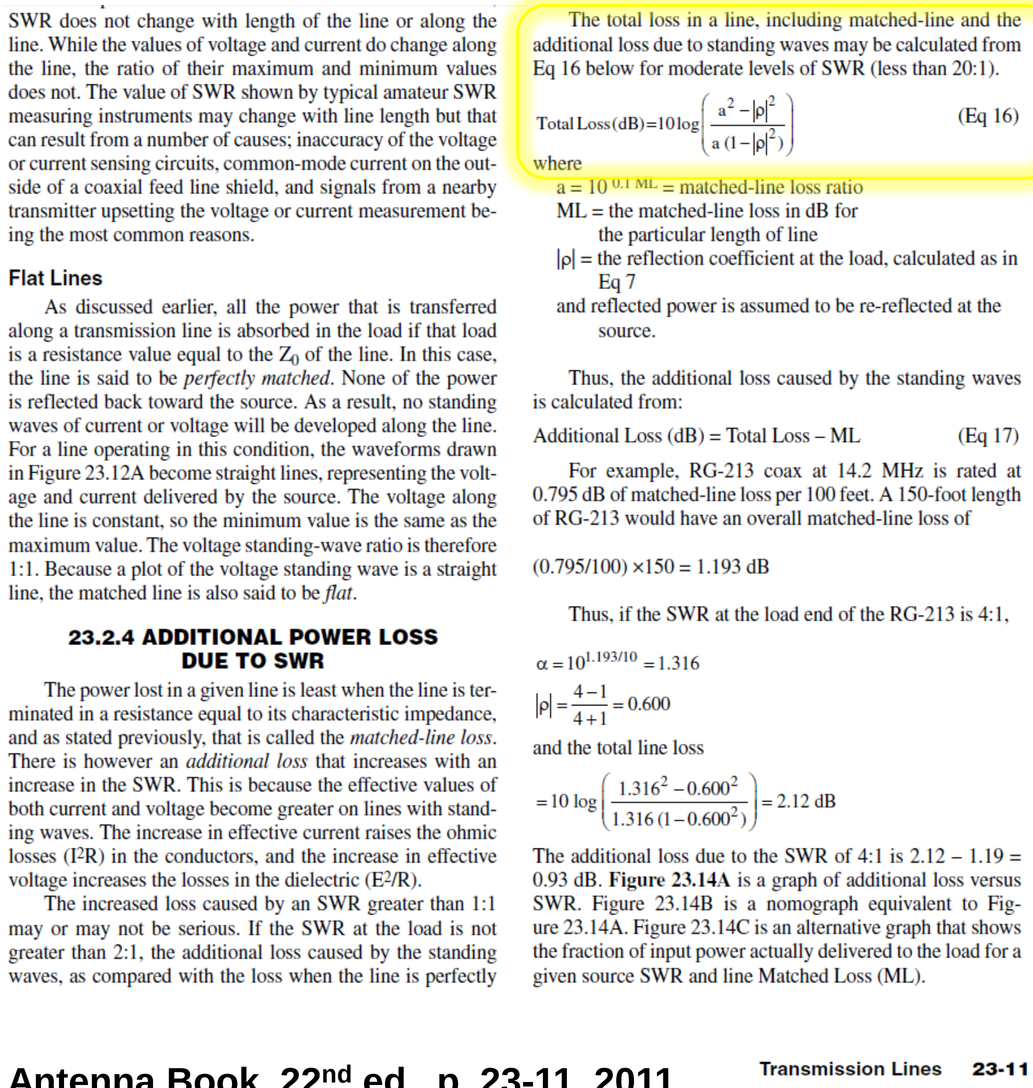
Ohmic and Dielectric Heating Alternate Along a Mismatched Line



For distortionless line, heating is uniform

Total Line Loss

Published Formula



SWR does not change with length of the line or along the line. While the values of voltage and current do change along the line, the ratio of their maximum and minimum values does not. The value of SWR shown by typical amateur SWR measuring instruments may change with line length but that can result from a number of causes; inaccuracy of the voltage or current sensing circuits, common-mode current on the outside of a coaxial feed line shield, and signals from a nearby transmitter upsetting the voltage or current measurement being the most common reasons.

Flat Lines

As discussed earlier, all the power that is transferred along a transmission line is absorbed in the load if that load is a resistance value equal to the Z_0 of the line. In this case, the line is said to be *perfectly matched*. None of the power is reflected back toward the source. As a result, no standing waves of current or voltage will be developed along the line. For a line operating in this condition, the waveforms drawn in Figure 23.12A become straight lines, representing the voltage and current delivered by the source. The voltage along the line is constant, so the minimum value is the same as the maximum value. The voltage standing-wave ratio is therefore 1:1. Because a plot of the voltage standing wave is a straight line, the matched line is also said to be *flat*.

23.2.4 ADDITIONAL POWER LOSS DUE TO SWR

The power lost in a given line is least when the line is terminated in a resistance equal to its characteristic impedance, and as stated previously, that is called the *matched-line loss*. There is however an *additional loss* that increases with an increase in the SWR. This is because the effective values of both current and voltage become greater on lines with standing waves. The increase in effective current raises the ohmic losses (I^2R) in the conductors, and the increase in effective voltage increases the losses in the dielectric (E^2/R).

The increased loss caused by an SWR greater than 1:1 may or may not be serious. If the SWR at the load is not greater than 2:1, the additional loss caused by the standing waves, as compared with the loss when the line is perfectly

The total loss in a line, including matched-line and the additional loss due to standing waves may be calculated from Eq 16 below for moderate levels of SWR (less than 20:1).

$$\text{Total Loss (dB)} = 10\log\left(\frac{a^2 - |\rho|^2}{a\,(1 - |\rho|^2)}\right) \qquad \text{(Eq 16)}$$

where

$a = 10^{0.1\,ML}$ = matched-line loss ratio

ML = the matched-line loss in dB for the particular length of line

$|\rho|$ = the reflection coefficient at the load, calculated as in Eq 7

and reflected power is assumed to be re-reflected at the source.

Thus, the additional loss caused by the standing waves is calculated from:

$$\text{Additional Loss (dB)} = \text{Total Loss} - \text{ML} \qquad \text{(Eq 17)}$$

For example, RG-213 coax at 14.2 MHz is rated at 0.795 dB of matched-line loss per 100 feet. A 150-foot length of RG-213 would have an overall matched-line loss of

$$(0.795/100) \times 150 = 1.193 \text{ dB}$$

Thus, if the SWR at the load end of the RG-213 is 4:1,

$$\alpha = 10^{1.193/10} = 1.316$$

$$|\rho| = \frac{4-1}{4+1} = 0.600$$

and the total line loss

$$= 10\log\left(\frac{1.316^2 - 0.600^2}{1.316\,(1 - 0.600^2)}\right) = 2.12 \text{ dB}$$

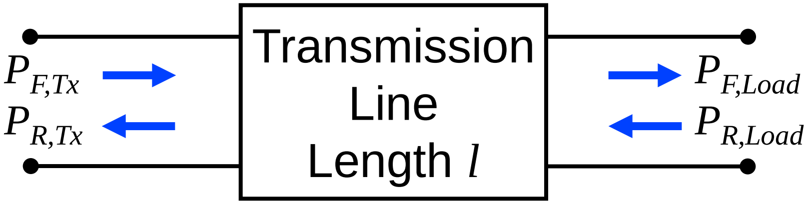
The additional loss due to the SWR of 4:1 is 2.12 – 1.19 = 0.93 dB. **Figure 23.14A** is a graph of additional loss versus SWR. Figure 23.14B is a nomograph equivalent to Figure 23.14A. Figure 23.14C is an alternative graph that shows the fraction of input power actually delivered to the load for a given source SWR and line Matched Loss (ML).

Why SWR < 20:1?

Source: ARRL Antenna Book, 22nd ed., p. 23-11, 2011

Matched Loss and Attenuation Constant

- **Power at load end in terms of power at transmitter end of line**



$$P_{F,Load} = \frac{1}{a} \cdot P_{F,Tx}$$

$$P_{R,Load} = a \cdot P_{R,Tx}$$

- a **is the matched loss or power attenuation ratio *in linear units***
  - A real number greater than one
  - Related to the line's attenuation constant α or scattering parameter s_{21}

Don't confuse
Latin a and Greek α

$$a = \begin{cases} e^{2\alpha l} & \text{for } \alpha \text{ in nepers/meter and } l \text{ in meters} \\ \quad \text{or} & \\ 10^{\alpha l/1000} & \text{for } \alpha \text{ in dB}/100 \text{ feet and } l \text{ in feet} \end{cases}$$

$$a = \frac{1}{|s_{21}|^2}$$

Derivation of Published Formula for Total Line Loss

$$\text{Total Loss (dB)} = 10\log_{10}\frac{P_{in}}{P_{out}}$$

$$= 10\log_{10}\frac{P_{F,Tx} - P_{R,Tx}}{P_{F,Load} - P_{R,Load}}$$

☜ **Key Assumption**

$$= 10\log_{10}\left(\frac{P_{F,Tx}}{P_{F,Load}}\right)\left(\frac{1-\dfrac{P_{R,Tx}}{P_{F,Tx}}}{1-\dfrac{P_{R,Load}}{P_{F,Load}}}\right)$$

$$= 10\log_{10} a\,\frac{1-|\Gamma_{in}|^2}{1-|\Gamma_{Load}|^2}$$

Matched Loss

$$= 10\log_{10} a \quad + \quad 10\log_{10}\frac{1-|\Gamma_{in}|^2}{1-|\Gamma_{Load}|^2}$$

Additional Loss

$$= \alpha l\,(\text{dB}) \quad + \quad 10\log_{10}\frac{1-|\Gamma_{in}|^2}{1-|\Gamma_{Load}|^2}$$

Total Line Loss in Terms of SWR at Load

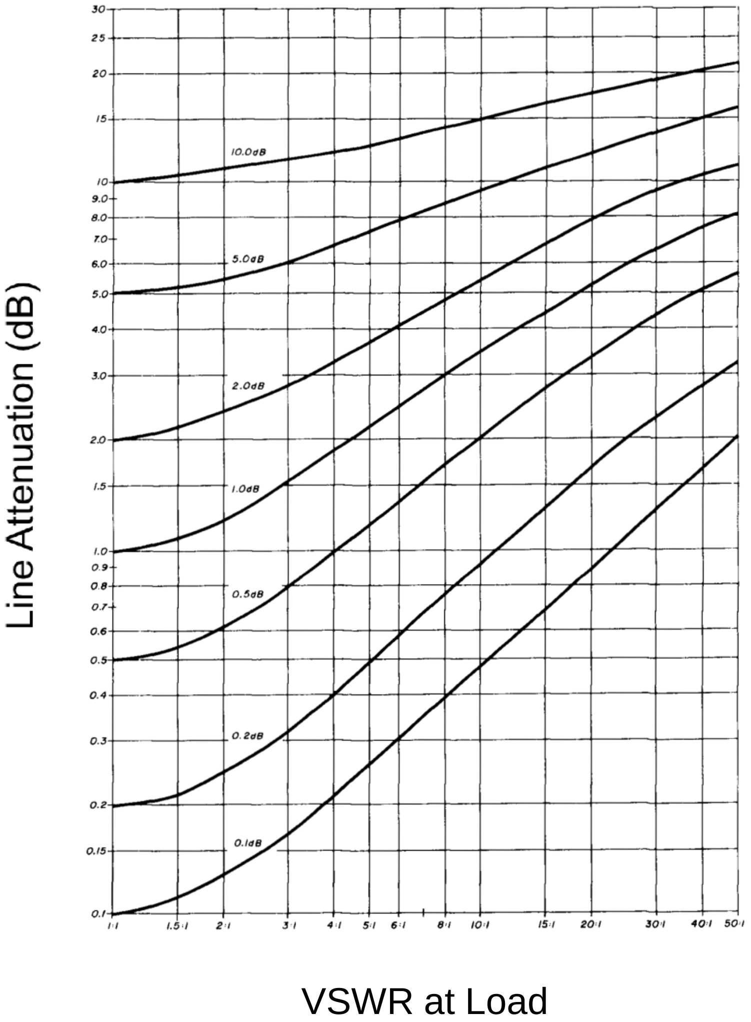
$$\text{Total Loss (dB)} = \alpha l \text{ (dB)} + 10\log_{10}\frac{(SWR_{Load}+1)^2 - \frac{1}{a^2}(SWR_{Load}-1)^2}{(SWR_{Load}+1)^2-(SWR_{Load}-1)^2}$$

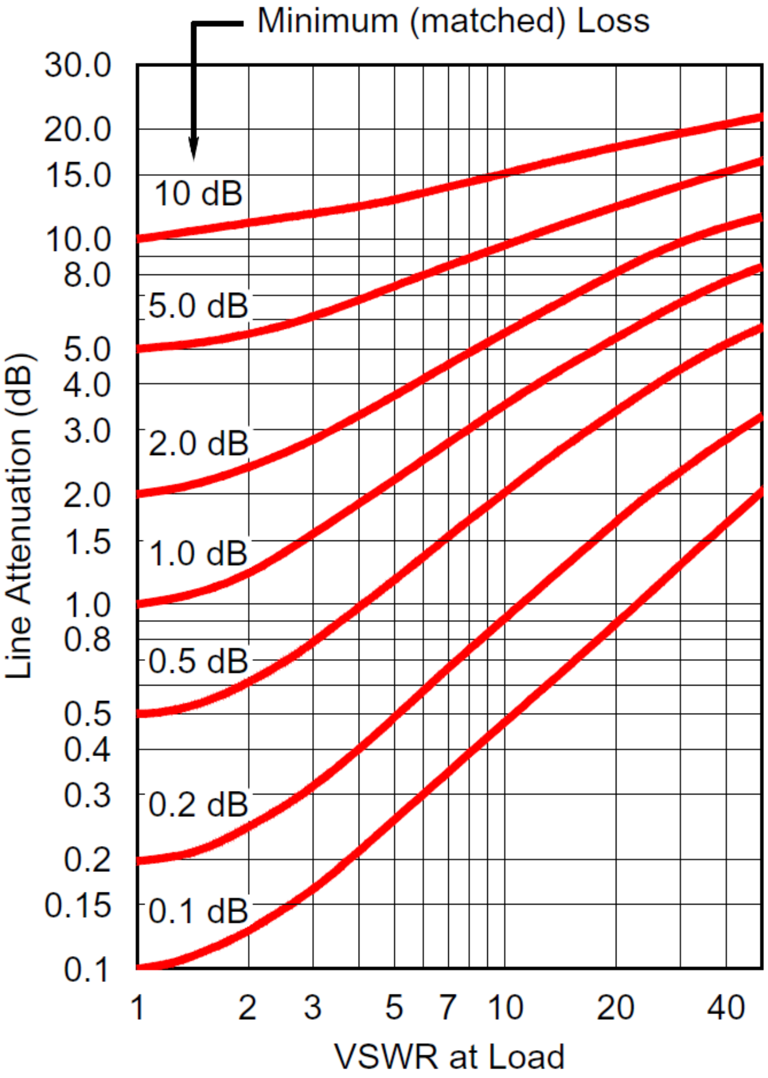
Matched Loss

Additional Loss

Published line loss formula depends on matched loss and SWR.

Transmission Line Total Loss





J. Reisert W1JR , *Ham Radio*, p. 27, Oct. 1987

ARRL Handbook, 91st ed., p. 20.5, 2014

Additional Loss Due to Mismatch

- **The additional loss can be expressed in terms of the SWR at the line's input or its output**

Input SWR

$$10\log_{10}\frac{1-|\Gamma_{in}|^2}{1-a^2|\Gamma_{in}|^2}=10\log_{10}\frac{(SWR_{Tx}+1)^2-(SWR_{Tx}-1)^2}{(SWR_{Tx}+1)^2-a^2(SWR_{Tx}-1)^2}$$

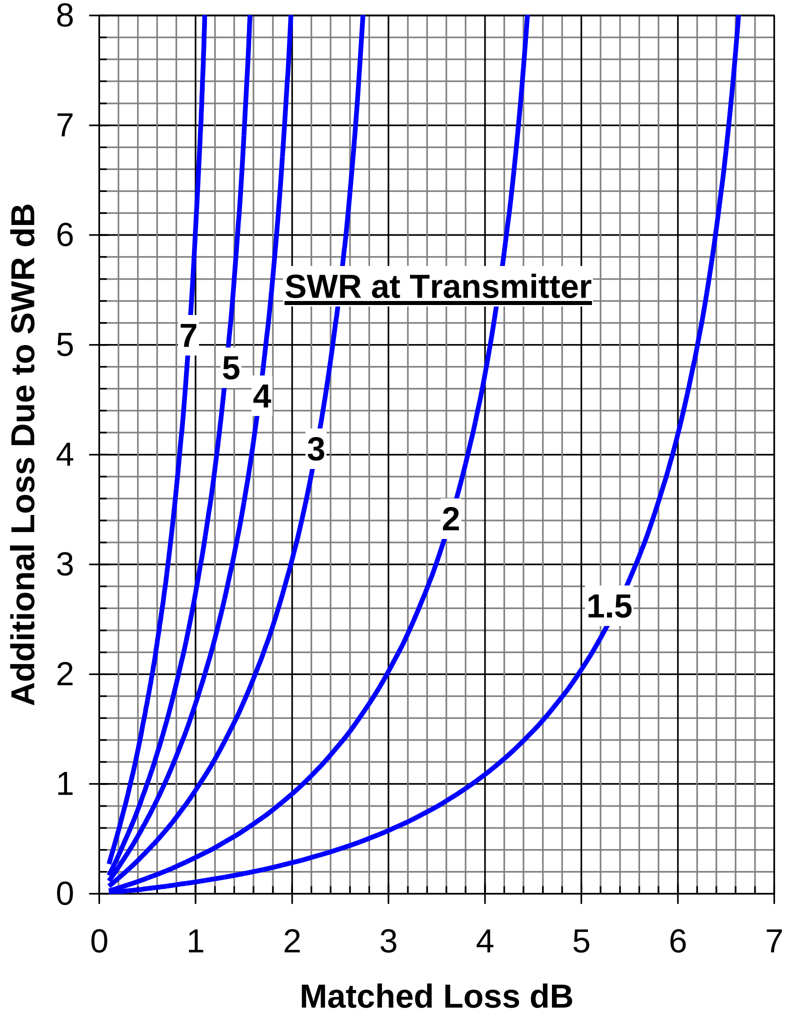
$$\text{Additional Loss (dB)} \quad =$$

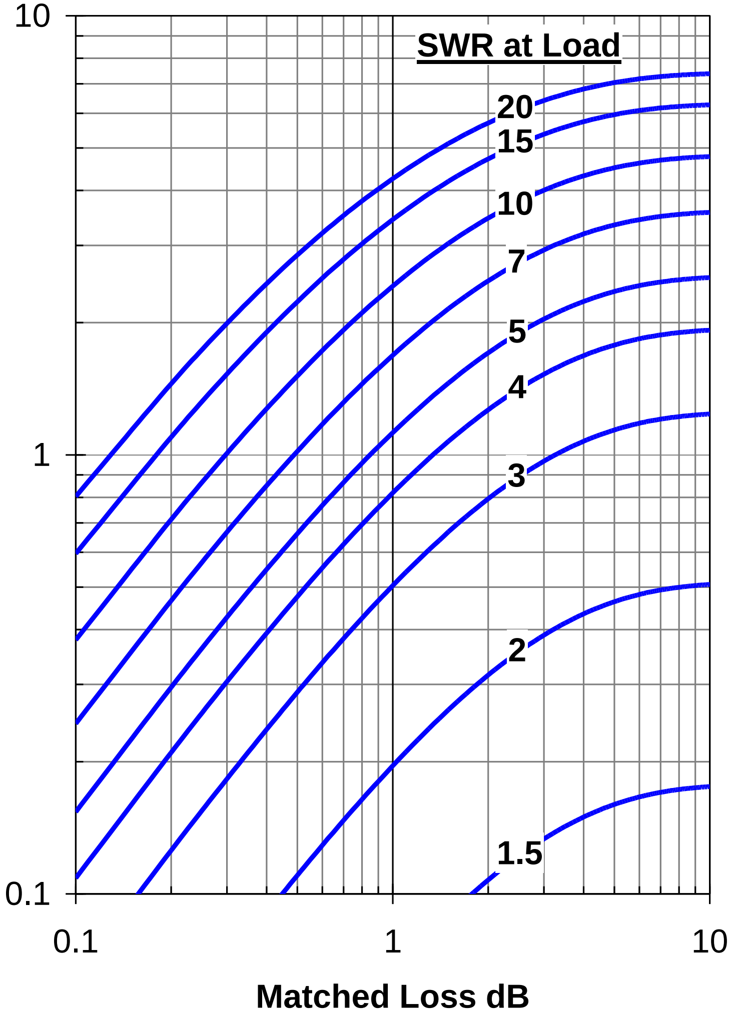
Output SWR

$$10\log_{10}\frac{1-\frac{1}{a^2}|\Gamma_{Load}|^2}{1-|\Gamma_{Load}|^2}=10\log_{10}\frac{(SWR_{Load}+1)^2-\frac{1}{a^2}(SWR_{Load}-1)^2}{(SWR_{Load}+1)^2-(SWR_{Load}-1)^2}$$

- **Next slide shows the additional loss graphed both ways**

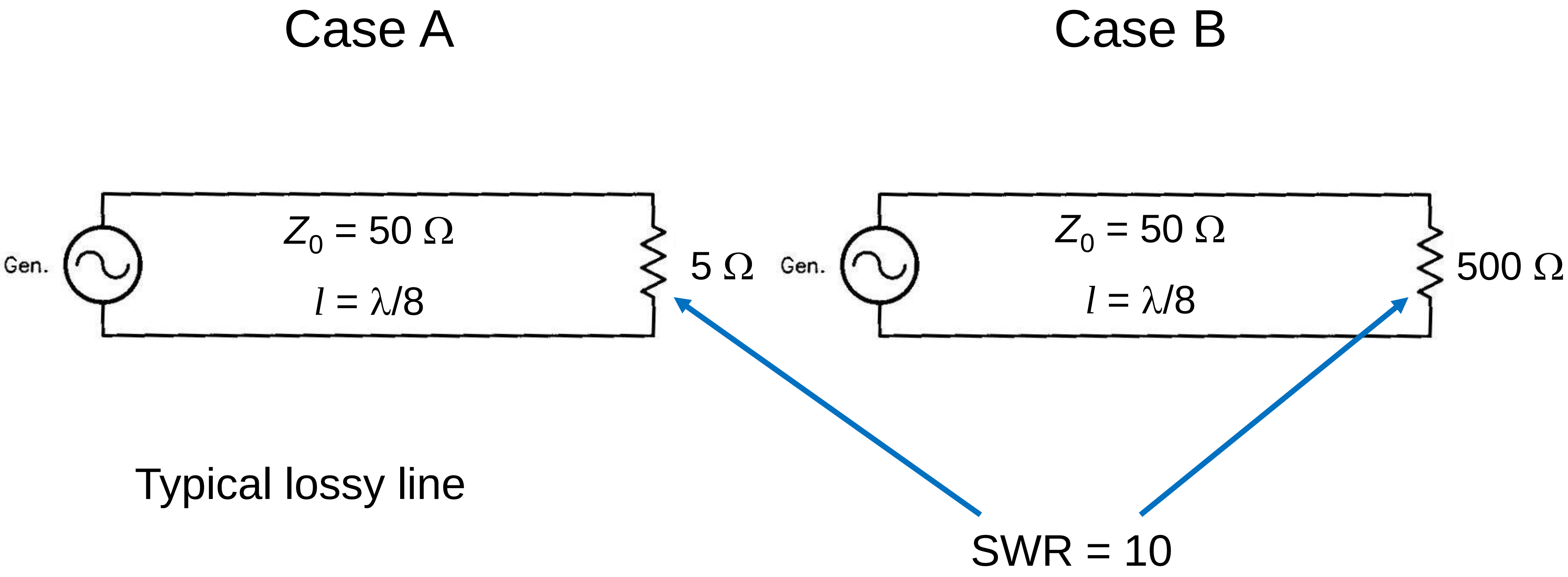
Additional Loss versus Matched Loss





The Loss Paradox

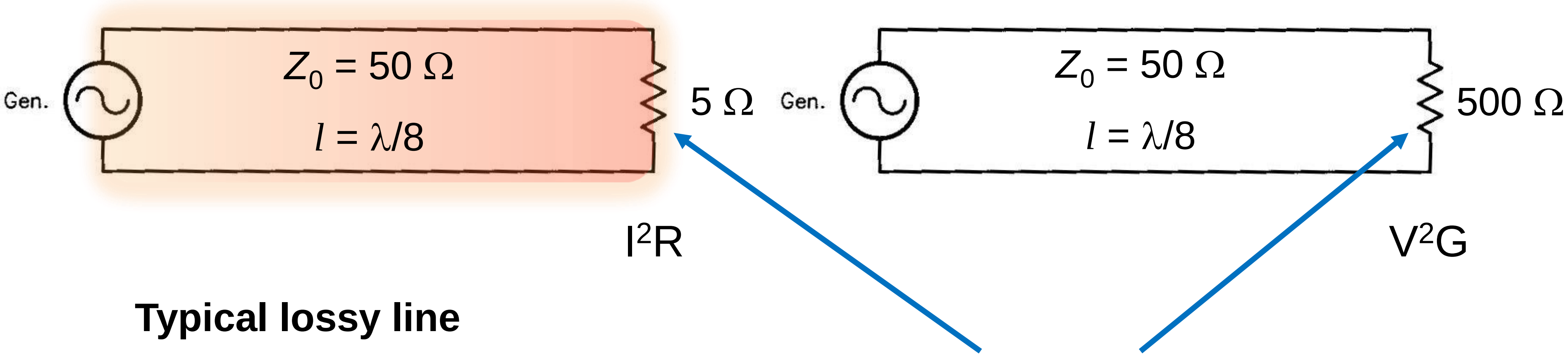
Which Case has Greater Loss?



A Clue

Case A

Case B



Z_0 = 50 Ω

l = λ/8

5 Ω

Z_0 = 50 Ω

l = λ/8

500 Ω

I^2R

V^2G

SWR = 10

Typical lossy line

Ohmic loss dominates

Dielectric loss negligible

Total Line Loss According to Published Formula

$$\text{Total Loss}\,(\text{dB}) = \alpha l\,(\text{dB}) \; + \; 10\log_{10}\frac{11^2 - \dfrac{1}{a^2}9^2}{11^2 - 9^2}$$

- Does not depend on R_L except through SWR !
- For a given SWR, does not depend on whether $R_L > Z_0$ or $R_L < Z_0$!
- For a given SWR, does not depend on where Z_L lies on Smith Chart's SWR circle !

Published line loss formula depends on matched loss and SWR.

The Paradox

- **Assume a lossy 50-Ω line in which copper loss dominates dielectric loss**
- **Case A**
  - A short lossy 50-Ω line terminated with a 5-Ω resistor
  - Line current is high at the load
- **Case B**
  - Same line terminated with a 500-Ω resistor
  - Line current is low at the load
- **SWR at load is 10:1 in both cases**
- **By published formula, line loss is the same in both cases**
- **By physical reasoning, line loss is greater in Case A than Case B**

A contradiction !

The Resolution

The Problem

- **The total line loss formula depends for its proof on the condition**
  - Delivered power is forward power minus reverse power

$$P_{Delivered} = P_{Forward} - P_{Reverse}$$

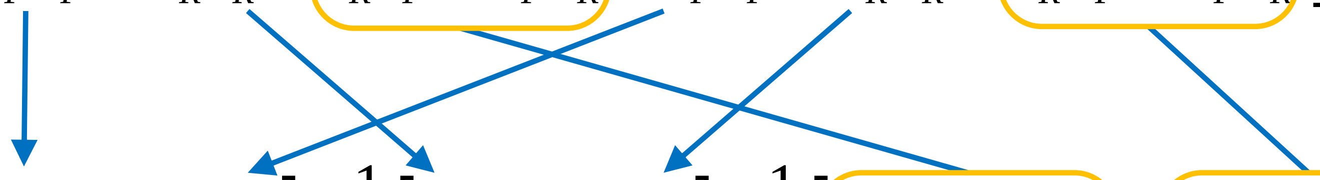
- **While true for distortionless line, this statement is not true for general transmission lines**
- **Certain kinds of power meters make a similar assumption**

Delivered Power Along a General Transmission Line

$$P_{Delivered} = \frac{1}{2}\mathrm{Re}\{V\,I^*\} = \frac{1}{4}\left[V\,I^* + V^* I\right]$$

$$= \frac{1}{4}\left[(V_F + V_R)(I_F^* - I_R^*) + (V_F^* + V_R^*)(I_F - I_R)\right]$$

$$= \frac{1}{4}\left[V_F I_F^* - V_R I_R^* + \boxed{V_R I_F^* - V_F I_R^*} + V_F^* I_F - V_R^* I_R + \boxed{V_R^* I_F - V_F^* I_R}\,\right]$$



$$= \frac{1}{4}\left[V_F I_F^* + V_F^* I_F\,\right] - \frac{1}{4}\left[V_R I_R^* + V_R^* I_R\right] + \frac{1}{4}\left[\boxed{V_R I_F^* - V_F I_R^*} + \boxed{V_R^* I_F - V_F^* I_R}\right]$$

$$= P_{Forward} - P_{Reverse} + \frac{1}{4}\underbrace{\left[V_R I_F^* - V_F I_R^* + V_R^* I_F - V_F^* I_R\right]}_{\textbf{Cross terms !}}$$

Cross Power Terms

$$\text{Cross Power Terms} = \frac{1}{4}\left[V_R I_F^* - V_F I_R^* + V_R^* I_F - V_F^* I_R\right]$$

$$= \frac{1}{4}\left[\frac{V_F^* V_R}{Z_0^*} - \frac{V_F V_R^*}{Z_0^*} + \frac{V_F V_R^*}{Z_0} - \frac{V_F^* V_R}{Z_0}\right]$$

$$= \frac{1}{4}\left(V_F V_R^* - V_F^* V_R\right)\left(\frac{1}{Z_0} - \frac{1}{Z_0^*}\right)$$

$$= 0 \quad \text{if } V_F V_R^* \text{ is real or } Z_0 \text{ is real}$$

- **Cross terms vanish if either of two conditions is met**
  - Real Z_0, e.g. distortionless line
  - Real reflection coefficient, i.e. real V_R / V_F or real Z_L / Z_0

Formulas for Total Loss of Terminated General Transmission Lines

- **Loss expressed in terms of the load impedance**

$$\text{Total Loss (dB)} = 20\ \log_{10}\left|\cosh\gamma L + \frac{Z_L}{Z_0}\sinh\gamma L\right| \quad + \quad 10\log_{10}\frac{R_{IN}}{R_L}$$

- **Loss expressed in terms of line input impedance**

$$\text{Total Loss (dB)} = -20\ \log_{10}\left|\cosh\gamma L - \frac{Z_{IN}}{Z_0}\sinh\gamma L\right| \quad + \quad 10\log_{10}\frac{R_{IN}}{R_L}$$

- **where Z_0 is complex characteristic impedance taken as**

$$\begin{aligned} Z_0 &= R_0 + jX_0 \\ &\approx R_0 - j\frac{\alpha}{\beta}R_0 \end{aligned}$$

Other useful forms are obtained from equations on Slides 20 and 21

Test Case for Numerical Validation of Loss Formulas

- **Input parameters (N6BV example)**
  - Line type: Wireman #551, 450-Ω window line
  - Length: 100 feet
  - Frequency: 1.83 MHz
  - Alpha: 0.095 dB/100 feet
  - Velocity factor: 0.915
  - R_0 = real part of Z_0 : 402.75 Ω
  - Load resistance R_L : 4.5 Ω
  - Load reactance X_L : −1,673 Ω

$Z = 4.5 - j\,1{,}673\ \Omega$ – What Antenna Is This?

- **Electrically small dipole – three step calculation**

$$R_{Rad} = 5(\beta L)^2 = 20\left(\frac{\pi f L}{c}\right)^2$$

$$\therefore \quad L = \frac{c}{2\pi f}\sqrt{\frac{R_{Rad}}{5}} = 24.735\,\text{m} = 81.15\,\text{ft}$$

$L \approx \lambda\,/6.6$

$L\,/d \approx 30$

Cage dipole?

$$X_{Ant} = -120\cot\left(\frac{\beta L}{2}\right)\ln\left(\frac{L}{d}-1\right) = -120\cot\left(\frac{\pi f L}{2c}\right)\ln\left(\frac{L}{d}-1\right)$$

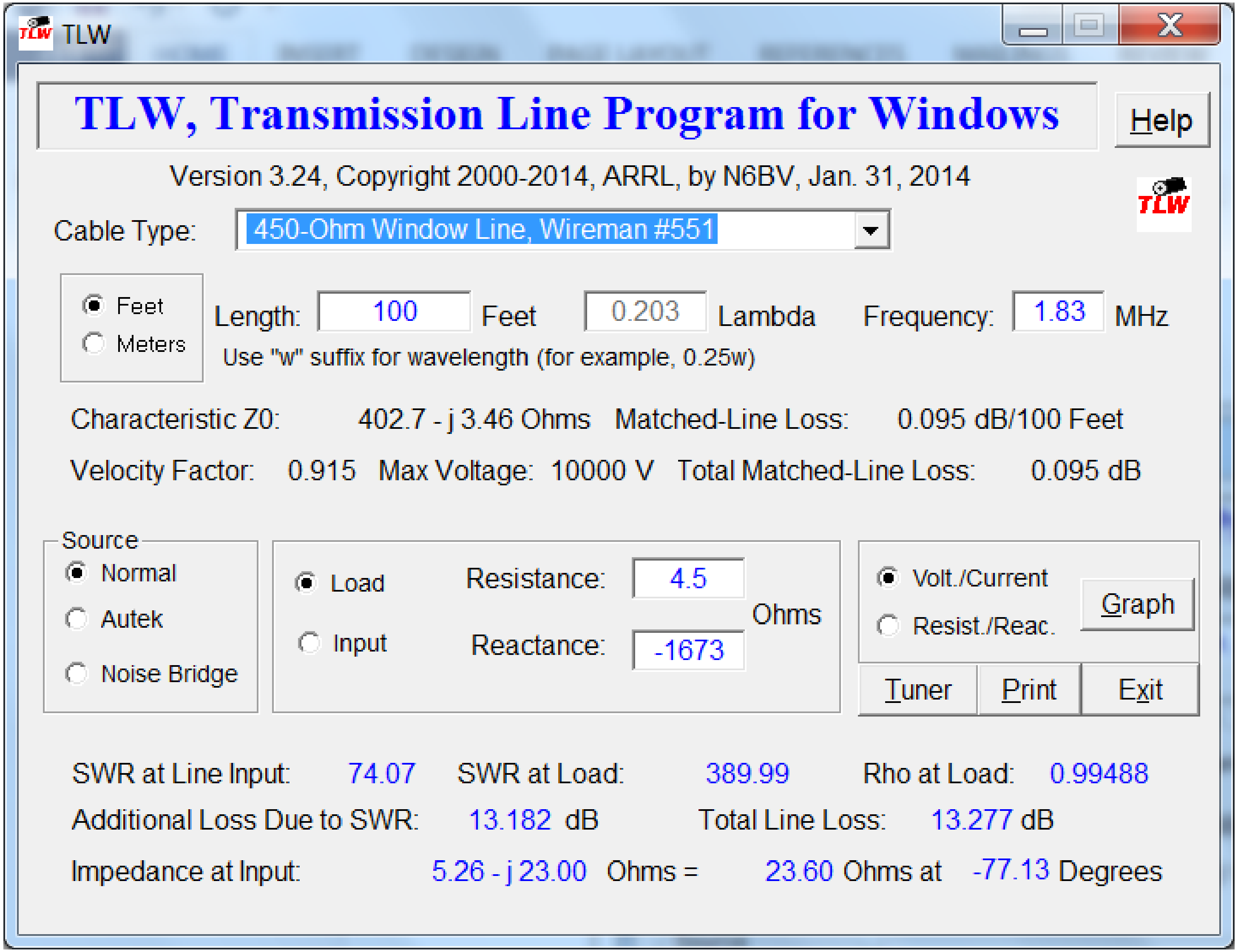
$$\therefore \quad d = \frac{L}{1+e^{\frac{-X_{Ant}}{120\cot\left(\frac{\pi f L}{2c}\right)}}} = \frac{24.735}{1+29.078} = 0.8224\,\text{m} = 32.38\,\text{in}$$

Efficiency if a single wire/tube, not a cage

$$R_{Loss} = \frac{2}{3}\sqrt{\frac{\pi \mu_0 f}{\sigma}}\,\frac{1}{\pi d} = \frac{4\times 10^{-7}}{3\sqrt{5.8}}\,\frac{\sqrt{f}}{d} = 91\,\mu\Omega$$

$$\therefore \quad \text{Efficiency (\%)} = \frac{100 R_{Rad}}{R_{Rad} + R_{Loss}} = \frac{4.5}{4.500091} = 99.998\,\%$$

Calculated Values from TLW v3.24



Version 3.24, Copyright 2000-2014, ARRL, by N6BV, Jan. 31, 2014

Cable Type: 450-Ohm Window Line, Wireman #551

● Feet
○ Meters

Length: 100 Feet 0.203 Lambda Frequency: 1.83 MHz

Use "w" suffix for wavelength (for example, 0.25w)

Characteristic Z0: 402.7 - j 3.46 Ohms Matched-Line Loss: 0.095 dB/100 Feet

Velocity Factor: 0.915 Max Voltage: 10000 V Total Matched-Line Loss: 0.095 dB

Source
● Normal
○ Autek
○ Noise Bridge

● Load
○ Input

Resistance: 4.5 Ohms
Reactance: -1673

● Volt./Current
○ Resist./Reac.

SWR at Line Input: 74.07 SWR at Load: 389.99 Rho at Load: 0.99488

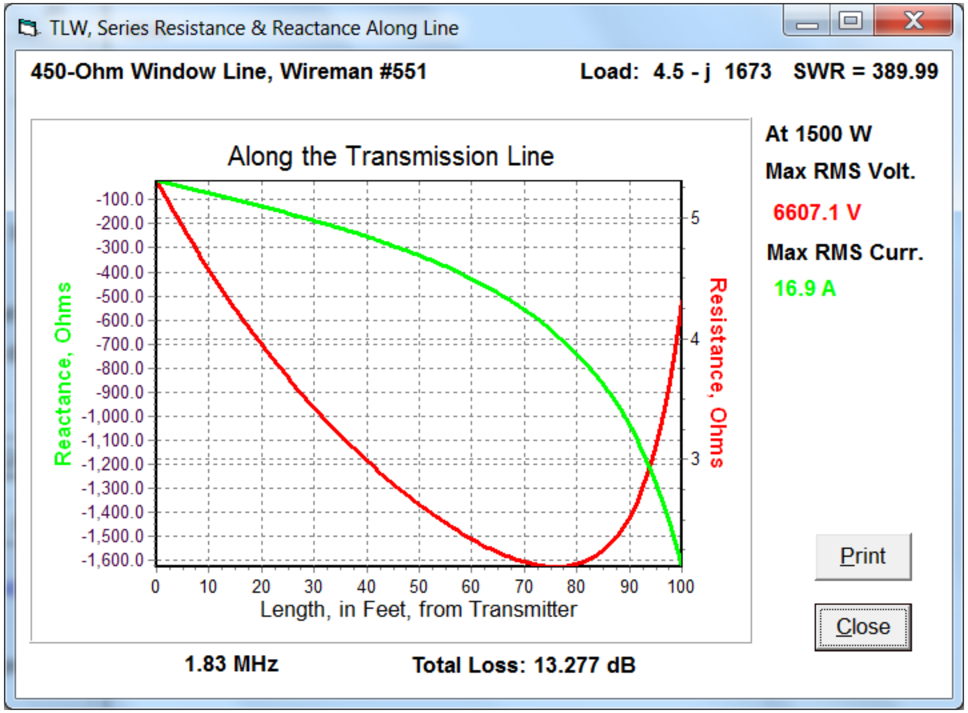
Additional Loss Due to SWR: 13.182 dB Total Line Loss: 13.277 dB

Impedance at Input: 5.26 - j 23.00 Ohms = 23.60 Ohms at -77.13 Degrees

Validation Results

	Published Formula	TLW	New Formula 1	New Formula 2
R_0 = (given)	402.75	402.7	402.75	402.75
X_0 =	-3.446913256	-3.46	-3.446913256	-3.446913256
R_L = (given)	4.5	4.5	4.5	4.5
X_L = (given)	1773	1773	1773	1773
$\|\Gamma\|$ at load	0.994894069	0.99488	0.994894069	0.994894069
SWR at load =	390.7013472	389.99	390.7013472	390.7013472
R_{in} =	5.255606806	5.26	5.255606806	5.255606806
X_{in} =	-22.99763828	-23.00	-22.99763828	-22.99763828
$\|Z_{in}\|$ =	23.59052287	23.60	23.59052287	23.59052287
Angle(Z_{in}) =	-77.12735721	-77.13	-77.12735721	-77.12735721
SWR at input =	74.11147916	74.07	74.11147916	74.11147916
Total Loss (dB) =	7.220380739	13.277	13.27435696	13.27435696
Additional Loss Due to Mismatch (dB) =	7.125380739	13.182	13.17935696	13.17935696

Resistance and Reactance Along the Transmission Line



450-Ohm Window Line, Wireman #551

Load: 4.5 - j 1673 SWR = 389.99

At 1500 W

Max RMS Volt.

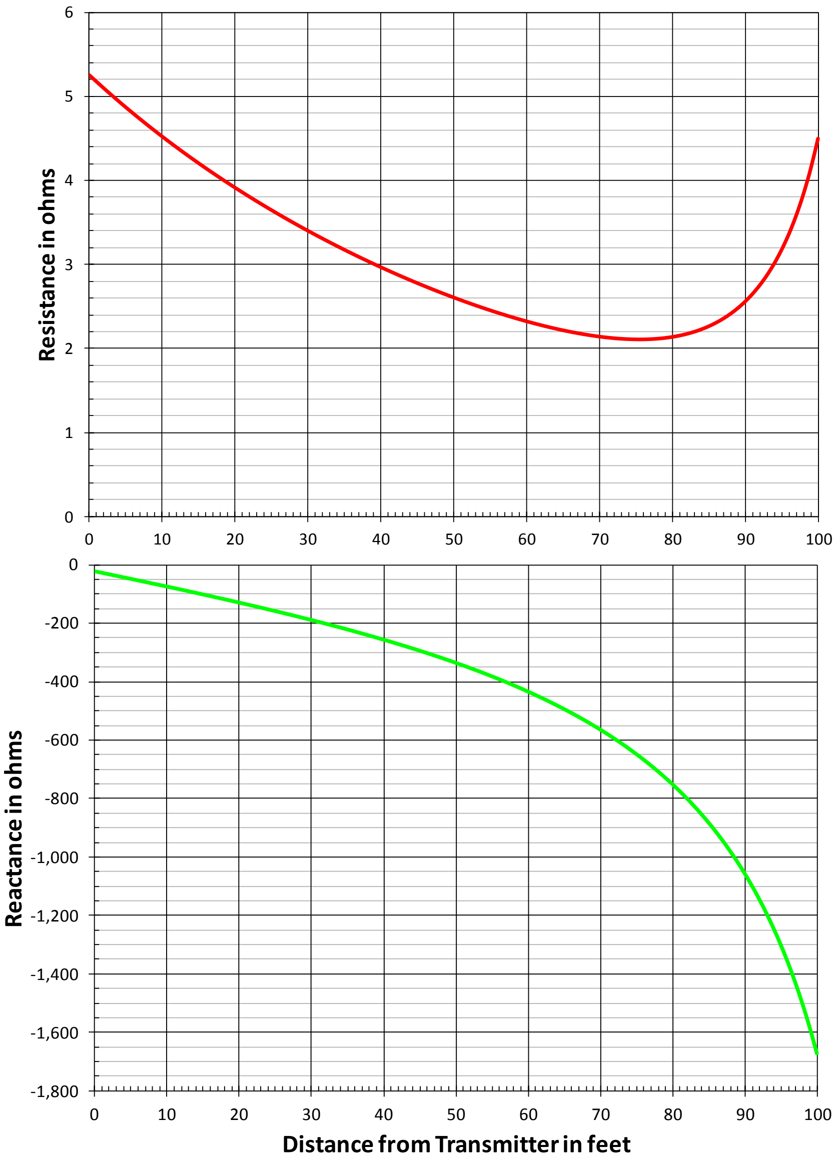
6607.1 V

Max RMS Curr.

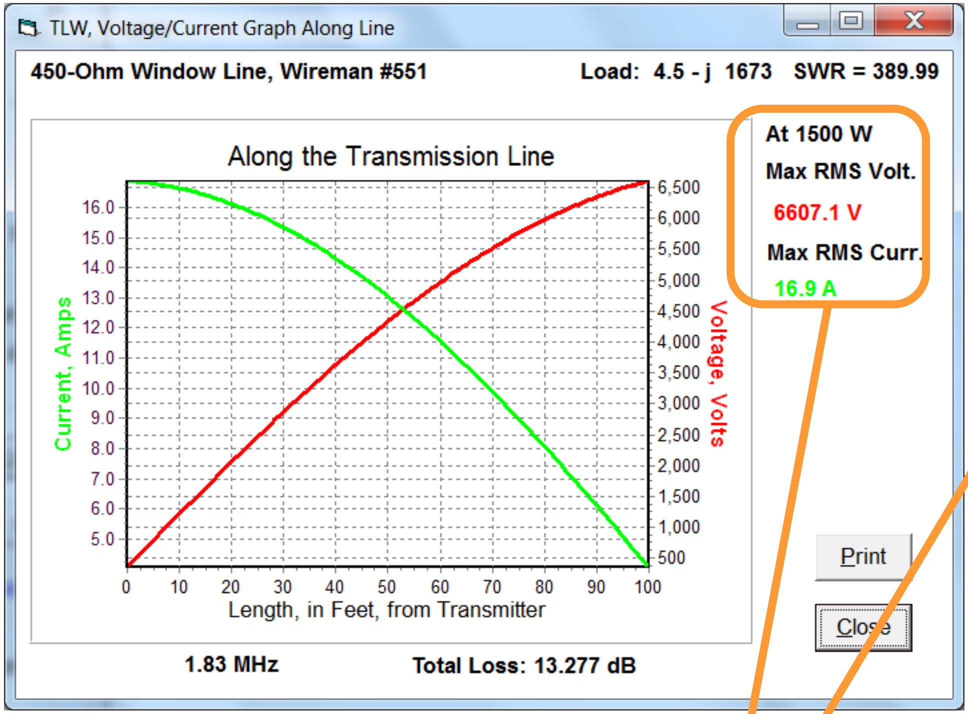
16.9 A

1.83 MHz

Total Loss: 13.277 dB



Voltage and Current Along Transmission Line



450-Ohm Window Line, Wireman #551

Load: 4.5 - j 1673 SWR = 389.99

At 1500 W

Max RMS Volt.

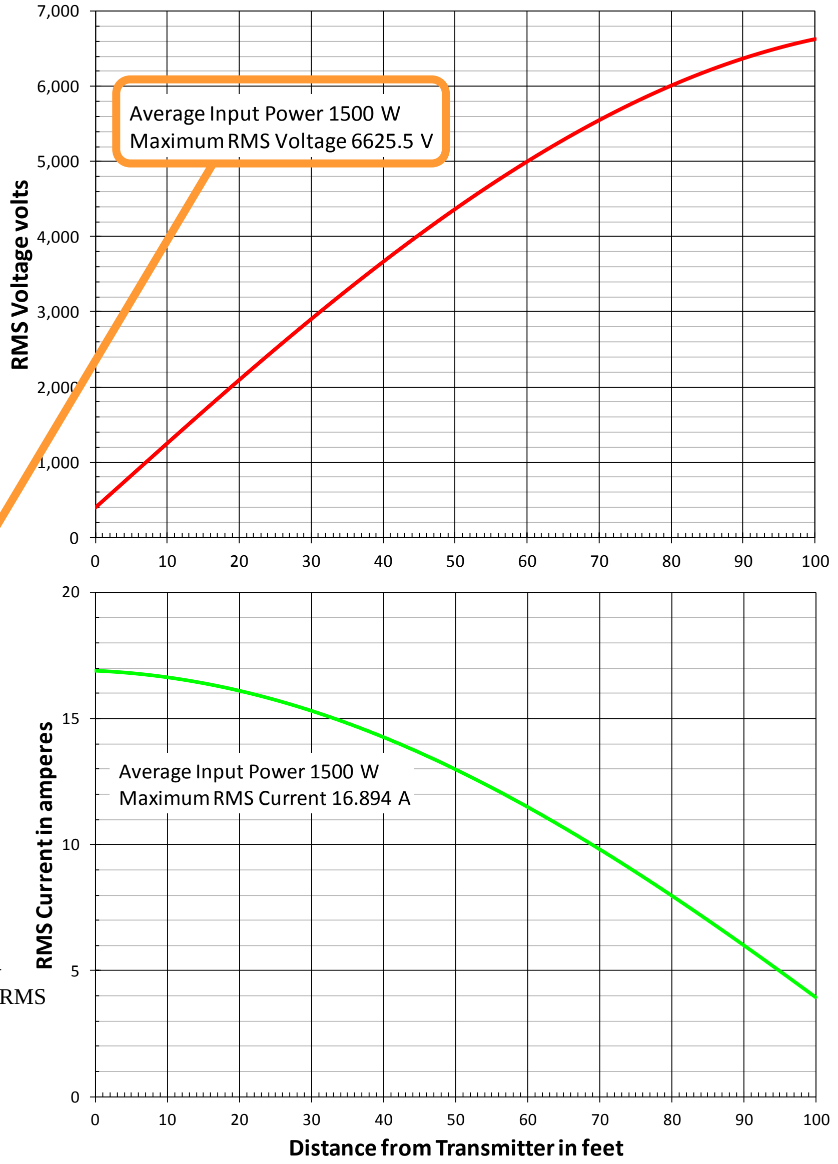
6607.1 V

Max RMS Curr.

16.9 A

1.83 MHz

Total Loss: 13.277 dB



Average Input Power 1500 W
Maximum RMS Voltage 6625.5 V

Average Input Power 1500 W
Maximum RMS Current 16.894 A

TLW load voltage is wrong

$$V_{Load} = \sqrt{\frac{P_{Load}}{G_{Load}}} = \sqrt{\frac{1500 \times 10^{-1.3274357}}{\frac{4.5}{4.5^2 + 1673^2}}} = \sqrt{\frac{70.5758\text{ W}}{1.60776\text{ mS}}} = 6625.5\text{ V}_{\text{RMS}}$$

Summary of Test Example

- **The published formula underestimates line loss by 6.05 dB**
- **Assuming 1500 watts input to line**
- **Published formula predicts incorrectly**
  - Total line loss = 7.22 dB
  - Power delivered to load = 284.5 W
  - Power dissipated in line = 1215.5 W
- **New formulas and TLW predict correctly**
  - Total line loss = 13.27 dB
  - Power delivered to load = 70.6 W
  - Power dissipated in line = 1429.4 W

Ignoring the seemingly small imaginary part of Z_0 of practical transmission lines can give large error in predicted delivered power.

Summary and Conclusion

- **Showed the published formula for total line loss has a fundamental problem – stemming from a hidden assumption**

$$P_{Delivered} = P_{Forward} - P_{Reverse}$$

- **Showed the published formula and graphs for mismatched line loss are correct only in two special cases**
    - Real Z_0, e.g. distortionless line
    - Real reflection coefficient, i.e. real V_R / V_F or real Z_L / Z_0
- **“SWR < 20” is not the correct condition to assure accuracy**
- **Gave improved formulas for the total loss of general lines**
- **Showed the new formulas agree with *Transmission Lines for Windows* (TLW)**

Ignoring the seemingly small imaginary part of Z_0 of practical transmission lines can give large error in predicted delivered power.

References – Papers and Books

- Papers
  - T.M. Papazoglou, “Review of the Maximum Efficiency of Transmission Lines,” *IEEE Trans. Education*, Nov. 1992
  - ★★★ ☞ R.J. Vernon and S.R. Seshadri, “Reflection Coefficient and Reflected Power on a Lossy Transmission Line,” *Proc. IEEE*, Jan. 1969
  - ★★★ ☞ G.E. Stannard, Calculation of Power on a Transmission Line, *Proc. IEEE*, Jan. 1967
  - K. Kurokawa, “Power Waves and the Scattering Matrix,” *IEEE Trans. Microwave Theory and Techniques*, Mar. 1965
  - W.W. Macalpine, “Computation of Impedance and Efficiency of Transmission Line with High Standing-Wave Ratio,” *Trans. AIEE*, July 1953
  - S.G. Lutz, “Transmission Line Load Impedance for Maximum Efficiency,” *Trans. AIEE*, July 1951
  - W.W. Macalpine, “Heating of Radio-Frequency Cables,” *Trans. AIEE*, July 1949
- Books
  - ☞ M.E. Van Valkenburg and W.M. Middleton, eds., *Reference Data for Engineers*, 9th ed., Newnes/Butterworth-Heineman, 2002
  - ★★★ ☞ R.A. Chipman, *Theory and Problems of Transmission Lines*, McGraw-Hill, 1968
  - R.W.P. King, *Transmission Line Theory*, McGraw-Hill, 1955. Republished by Dover 1965
  - ★★★ ☞ H.H. Skilling, *Electric Transmission Lines*, McGraw-Hill, 1951
  - W.C. Johnson, *Transmission Lines and Networks*, McGraw-Hill, 1950

References – Books continued

- J.J. Karakash, *Transmission Lines and Filter Networks*, Macmillan, 1950
- ★★★ ☞ E.W. Kimbark, *Electrical Transmission of Power and Signals*, Wiley, 1949
- J.D. Ryder, *Networks, Lines, and Fields*, Prentice-Hall, 1949
- S. Ramo and J.R. Whinnery, *Fields and Waves in Modern Radio*, Wiley, 1949
- ☞ W. Jackson, *High Frequency Transmission Lines*, Methuen, 1945.
- R.I. Sarbacher and W.A. Edson, *Hyper and Ultrahigh Frequency Engineering*, Wiley, 1943, Chapter 9.
- F.E. Terman, *Radio Engineering*, McGraw-Hill, 1937
- E.A. Guillemin, *Communication Networks, Vol. II: The Classical Theory of Long Lines, Filters and Related Networks*, Wiley, 1935
- W.L. Everitt, *Communication Engineering*, McGraw-Hill, 1932
- A.E. Kennelly, *The Application of Hyperbolic Functions to Electrical Engineering Problems*, McGraw-Hill and Univ. London Press, 1916; *Tables of Complex Hyperbolic and Circular Functions*, Harvard Univ. Press, 1914; *Chart Atlas of Complex Hyperbolic and Circular Functions*, Harvard Univ. Press, 1914
- C.P. Steinmetz, *Elementary Lectures on Electric Discharges, Waves and Impulses, and Other Transients*, McGraw-Hill, 1914
- H.B. Dwight, *Transmission Line Formulas for Electrical Engineers and Engineering Students*, Constable, 1913
- J.A. Fleming, *The Propagation of Electrical Currents in Telephone and Telegraph Conductors*, Constable, 1911
- O. Heaviside, *Electromagnetic Theory*, London, 1893-1912
- O. Heaviside, *Electrical Papers*, London, 1872-1892

The End

This presentation will be archived at

http://www.fars.k6ya.org/docs/k6oik